


# Optimizing maritime travel time reliability

Proc IMechE Part M:  
*J Engineering for the Maritime Environment*  
227(2) 167–176  
© IMechE 2013  
Reprints and permissions:  
sagepub.co.uk/journalsPermissions.nav  
DOI: 10.1177/1475090212460626  
pim.sagepub.com  


Rajesh S Prabhu Gaonkar<sup>1</sup>, Min Xie<sup>2,3</sup> and Hong-Zhong Huang<sup>4</sup>

## Abstract

Travel time reliability optimization problems generally cannot be formulated in direct mathematical form due to complexity of precise information. Classically, qualitative form of objective function and/or constraints does not allow the decision maker to represent it in a typical form of standard optimization model. This article attempts to resolve travel time reliability optimization problem of maritime transportation. The marine vessel's travel time consists of seven time components. The travel time reliability is considered in a possibilistic manner, and then the reliability optimization problem with budgetary constraints and stage-time limitations is formulated. Next, algorithmic framework for solution of the possibilistic programming has been proposed, followed by a suitable illustration. Finally, this article investigates the parameter sensitivity that has consequences in the marine vessel's transportation decision making. The proposed programming model is useful in the decision support system with regard to decision about the individual marine vessel's stage durations and their scheduling, as well as the optimal arrival time of the vessel at the destination harbor.

## Keywords

Travel time reliability, maritime transportation system, possibilistic programming, possibility measures

Date received: 11 May 2012; accepted: 30 July 2012

## Introduction

Day by day, there is continually increasing traffic at the sea and at the oceans, and thus, there is a wide scope for research investigations into different dimensions of reliability of maritime transportation system. The maritime sector system complexity and the data availability problems pose significant challenge in reliability assessment and its optimization. Conventionally, the randomness in variables in reliability evaluation has been modeled using probability concepts and stochastic programming techniques. However, a diverse approach dealing with ambiguity and vagueness-related imprecision in data was evolved in the form of the fuzzy sets theory, which later on progressed to fuzzy linear programming and possibilistic programming.<sup>1–4</sup>

Optimization of reliability in possibilistic or fuzzy domain is of researcher's interest too. Utkin et al.<sup>5</sup> presented a method to solve the fuzzy reliability optimization problem that can be used for systems described by possibility measures. Huang<sup>6</sup> proposed and illustrated fuzzy multiobjective reliability optimization decision making with two or more objectives. In the study by Ravi et al.,<sup>7</sup> the problem of optimizing the reliability of complex systems has been modeled as a fuzzy multiobjective optimization problem where system reliability,

system cost, weight, and volume are all considered as fuzzy goals. Furthermore, Huang et al.<sup>8</sup> presented a coordination method for fuzzy multiobjective optimization of system reliability. Mahapatra and Roy<sup>9</sup> introduced a new fuzzy multiobjective method for the system reliability optimization problem where reliability enhancement is involved with several mutually conflicting objectives such as to reduce the cost of the system and improve the reliability of the same system simultaneously. Verma et al.<sup>10</sup> carried out possibilistic optimization incorporating fuzzy dynamic reliability.

<sup>1</sup>Mechanical Engineering Department, Goa College of Engineering, Ponda, Goa, India

<sup>2</sup>Department of Industrial and Systems Engineering, National University of Singapore, Singapore

<sup>3</sup>Department of Systems Engineering and Engineering Management, City University of Hong Kong, Kowloon, Hong Kong

<sup>4</sup>School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, Sichuan, People's Republic of China

### Corresponding author:

Rajesh S Prabhu Gaonkar, Mechanical Engineering Department, Goa College of Engineering, Farmagudi, Ponda, Goa 403401, India.  
Email: rpg@gec.ac.in

Different facets of transportation reliability with applications mostly to road, rail, and air have been in research focus in the last two decades.<sup>11–24</sup> In a guest editorial, Sumalee and Kurauchi<sup>25</sup> discussed the reliability and emergency issues in transportation network analysis. Innamaa<sup>26</sup> investigated the predictability aspects of travel time using neural networks on an inter-urban highway. Van Lint<sup>27</sup> suggested online learning solutions for freeway travel time prediction, whereas recently, Yang et al.<sup>28</sup> proposed an approach for forecasting the reliability of travel time. Margulici and Ban<sup>29</sup> proposed benchmarking of travel time estimates. Zheng and McDonald<sup>30</sup> estimated travel time using fuzzy clustering method. There are few more diverse articles describing travel time domain.<sup>31,32</sup> Rakha et al.<sup>33</sup> investigated trip travel time reliability issues and proposed some solutions to the problem. Prabhu Gaonkar et al.<sup>34</sup> presented a methodology of maritime transportation system reliability evaluation by the application of fuzzy sets and fuzzy logic techniques.

Zhen and Chang<sup>35</sup> studied the methodology to develop a robust schedule for berth allocation that incorporates a degree of anticipation of uncertainty (e.g. vessels' arrival time and operation time) during the schedule's execution. Xu et al.<sup>36</sup> studied a robust berth scheduling problem with uncertain vessel delay and handling time. Arango et al.<sup>37</sup> addressed the berth allocation planning problems using simulation and optimization to improve the berth management strategy. Zhen et al.<sup>38</sup> developed a decision model for berth allocation under uncertainty. Golias et al.<sup>39</sup> dealt with the berth allocation problem with focus on optimizing the vessel's arrival time. Lee and Chen<sup>40</sup> developed an optimization heuristic for the berth scheduling problem, whereas Fagerholt<sup>41</sup> proposed an optimization-based approach for ship scheduling problem. Wang and Meng<sup>42</sup> developed ship route schedule design with sea contingency time and port time uncertainty. Shao et al.<sup>43</sup> developed forward dynamic programming method for weather routing during a voyage. Kosmas and Vlachos<sup>44</sup> presented a simulated annealing-based algorithm for the determination of optimal ship routes through the minimization of a cost function. Korsvik et al.<sup>45</sup> solved a planning problem by developing an efficient tabu search algorithm, and the solver is integrated in a prototype decision support system used by several shipping companies. Fancello et al.<sup>46</sup> presented an approach for predicting ships' delays and then proposed two algorithms for resource allocation.

As seen from the literature, the problems in these domains are tackled earlier using the probabilistic approach that considers randomness in the variables. This article attempts a possibilistic optimization problem of the maritime transportation vessel's travel time reliability. Section "Possibilistic programming" explains the possibilistic programming and possibility indices used in the development of the problem and its solution (refer Appendix 1 for a description of notation and abbreviations used). Section "Problem and proposed

methodology" initially discusses the marine vessel's travel time components and possibilistic reliability. This is followed by the reliability optimization problem formulation with possibilistic constraints of budget and stage times. Then, algorithmic methodology/framework for solving the possibilistic programming problem is proposed in the same section. A suitable illustrative example is presented in section "Illustration and sensitivity." The sensitivity analysis with regard to the variables and its parameters is carried out at the end of section "Illustration and sensitivity." This article concludes with comments regarding the usefulness of the model in marine vessel decision support system with regard to the decisions about stage durations and scheduling at the destination harbor.

### Possibilistic programming

For quite a few initial years in the operations research arena, linear programming technique was widely used and accomplished numerous milestones in theory as well as in applications. However, in real-life decision making, data validity and its precision problem made linear programming relevance and exercise inadequate. After the emergence of fuzzy sets a few decades back, fuzzy linear programming and possibilistic programming have gained a wider acceptance to solve such problems.<sup>1–4</sup>

The possibilistic programming problem comprises linear functions with possibilistic variables or their coefficients that are ambiguous in nature, and consequently, the function values that are also ambiguous. A possibilistic linear function optimal value cannot be determined exclusively as it involves ambiguity or imprecision. Hence, the resolution of objective of minimizing or maximizing a possibilistic function under the constraints of similar nature requires fuzzy inequalities or relations/indices that are defined by possibility and necessity measures. In our article, the constraint inequalities have right-hand side as crisp values; accordingly, requisite possibility (and necessity) measures are stated below.<sup>1,10,47</sup>

Consider  $\psi$  as a possibilistic variable and  $A$  and  $B$  are fuzzy sets. Under possibilistic distribution  $\mu_A$  of a possibility variable  $\psi$ , possibility and necessity measures of the event that  $\psi$  is in fuzzy set  $B$  are defined as

$$\Pi_A(B) = \sup_r \min(\mu_A(r), \mu_B(r)) \quad (1)$$

$$N_A(B) = \inf_r \max(1 - \mu_A(r), \mu_B(r)) \quad (2)$$

where  $\mu_B$  is the membership function (MF) of the fuzzy set  $B$ .  $\Pi_A(B)$  evaluates to what extent it is possible that  $\psi$  restricted by the possibility distribution  $\mu_A$  is in the fuzzy set  $B$ , and  $N_A(B)$  evaluates to what extent it is certain that  $\psi$  restricted  $\mu_A$  is in the fuzzy set  $B$ .

Now consider  $B$  as nonfuzzy (crisp set of real numbers) that is not greater than  $g$ ,  $B = (-\infty, g]$ . Possibility and necessity indices are defined as follows

$$Pos(\psi \leq g) = \Pi_A((-\infty, g]) = \sup\{\mu_A(r) | r \leq g\} \tag{3}$$

$$Nes(\psi \leq g) = N_A((-\infty, g]) = 1 - \sup\{\mu_A(r) | r > g\} \tag{4}$$

$Pos(\psi \leq g)$  and  $Nes(\psi \leq g)$  show the possibility and certainty degrees to which extent  $\psi$  is not greater than  $g$ . Both these indices are shown in Figure 1.

Further considering  $B = [g, +\infty)$ , both the indices could be obtained as

$$Pos(\psi \geq g) = \Pi_A([g, +\infty)) = \sup\{\mu_A(r) | r \geq g\} \tag{5}$$

$$Nes(\psi \geq g) = N_A([g, +\infty)) = 1 - \sup\{\mu_A(r) | r \geq g\} \tag{6}$$

$Pos(\psi \geq g)$  and  $Nes(\psi \geq g)$  show the possibility and certainty degrees of the extent that  $\psi$  is not smaller than  $g$ . Both these indices are shown in Figure 2.

It is worthwhile to note that possibility distribution  $\mu_A$  can be of any type such as trapezoidal (TMF), Gaussian membership function (GMF), and so on. The only criterion for adopting the methodology proposed in this article is that  $\mu_A$  should be a valid convex, normal possibilistic distribution or a fuzzy number.

### Problem and proposed methodology

Initially, this section explains marine vessel’s travel time composition and the concept of its possibilistic travel time reliability. Then, possibilistic formulation of the problem in mathematical form is presented. This is

followed by the proposed algorithm suggesting the solution methodology.

### Possibilistic travel time reliability

Marine vessel’s travel time is composed of a variety of different times that are uncertain in nature. The uncertainty may be due to randomness or may be due to the imprecision or ambiguity. The diverse components of vessel travel times have been considered as per the distinct stages of marine transportation system. The orderly stages considered here are preparations for departure at source harbor, subsequent waiting, navigation at sea, delay due to unforeseen events, waiting just before docking at destination harbor, actual docking, and finally clearance activities at destination harbor. Accordingly, the stage travel times of the marine vessel mission are  $T_{psh}$ ,  $T_{wsh}$ ,  $T_n$ ,  $T_d$ ,  $T_{wdh}$ ,  $T_{ddh}$ , and  $T_{cdh}$ . This article considers a possibilistic approach considering the fact about scarcity of statistical data. This makes the decision maker rely on the information about all the above-mentioned times obtained from the people working on the particular marine vessel. As a result, the information or the data are solely as per the personnel’s experience and associated judgment and is modeled using fuzzy sets. The travel time, which is the sum of all stage travel times and its various stage components, is modeled as fuzzy variables,  $\tilde{T} = \tilde{T}_{psh} + \tilde{T}_{wsh} + \tilde{T}_n + \tilde{T}_d + \tilde{T}_{wdh} + \tilde{T}_{ddh} + \tilde{T}_{cdh}$ . The MFs for all the times are considered as GMFs, and all travel time components are assumed as independent fuzzy variables. Gaussian functions are the most accurate choice of MFs for representing uncertainties in measurement,<sup>48</sup> and the procedure to obtain

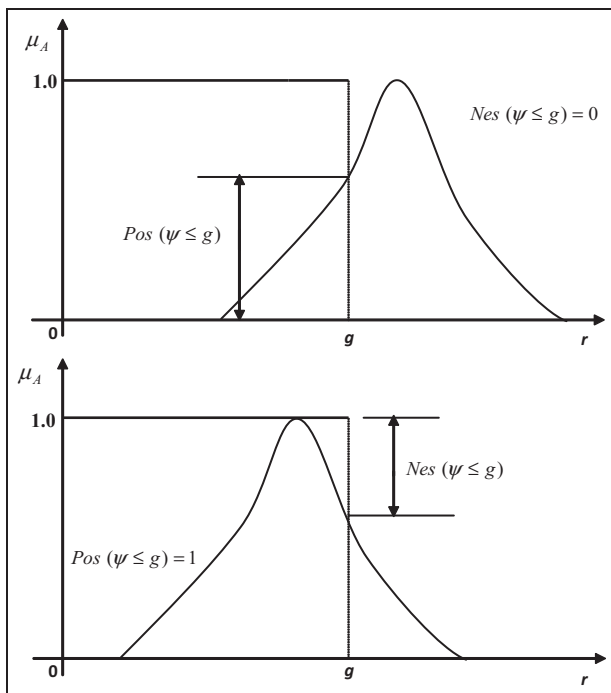


Figure 1. Possibility and necessity degrees of  $\psi \leq g$ .<sup>47</sup>

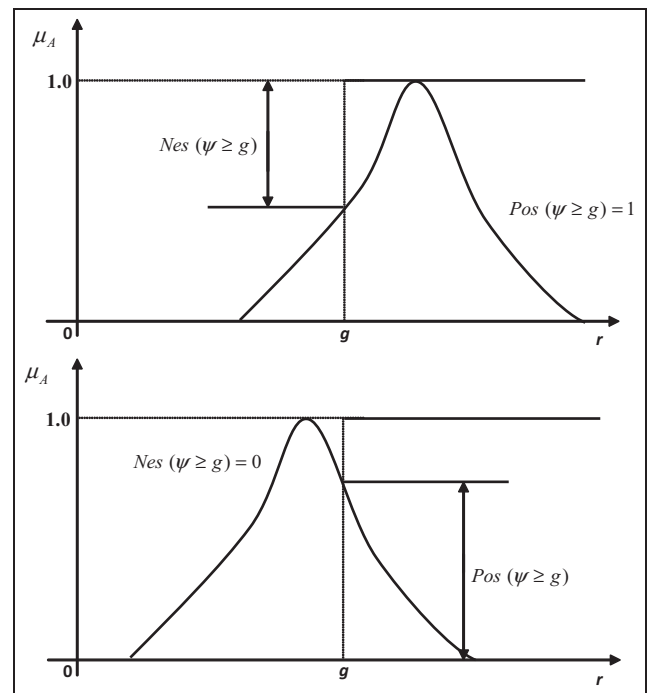


Figure 2. Possibility and necessity degrees of  $\psi \geq g$ .<sup>47</sup>

parameters of *GMF* from expert’s imprecise estimates is given by Kreinovich et al.<sup>48</sup> Uncertainties such as measurement imprecision, linguistic/subjective assessments, expert’s judgments/opinions, and ambiguity are well depicted with the help of MF in the fuzzy sets domain. *GMF* is a non-linear MF used for uncertainty quantification and for modeling of travel time and its components in this article.

The MF  $\mu(x)$  of symmetric Gaussian function with two parameters, that is, central value “*a*” and deviation “ $\delta$ ” is

$$\mu(x) = e^{-\beta \left\{ \frac{(x-a)^2}{\delta^2} \right\}}; \quad \text{for } \beta > 0 \tag{7}$$

The parameter  $\beta$  in the above equation decides the shape of the MF, in particular, the fuzziness content or imprecision in the travel time.

The possibilistic travel time reliability in context with maritime transportation system is the possibility that the marine vessel would reach the destination harbor in scheduled time with accomplishment of the intended mission satisfactorily. Intended mission would be different for dissimilar types of vessels. For example, container ship, iron ore barge, trans-shipper, ferry, oil tanker, and cargo ship, each has a different proposed mission. In mathematical terms, possibilistic reliability is defined as follows

$$\tilde{R} = Pos(\tilde{T} \leq t); \quad t \geq 0 \tag{8}$$

While solving equation (8), the possibility measure, explained in earlier section, needs to be assessed as travel time, which is a fuzzy variable, whereas its expected or planned/scheduled time at destination harbor is nonfuzzy, that is, crisp numerical value.

**Optimization problem formulation**

Optimization of marine vessel’s travel time reliability is a multifaceted and complex task, mainly because of the requirement of quite voluminous and inaccurate data. Although some quantitative data values could be obtained precisely, for various other data values, decision makers have to rely on information provided by the experts on the particular marine vessel. Such information is inevitably of qualitative nature and hence needs to be modeled with the help of fuzzy sets.

The problem attempted in this article is to achieve maximum possible reliability under budgetary constraints. The vessel’s travel time stage components, related reliability, and individual stage costs are fuzzy variables; however, scheduled time, individual stage cost limits, and total cost budget are crisp, that is, precise values. As there is interaction of fuzzy variables and crisp values, possibilistic measures have been introduced in the constraints in terms of possibilistic limits for reliability and the costs. The intention of the problem solving is to obtain various stage optimal individual travel times, which are the decision variables. The

problem formulation proposes two kinds of constraints, namely, hard and soft constraints. The possibilistic optimization formulation of the problem with hard constraints is as follows

$$\left. \begin{aligned} &Max \tilde{R} \\ &\text{subject to} \\ &\sum C_{bi} \leq C_b; && \text{for all } i \\ &Pos(\tilde{T} \leq t) \geq RPL \\ &Pos(\tilde{C} \leq C_{bi}) \geq CPL_i; && \text{for all } i \\ &Pos(\sum \tilde{C} \geq C_b) \geq CPL_b; && \text{for all } i \end{aligned} \right\} \tag{9}$$

where “*i*” would take the value of individual stages, that is, *psh*, *wsh*, *n*, *d*, *wdh*, *ddh*, and *cdh*. The first constraint is totally crisp and relates to the budget allocated for the seven stages and total cost budget for the mission. Possibility measures have been incorporated in the remaining three constraints. The second constraint equation states that possibility of marine vessel reaching the destination harbor should be greater than the specified limit, that is, *RPL*. The following two constraints are about the costs and take care that the possibility of costs exceeding the bounds should be as less as possible. The third constraint equation imposes the bound on the stage costs, whereas the last one forces a similar restrictive value on the total cost of marine vessel travel.

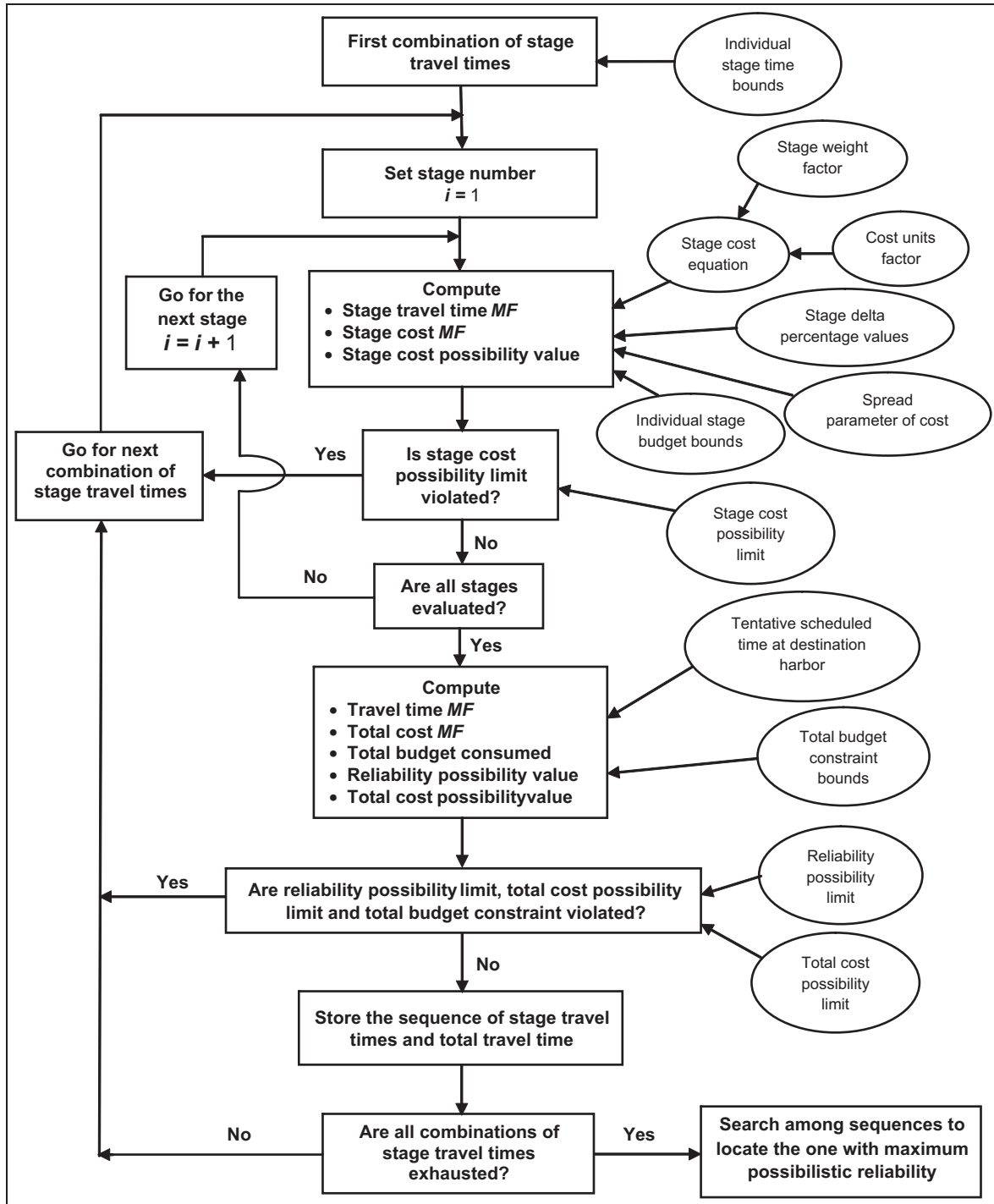
In real implementation of the algorithm proposed in the next section, for solution of the above mathematical problem formulation, few more additional trivial constraints have been brought in. These are soft constraints as stated below

$$\left. \begin{aligned} &C_{Li} \leq C_{bi} \leq C_{Ui}; && \text{for all } i \\ &T_{Li} \leq T_i \leq T_{Ui}; && \text{for all } i \end{aligned} \right\} \tag{10}$$

The sole purpose of the soft constraints is to trim down the entire search space and reduce computational time, and these would have no influence on the optimal solutions that are to be attained. Moreover, these supplementary restrictions or bounds have been imposed on individual stage time and stage cost budget components based on most optimistic and pessimistic times acquired from experts’ past experience.

**Proposed algorithmic framework**

The problem stated in the earlier subsection is attempted using possibilistic programming approach. An algorithmic framework has been proposed here so as to acquire the optimal values through iterative process. The flowchart of the same is depicted in Figure 3. The intention behind the proposed possibilistic optimization routine is to get the highest possible reliability making use of limiting values of the constraints. The approach of optimization is to get utmost gain under the given limitations. The nature of the optimization routine is fairly dynamic in the sense that when a program is run for the initially provided data values, it



**Figure 3.** Proposed possibilistic reliability optimization flowchart. MF: membership function.

may happen that constraints are not violated and optimal values have been outputted. However, the program would still be run further as the constraints are not violated to the maximum extent. It would iterate further till all constraints are satisfied to the limiting values. The proposed procedure would as well keep track of all the possible optimal sequences thus obtained and compare the subsequent optimal sequence with regard to maximum possibilistic reliability. This process is continued till maximum reliability value is attained, and

travel time and its components (i.e. stage times/durations) obtained during this iteration would be the one with maximum objective function value.

The data necessitated for solving the problem under investigation are quite large as seen from the problem formulation itself and also because the marine vessel activities are of complex nature. The decision makers' job is thus challenging and involves correct data (fuzzy and nonfuzzy) compilation, accuracy checks, and confidence in data, as it need to be collected from various

diverse sources. The eventual aim of decision makers is to arrive at the most optimal travel time-based plan/schedule depending on various facts, information/data, and with no violation of the limiting bounds. For execution of the proposed possibilistic optimization algorithm, few more details need to be provided to the program such as individual stage cost equation and a small number of parameters. The decision about the type of MFs for stage travel times and costs are as well required.

There are three basic loops in the proposed algorithm, as seen from its flowchart shown in Figure 3. The first loop computes the MFs for the stage travel times and stage costs for the particular stage travel time combination and checks for violation of stage cost possibility limit, that is, constraint number 3. In case of its violation, it iterates for the next combination. The next loop obtains the MFs for the travel time and total cost for the combination satisfying constraint number 3. It confirms that both travel time and total cost satisfy the respective possibility limits. Moreover, it checks for lonely crisp constraint as well, that is, total budgetary bound is not crossed. Therefore, this loop takes care of other three hard constraints. Once all hard constraints are within the approved bounds, then the combination under investigation is the one satisfying all four constraints. This is stored as a sequence and all above steps are reiterated as final loop for all remaining stage travel time combinations. At the end, the sequence with maximum possibilistic reliability is identified as the most optimum one. The two soft constraints are shown as part of data (information) inputs in the flowchart for the reasons explained earlier and these may be duly included while programming. The explanation given here is illustrated with suitable data values and intrinsic particulars in the next section.

## Illustration and sensitivity

### Example

This section presents an illustration of the possibilistic optimization problem under investigation and the solution thereof by the proposed algorithm by means of suitable example with reasonable data values. The main objective function is to maximize the possibilistic maritime travel reliability with one crisp and other possibilistic constraints, as stated in the earlier sections. All those constraints that are hard constraints are the ones that must be satisfied by the computer program which is to be implemented. The computer programming for this is done using MATLAB. The units for time and cost have been assumed as respective general units.

The cost equation for individual travel stages, designed on the analogous thought from the study by Verma et al.<sup>10</sup> is given by

$$\tilde{C}_i = e^{\{f_c \times (1-\omega_i) \times \tilde{T}_i\}} \quad (11)$$

where  $f_c$  takes care of manageable cost units and takes values such as 0.01, 0.1, and so on, whereas weight factor  $\omega_i$  for stage “ $i$ ” need to be obtained by fuzzy analytic hierarchy process (FAHP). For this, each pair of stages need to be compared under the cost criteria using fuzzy linguistics, and final crisp weights can be obtained using any FAHP method.<sup>49</sup> It may be noted that the value obtained by using equation (11) is the midvalue of the stage cost  $TMF$ . Other extreme lower (left) and upper (right) values of  $TMFs$  are obtained by utilizing cost spread parameter  $f_s$ . It signifies the percentage spread on both sides about the middle value to get the extreme values of  $TMFs$ . As such,  $f_s$  decides the support set of cost  $TMF$ , that is, the difference between right and left extensions, which denote the quantum of fuzziness. Larger the value of absolute difference, fuzzier the judgment.

For this illustration, data values that are assumed to be acquired from the experts are as follows

- $C_b = 10$  cost units
- $t = 72$  time units
- $RPL = 0.90$
- $CPL_{psh} = CPL_{cdh} = 0.15$ ,  $CPL_{wsh} = CPL_{wdh} = 0.1$ ,  
 $CPL_n = CPL_{ddh} = 0.20$ ,  $CPL_d = 0.25$
- $CPL_b = 0.10$
- $\omega_{psh} = \omega_{ddh} = 0.2$ ,  $\omega_{wsh} = \omega_d = \omega_{wdh} = 0.1$ ,  $\omega_n = \omega_{cdh} = 0.15$

Moreover, as mentioned earlier, few soft constraints have been introduced as below with solitary intention of reducing the programming time and the search space. These are mainly the bounds for individual stage cost budgets. These are most pessimistic and optimistic values based on the past experience of the experts working onto the marine vessel. Incorporation of these would in no way differ the final optimal solution. Following are few of such soft constraints related to stage costs, and similar bounds could be imposed on the individual stage times as well

$$1.1 \leq C_{bpsh}, C_{bwsh}, C_{bwdh}, C_{bcdh} \leq 1.4$$

$$1.8 \leq C_{bn} \leq 2.3$$

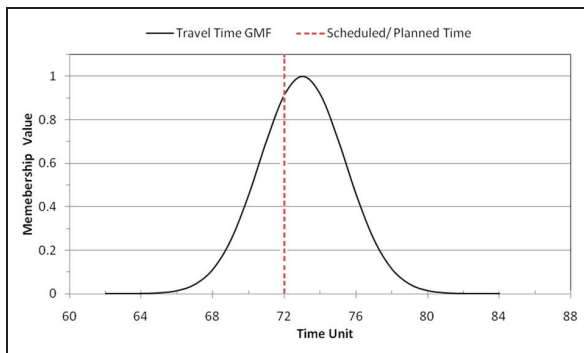
$$1.2 \leq C_{bd} \leq 1.5$$

$$1.2 \leq C_{bddh} \leq 1.4$$

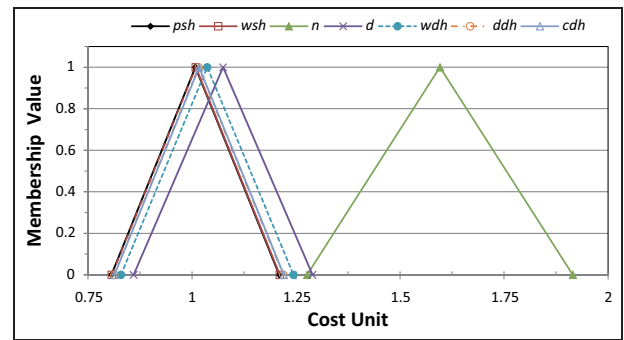
The MFs for stage travel times and resulting total time have been modeled using  $GMFs$ . The standard deviation for stage travel times has been considered as a percentage of the midvalue of the  $GMF$ . The illustrative delta percent values assumed are 0.11, 0.29, 0.03, 0.14, 0.20, 0.20, and 0.21 for  $psh$ ,  $wsh$ ,  $n$ ,  $d$ ,  $wdh$ ,  $ddh$ , and  $cdh$ , respectively. As indicated earlier, individual stage cost and ensuing total cost are  $TMFs$ , and to get the middle value (at which  $\mu = 1.0$ ) of stage cost  $TMF$ , value of  $f_c = 0.01$  is assumed. Furthermore, for obtaining the support set of the  $TMF$ 's fuzziness spread value,  $f_s = 0.20$  is presumed. The MATLAB program (code

**Table 1.** Optimal sequence outputs for the illustrated example.

Total budget utilized			9.3000	
$\tilde{R}$			0.9165	
Possibility value for total cost			0.0059	
$\tilde{T}$			[73.0000, 4.7900]	
$\sum \tilde{C}$			[6.2061, 7.7577, 9.3092]	
Stage	Optimal stage time [ $T_i, d_i$ ]	$\tilde{C}_i$	Possibility value for stage cost	Stage budget utilized
<i>psh</i>	[1.0000, 0.1100]	[0.8064, 1.0080, 1.2096]	0.0478	1.2000
<i>wsh</i>	[1.0000, 0.2900]	[0.8072, 1.0090, 1.2108]	0.0538	1.2000
<i>n</i>	[55.0000, 1.6500]	[1.2768, 1.5960, 1.9152]	0.0476	1.9000
<i>d</i>	[8.0000, 1.1200]	[0.8597, 1.0747, 1.2896]	0.0000	1.3000
<i>wdh</i>	[4.0000, 0.8000]	[0.8293, 1.0367, 1.2440]	0.0000	1.3000
<i>ddh</i>	[2.0000, 0.4000]	[0.8129, 1.0161, 1.2194]	0.0952	1.2000
<i>cdh</i>	[2.0000, 0.4200]	[0.8137, 1.0171, 1.2206]	0.1011	1.2000



**Figure 4.** Travel time GMF and possibilistic reliability for  $t = 72$  time units.  
GMF: Gaussian membership function.



**Figure 5.** TMFs of utilized stage cost budget.  
TMF: triangular membership function.

available from the first author on request) is run with increment of 0.1 for each stage budgetary cost, and total 158 sequences satisfying all the constraints are obtained. Out of these sequences, a single sequence having the highest possibilistic reliability is identified, which is tabulated with all other outputs in Table 1. This sequence is the optimal sequence with the highest possibilistic reliability of 91.65%.

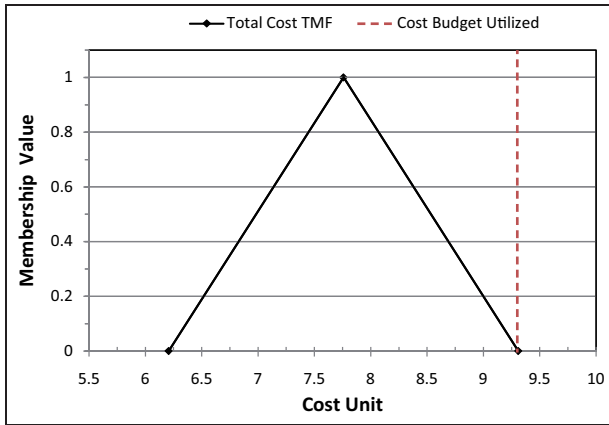
The travel time GMF obtained for the values in Table 1 is depicted in Figure 4. It can be seen that for a scheduled time of 72 time units, the possibilistic reliability value is 91.65%. This is the highest possibilistic reliability of marine vessel obtained under the constraints stated earlier. The individual stage budget that is utilized in achieving this reliability is shown in Figure 5. All the cost components are in the form of TMFs, as seen. Figure 6 portrays the total cost TMF and reveals cost possibilistic value that is very small (i.e. equal to 0.0059).

**Comments on sensitivity**

In the previous illustrative section, one case with data and the output acquired after running the proposed algorithm was explained. A sensitivity analysis is

conducted for the same data with variations in few parameters. Possibilistic reliability of marine vessel for cost spread parameter  $f_s$  values equal to 0.2 and 0.3 with different values of  $t$  varying from 69 to 83 are given in Table 2. No optimal sequences could be got outside this range of  $t$ . Table 2 also shows the total cost units utilized in terms of budget utilization factors  $f_u$  in the optimization process, which is the ratio of  $C_u$  to  $(C_u + C_{um})$ . Table 2 provides valuable information in the sense of scheduling of the travel time and would also be useful in further decision making. For example, for  $t = 75$  time units,  $\tilde{R}$  takes maximum values of 92.43% and 92.54% for  $f_u = 0.93$  and 0.99, respectively. Fuzziness spread parameter too plays a significant role in this. The shaded values are the redundant values, as same  $\tilde{R}$  values have been obtained with lesser cost budget utilization.

Furthermore, the decision maker can choose the tactic based on increase in reliability with respect to cost utilization. Therefore, compromise strategy could be decided as per the circumstances. Charts such as the one given in Figure 7 could be developed that would give a ready reckoner about the cost and possibilistic reliability trade-offs. If one observes the value of  $\tilde{R}$  obtained for  $t = 78$  time units, it is same for  $f_s = 0.2$  and 0.3 with  $f_u = 0.94$  and 1.0. This portrays the effect of ambiguity, that is, fuzziness in data values. The value

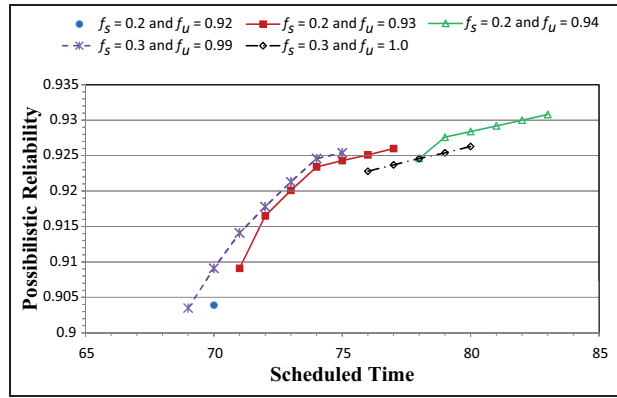


**Figure 6.** TMF of total cost and related possibilistic value. TMF: triangular membership function.

of possibilistic reliability,  $\tilde{R} = 0.9246$ , is achieved by 100% utilization of the budget. Moreover, this value is also acquired with more precise data, that is, with lesser ambiguity in cost values.

**Summary and conclusion**

In real-life problems, it is not easy to formulate a problem in mathematical form owing to the qualitative nature of the objective function and/or constraints. In this article, a possibilistic reliability optimization problem is formulated initially with focus on maritime transportation system. Possibilistic measures are used in the mathematical formulation of the problem investigated here. The possibilistic programming approach is utilized in obtaining the optimal travel time reliability



**Figure 7.** Optimal possibilistic reliability variations.

value under the crisp as well as possibilistic constraints. Travel times and costs are modeled with *GMFs* and *TMFs*, and a dynamic type of optimization algorithm has been proposed as a part of solution methodology. The approach of optimization is to derive the maximum with full exploitation of limiting constraints. The algorithm first computes all the sequences of stage times satisfying the constraints and then identifies the one among those with maximum reliability. An example with numerical solution obtained through MATLAB program is presented as illustration. A sensitivity analysis with parameter variations is also carried out at the end. The proposed methodology would be helpful for decision makers in optimal scheduling of the marine transportation vessel with regard to stage travel times, total vessel travel time, arrival time at destination harbor, in making budgetary provisions and decisions requiring reliability–cost trade-offs.

**Table 2.** Optimal possibilistic reliability for different combinations.

t (time units)	$f_u$				
	0.92 $f_s = 0.2$	0.93	0.94	0.99 $f_s = 0.3$	1.0
68	—	—	—	—	—
69	—	—	—	0.9035	0.9035
70	0.9039	—	—	0.9091	0.9091
71	—	0.9091	—	0.9141	0.9141
72	—	0.9165	—	0.9178	0.9178
73	—	0.9201	—	0.9213	0.9213
74	—	0.9234	—	0.9246	0.9246
75	—	0.9243	—	0.9254	0.9254
76	—	0.9251	—	—	0.9228
77	—	0.926	—	—	0.9237
78	—	—	0.9246	—	0.9246
79	—	—	0.9276	—	0.9254
80	—	—	0.9284	—	0.9263
81	—	—	0.9292	—	—
82	—	—	0.93	—	—
83	—	—	0.9308	—	—
84	—	—	—	—	—

Shaded values are the redundant values, as same  $\tilde{R}$  values have been obtained with lesser cost budget utilization.



## Funding

The work in this article is partially done under the Singapore–Poland Joint Research Project entitled “Safety and Reliability of Complex Industrial Systems and Process” (SERC grant number 072 1340050) granted by Singapore’s Agency for Science, Technology and Research (A\*STAR) and Poland’s Ministry of Science and Higher Education (MSHE). The work described in this article was also partially supported by a grant from City University of Hong Kong (project number 9380058). Professor Huang’s research is partially supported by National Natural Science Foundation of China under the contract number 51075061.

## References

- Inuiguchi M and Ramik J. Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. *Fuzzy Set Syst* 2000; 111: 3–28.
- Jensen HA and Maturana S. A possibilistic decision support system for imprecise mathematical programming problems. *Int J Prod Econ* 2002; 77: 145–158.
- Parra MA, Terol AB, Gladish BP, et al. Solving a multi-objective possibilistic problem through compromise programming. *Eur J Oper Res* 2005; 164: 748–759.
- Sadi-Nezhad S and Akhtari P. Possibilistic programming approach for fuzzy multidimensional analysis of preference in group decision making. *Appl Soft Comput* 2008; 8: 1703–1711.
- Utkin LV, Gurov SV and Shubinsky IB. A method to solve fuzzy reliability optimization problem. *Microelectron Reliab* 1995; 35: 171–181.
- Huang HZ. Fuzzy multi-objective optimization decision-making of reliability of series system. *Microelectron Reliab* 1997; 37: 447–449.
- Ravi V, Reddy PJ and Zimmermann H-J. Fuzzy global optimization of complex system reliability. *IEEE T Fuzzy Syst* 2000; 8: 241–248.
- Huang HZ, Wu WD and Liu CS. A coordination method for fuzzy multi-objective optimization of system reliability. *J Intell Fuzzy Syst* 2005; 16: 213–220.
- Mahapatra GS and Roy TK. Fuzzy multi-objective mathematical programming on reliability optimization model. *Appl Math Comput* 2006; 174: 643–659.
- Verma AK, Srividya A and Prabhu Gaonkar RS. Maintenance and replacement interval optimization using possibilistic approach. *Int J Model Simul* 2007; 27: 193–199.
- Mahmassani HS and Chen PST. An investigation of the reliability of real-time information for route choice decisions in a congested traffic system. *Transportation* 1993; 20: 157–178.
- Senna L. The influence of travel-time variability on the value of time. *Transportation* 1994; 21: 203–228.
- Clark S and Watling D. Modelling network travel time reliability under stochastic demand. *Transport Res B: Meth* 2005; 39: 119–140.
- Sanchez-Silva M, Daniels M, Lleras G, et al. A transport network reliability model for the efficient assignment of resources. *Transport Res B: Meth* 2005; 39: 47–63.
- Tzannatos ES. Technical reliability of the Greek coastal passenger fleet. *Mar Policy* 2005; 29: 85–92.
- Al-Deek HM and Emam EB. Computing travel time reliability-in transportation networks with multistates and dependent link failures. *J Comput Civil Eng* 2006; 20: 317–327.
- Bhat CR and Sardesai R. The impact of stop-making and travel time reliability on commute mode choice. *Transport Res B: Meth* 2006; 40: 709–730.
- Ettema D and Timmermans H. Costs of travel time uncertainty and benefits of travel time information: conceptual model and numerical examples. *Transport Res C: Emer* 2006; 14: 335–350.
- Fan YY and Nie Y. Optimal routing for maximizing the travel time reliability. *Netw Spat Econ* 2006; 6: 333–344.
- Kolowrocki K and Soszynska J. Reliability and availability analysis of complex port transportation systems. *Qual Reliab Eng Int* 2006; 22: 79–99.
- Batley R. Marginal valuations of travel time and scheduling, and the reliability premium. *Transport Res E: Log* 2007; 43: 387–408.
- Wu WW, Ning A and Ning XX. Evaluation of the reliability of transport networks based on the stochastic flow of moving objects. *Reliab Eng Syst Safe* 2008; 93: 838–844.
- Chang JS. Assessing travel time reliability in transport appraisal. *J Transp Geogr* 2010; 18: 419–425.
- Fosgerau M and Karlstrom A. The value of reliability. *Transport Res B: Meth* 2010; 44: 38–49.
- Sumalee A and Kurauchi F. Guest editorial: reliability and emergency issues in transportation network analysis. *Netw Spat Econ* 2006; 6: 169–172.
- Innamaa S. Short-term prediction of travel time using neural networks on an interurban highway. *Transportation* 2005; 32: 649–669.
- Van Lint JWC. Online learning solutions for freeway travel time prediction. *IEEE T Intell Transp* 2008; 9: 38–47.
- Yang ML, Liu YG and You ZS. The reliability of travel time forecasting. *IEEE T Intell Transp* 2010; 11: 162–171.
- Margulici JD and Ban X. Benchmarking travel time estimates. *IET Intell Transp Sy* 2008; 2: 228–237.
- Zheng P and McDonald M. Estimation of travel time using fuzzy clustering method. *IET Intell Transp Sy* 2009; 3: 77–86.
- El Faouzi N-E, Billot R and Bouzebda S. Motorway travel time prediction based on toll data and weather effect integration. *IET Intell Transp Sy* 2010; 4: 338–345.
- Vanajakshi L, Subramanian SC and Sivanandan R. Travel time prediction under heterogeneous traffic conditions using global positioning system data from buses. *IET Intell Transp Sy* 2009; 3: 1–9.
- Rakha H, El-Shawarby I and Arafef M. Trip travel-time reliability: issues and proposed solutions. *J Intell Transportation S* 2010; 14: 232–250.
- Prabhu Gaonkar RS, Xie M, Ng KM, et al. Subjective operational reliability assessment of maritime transportation system. *Expert Syst Appl* 2011; 38: 13835–13846.
- Zhen L and Chang DF. A bi-objective model for robust berth allocation scheduling. *Comput Ind Eng* 2012; 63: 262–273.
- Xu Y, Chen Q and Quan X. Robust berth scheduling with uncertain vessel delay and handling time. *Ann Oper Res* 2012; 192: 123–140.

37. Arango C, Cortes P, Munuzuri J, et al. Berth allocation planning in Seville inland port by simulation and optimization. *Adv Eng Inform* 2011; 25: 452–461.
38. Zhen L, Lee LH and Chew EP. A decision model for berth allocation under uncertainty. *Eur J Oper Res* 2011; 212: 54–68.
39. Golias MM, Saharidis GK, Boile M, et al. The berth allocation problem: optimizing vessel arrival time. *Marit Econ Logist* 2009; 11: 358–377.
40. Lee YS and Chen CY. An optimization heuristic for the berth scheduling problem. *Eur J Oper Res* 2009; 196: 500–508.
41. Fagerholt K. Ship scheduling with soft time windows: an optimisation based approach. *Eur J Oper Res* 2001; 131: 559–571.
42. Wang SA and Meng Q. Liner ship route schedule design with sea contingency time and port time uncertainty. *Transport Res B: Meth* 2012; 46: 615–633.
43. Shao W, Zhou P and Thong SK. Development of a novel forward dynamic programming method for weather routing. *J Mar Sci Technol* 2012; 17: 239–251.
44. Kosmas OT and Vlachos DS. Simulated annealing for optimal ship routing. *Comput Oper Res* 2012; 39: 576–581.
45. Korsvik JE, Fagerholt K and Laporte G. A tabu search heuristic for ship routing and scheduling. *J Oper Res Soc* 2010; 61: 594–603.
46. Fancello G, Pani C, Pisano M, et al. Prediction of arrival times and human resources allocation for container terminal. *Marit Econ Logist* 2011; 13: 142–173.
47. Klir GJ, Clair US and Yuan B. *Fuzzy set theory: foundations and applications*. New Jersey: Prentice Hall, 1997.
48. Kreinovich V, Quintana C and Reznik L. Gaussian membership functions are most adequate in representing uncertainty in measurements. In: *North American Fuzzy Logic Processing Society conference*, 1992, vol. 2., pp. 618–624. Houston, TX: NASA Johnson Space Center.
49. Kahraman C (ed.) *Fuzzy multi-criteria decision making: theory and applications with recent developments*. New York: Springer, 2008, pp. 53–83.

$C_{Li}$	lower limit/bound of cost budget for stage “ $i$ ”
$C_u$	cost budget utilized
$C_{Ui}$	upper limit/bound of cost budget for stage “ $i$ ”
$C_{un}$	cost budget unutilized
$CPL_b$	cost possibilistic limit for total budget
$CPL_i$	cost possibilistic limit for individual stage “ $i$ ”
$d$	delay due to the unforeseen events
$ddh$	docking at destination harbor
$f_c$	cost unit factor
$f_s$	fuzzy parameter or spread of cost (%)
$f_u$	budget utilization factor (%)
$GMF$	gaussian membership function
$n$	navigation at sea or ocean
$pos$	possibility
$psh$	preparation for departure at source harbor
$\tilde{R}$	possibilistic reliability
$RPL$	reliability possibilistic limit
$t$	scheduled or planned time at destination harbor
$T_{Li}$	lower limit/bound of time for stage “ $i$ ”
$T_{Ui}$	upper limit/bound of time for stage “ $i$ ”
$\tilde{T}$	marine vessel’s travel time
$\tilde{T}_i$	time or duration of stage “ $i$ ”
$TMF$	triangular membership function
$\sim$	denote a fuzzy variable
$wdh$	waiting at sea or ocean before docking at destination harbor
$wsh$	waiting subsequent to preparation at source harbor
$\mu$	possibilistic distribution or membership function
$\psi$	possibilistic variable
$\omega_i$	weight factor of the stage “ $i$ ” obtained under cost criteria

## Appendix

### Notation and abbreviations

$cdh$	clearance activities at destination harbor
$C_b$	total cost budget
$C_{bi}$	cost budget of stage “ $i$ ”
$\tilde{C}_i$	cost of stage “ $i$ ”