Remaining useful life estimation under degradation and shock damage

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Abstract

This article presents a prognostic approach to estimate remaining useful life for systems subjected to dependent competing failure processes. In the literature, shock damage is the damage to a soft failure process caused by a shock process. However, how the degradation process causes damage to a hard failure process has not been well studied. In this article, the degradation damage is modeled as the damage to a hard failure process from a degradation process. Degradation and shock processes, as "elemental processes," result in failures via either a soft failure or a hard failure process, namely, "compound processes." Instead of leading to a direct failure, elemental processes construct compound processes: the soft failure process consists of a degradation process and shock damage, and the hard failure process consists of a shock process and degradation damage. In this way, the damage in this article especially represents the effect of an elemental process on other compound processes. Furthermore, a particle filter is applied based on the established model for system statement estimation and on-line prediction of remaining useful life distribution with and without measurement noise in prognostics. Finally, a numerical example is presented with sensitivity analysis.

Introduction

The dependency and competitiveness of multiple failure mechanisms increase the difficulty of remaining useful life (RUL) prognostics for complex systems, which suffer from operation and external damage sources, including degradation, shock loads, and so on.[1] In the literature, failure processes are classified into two basic failure models:[2] (1) hard failures by sudden shock processes from incidental external discrete occasions and (2) soft failures by degradation processes from physical deterioration and aging. The fusion of both processes improves the condition monitoring information for maintenance decision making.[3–7] In this article, degradation and shock process, as "elemental processes," result in failures via either a soft failure process or a hard failure process, as "compound processes." They indicate failure occurrences by crossing the soft failure threshold or the hard failure threshold. The shock damage is well studied[8–10] and involves a soft failure process being impacted by a shock process. However, there are a limited number of studies about degradation damage that impacts hard failure processes. For example, the causes of bearing failures include normal fatigue as a degradation process and shock loads as a shock process. A soft failure is referred to as spalling in balls, inner rings, or outer rings. It is the result of both normal fatigue and shock damage from excessive loads. The degradation damage to a hard failure process by the increase in vibration and shock is due to small discrete particles from surfaces caused by spalling from normal fatigue. Therefore, a hard failure is the result of both the shock process and degradation damage from normal fatigue. It is obvious that fatigue and shock as causes are independent, whereas soft failures and hard failures as results or failure modes are dependent and competing. An example we present explains why the

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hard failure process is influenced by the degradation level.

The main objectives of this study are as follows. First, we seek to illustrate the procedures of dependent competing processes and their corresponding role in the framework of prognostics and health management (PHM). Second, we seek to establish a model with shock damage, and especially with the two models of degradation damage, considering the degradation process and the shock process as elemental processes and considering the soft failure process and the hard failure process as compound processes. Third, we seek to apply the established model for RUL estimation by a particle filter with and without measurement noise in prediction. Finally, we provide an example with practical and simulated data to demonstrate the proposed method.

The data source is one of the major distinctions between traditional life prediction and RUL estimation in the PHM framework. The life data from destructive experiments may be costly. The PHM methodology obtains degradation data from the monitoring sensors with uncertain measurement.[11] At the same time, the degradation data offer information about the potential physical processes[12] and identify critical components[13] and multiple performance characteristics of the systems.[14] The training signals for the off-line process and real-time signals for the on-line process are synthesized to predict RUL distribution.[15] On-line information with noisy signals is integrated to supply meaningful numerical results for prediction with error control.[16] Off-line data and on-line data are connected in order to update the measured underlying damage process states, which provide early warning to avoid expensive breakdowns.[17]

The newly observed degradation data update RUL distribution prediction by Bayesian updating and the expectation–maximization algorithm in PHM.[18] Recently, the on-line prediction of RUL with various kinds of failure modes[19] and prognostic models[20] has been emphasized in industry for the purposes of monitoring real-time reliability and ensuring system availability. Prognostics methods usually are classified into three approaches:[21] (1) physics-of-failure (PoF) approaches, (2) data-driven approaches,[22] and (3) hybrid approaches. PoF approaches apply information of in situ life-cycle loads and devices for failure precursors to estimate the RUL for devices and prevent failure.[23] For example, as one of the devices for failure precursors, canary devices usually have the same critical failure modes of target devices, but have more easily accessible features to alert failures in advance. Model-based approaches amplify the imprecision of prognostics in uncertainty analysis.[24] Data-driven approaches[25] contain the directly observed state processes, such as regression-based models,[26] Brownian motion with drift (Wiener processes),[27] Gamma processes,[28] Markovian-based models,[29] and indirectly observed state processes, including stochastic filtering-based models, especially the Kalman filtering approach[30] and the particle filter approach.[31] Multiple member algorithms are integrated with a weighted-sum formulation to lower the prediction error based on state estimation.[32] Furthermore, some data-based models, such as adaptive neuro-fuzzy inference systems[33,34] and interval analysis approaches,[35] are integrated with a particle filter to forecast the fault indicator and RUL prediction in a nonlinear system[36] with measurement error.[37]

Derived from sequential importance sampling and Bayesian theory, an on-line particle filter–based framework performs fault diagnosis and failure prognosis in real time by a swarm of particles for points and weights for probability mass.[38] Using state probability distribution function (PDF) enables the uncertainty propagation management of estimation and prediction by recursively inferred particles and weights.[39] A state dynamic model and a measurement model construct the particle filter state estimation and prognostics of the RUL of nonlinear components and non-Gaussian processes with high accuracy,[40] converged in most cases as reported by Crisan and Doucet.[41] The resampling algorithm solves the effects of the degeneracy problem in the particle filter.[42,43] Tutorials on the particle filter provide MATLAB codes for these methods.[44,45] In an estimation process, it performs well for the fault diagnosis of operational mode change in a complex dynamic system within a log-likelihood ratio approach.[46] In the particle filter framework for prognostics, one typical application is battery RUL prediction.[47,48] Furthermore, the particle filter can also be used in insulated gate bipolar transistors (IGBTs) with the system model obtained from a least square regression.[49] Details of the particle filter can be found in Orchard.[38] In this article, we employ a particle filter to estimate the system states and future behavior of elemental processes and compound processes in model-based prognostics.

This article is organized as follows. In section "PHM framework for dependent and competing processes," a framework of PHM for dependent competing failure offers a general understanding of the proposed model. In section "Dependent competing failure system," the system model is established for a system with compound processes, which consist of elemental processes with the shock damage model and the proposed degradation damage model. In section "Prognostics method," a generic particle filter is applied for on-line diagnosis and prognosis of RUL distribution with and without the measurement noise in prediction. Section "Numerical example" presents an example to demonstrate the proposed methods. Section "Conclusion and future directions" presents the conclusions and future directions.

PHM framework for dependent and competing processes

Figure 1 describes the generic framework of PHM. In order to optimize the health management policy, it predicts the RUL by estimation of current state and prediction of future state.[50] The framework is divided into
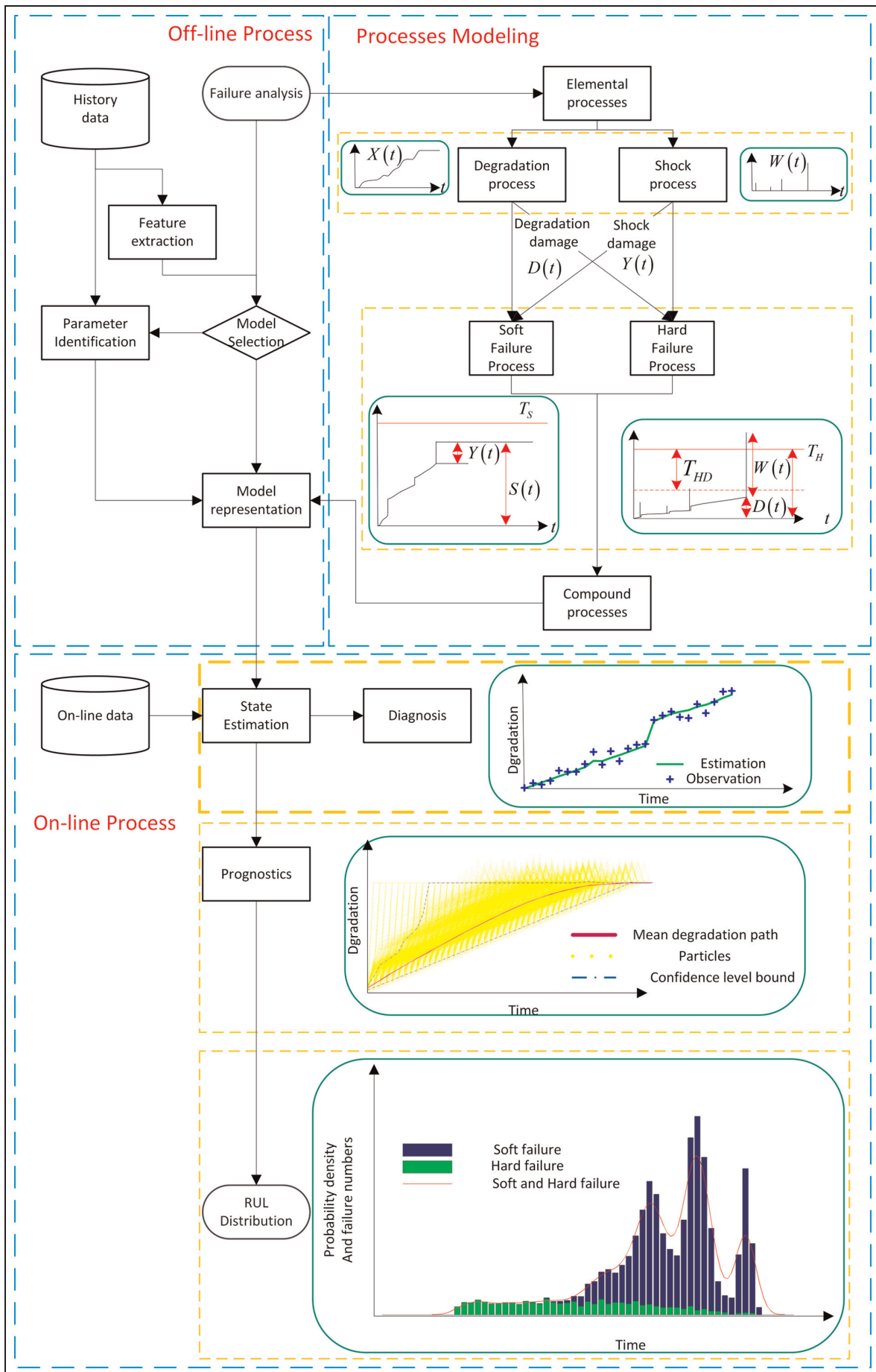
Figure 1. Prognostics framework for dependent and competing processes.

three parts: an off-line process, an on-line process, and a process modeling. In the off-line process, the historical data for feature extraction and failure analysis for model selection are combined to identify the parameters for model representation. In the on-line process, estimated state in real time serves as a bridge between the model in history and RUL in future. It is conceivable that an accurate estimation of RUL improves the asset's health level with corresponding management approaches.

In the process modeling part of Figure 1, the system experiences dependent competing processes. The degradation process and the shock process are elemental processes. The soft failure process and the hard failure process are compound processes. Compound processes are made of elemental processes. A soft failure process consists of the degradation process and the shock damage. A hard failure process consists of the shock process and the degradation damage. Elemental processes explain the root causes of failures and are independent. Compound processes explain the failures modes and are dependent on one or more elemental processes, or the other compound processes. The advantages of elemental process and compound process framework lie in three parts. First, elemental processes and compound processes are mutually separated. There are usually root causes and the direct reason for a failure, which are separated now. Elemental processes are for root reasons and compound processes are for direct reasons. Second, the damage is the effect of an elemental process on other compound processes, which stands for the dependency between them and could be measured. In this way, it explains a situation that if a system is susceptible to hard failure, we not only need to take measures against the shock process but also need to

Soft failure process

Degradation process. A degradation process, with various degradation paths, is due to corrosion, wear, crack growth, and other aging or usage-based degradation.[51] For illustration, a linear degradation path is applied, $X(t) = \varphi + \beta \cdot t$, where in this article the initial value φ is constant 0 and the degradation rate $\beta \sim \text{Normal}(\mu_\beta, \sigma_\beta)$. In particular, the initial value in the particle filter is $X(1*\Delta t)$ instead of $X(0)$, where the initial values are required to be random. Then

$$X(t) = X(t-1) + \beta \cdot \Delta t \tag{1}$$

Shock damage. Shock damage accelerates the soft failure process based on a shock process. The shock time follows a Poisson process $\{N(t), t \geqslant 0\}$, with rate λ. At the ith shock, the shock damage size y_i is s-dependent on the Poisson process. At time t, when there is a shock $N(t) - N(t - \Delta t) = 1$, shock damage equals the summation of past shock damage $Y(t - \Delta t)$ and the shock damage $y_{N(t)}$ at time t. Then, the shock damage could be written as

$$Y(t) = \begin{cases} Y(t - \Delta t) + y_{N(t)}, & \text{if } N(t) - N(t - \Delta t) = 1 \\ Y(t - \Delta t), & \text{if } N(t) - N(t - \Delta t) = 0 \end{cases} \tag{2}$$

where $y_{N(t)} \sim \text{Normal}(\mu_Y, \sigma_Y^2)$, $\mu_Y = a\mu_W$, a is a known constant, and μ_W is the mean size of shock load.

Soft failure process. A soft failure process consists of a degradation process and shock damage, that is, $S(t) = X(t) + Y(t)$. If soft failure state exceeds soft failure threshold T_S, the system fails and we set $S(t) = T_S$. By using equations (1) and (2), $S(t)$ is the soft failure process state, which is derived as

$$S(t) = \begin{cases} S(t - \Delta t) + \beta * \Delta t & \text{if } N(t) - N(t - \Delta t) = 0, \text{ and } S(t - \Delta t) + \beta * \Delta t < T_S \\ S(t - \Delta t) + \beta * \Delta t + y_{N(T)} & \text{if } N(t) - N(t - \Delta t) = 1, \text{ and } S(t - \Delta t) + \beta * \Delta t + y_{N(t)} < T_S \\ T_S & \text{else} \end{cases} \tag{3}$$

reduce degradation damage to the hard failure process. Third, in the applications, it could be applied in situations when there are more than one degradation processes or more than one shock processes.

Dependent competing failure system

The system in this article is subjected to a soft failure process and a hard failure process. The soft failure process comprises a degradation process and shock damage. The hard failure process comprises a shock process and degradation damage. The degradation process and the shock process are assumed mutually independent, and both of them represent the failure causes. Soft failure and hard failure are failure modes, both of which are competing for system malfunction. We set the minimal time interval or the time period $\Delta t = 1$.

Hard failure process

Shock process. A shock process is due to fracture, earthquake, and other external forces, with shock time and the magnitude of shock load. The sizes of shock load w_i for the ith shock are assumed to be independent and identically distributed (i.i.d.) normal random variables. For a particular system, one shock happens at t, the shock numbers increase $N(t) - N(t - \Delta t) = 1$, and the size of shock load is $w_{N(t)}$. In time interval Δt, no more than one shock could happen. Therefore, without considering degradation, the shock process state is

$$W(t) = \begin{cases} 0 & N(t) - N(t - \Delta t) = 0 \\ w_{N(t)} & N(t) - N(t - \Delta t) = 1 \end{cases} \tag{4}$$

where $w_{N(t)} \sim \text{Normal}(\mu_W, \sigma_W^2)$.

Degradation damage. Degradation damage to the shock failure process is based on the degradation process. Then, the exact model $D_D(t)$ for degradation damage $D(t)$ is presented. In this case, degradation damage requires measuring the resistance to hard failure at different degradation state levels. However, sometimes it is hard to divide the shock damage and the degradation process by measurement in field. Therefore, a soft failure damage model $D_S(t)$ is provided as the degradation damage $D(t)$, which does not need to distinguish the degradation process from shock damage.

1. Degradation damage model

   The ratio of $X(t)$ to T_S indicates consumption of resistance to soft failure. When the degradation state is in the highest level of the degradation process, the resistance to the hard failure is T_{HD}. The degradation damage is

$$D_D(t) = (T_H - T_{HD}) \cdot \left(\frac{X(t)}{T_S} \right) \tag{5}$$

2. Soft failure damage model for degradation damage

In practice, it is hard to separate the degradation state and shock damage to the soft failure process exactly. Also, shock damage is proved to affect the resistance to hard failure, so soft failure state is applied instead of the degradation state. The ratio of $S(t)$ to T_S indicates consumption of resistance to soft failure. When the soft failure state is in the highest level of the soft failure process, the resistance to hard failure is T_{HS}. The degradation damage to hard failure process is increasing with the soft failure state in this case. The soft failure damage model for degradation damage is

$$D_S(t) = (T_H - T_{HS}) \cdot \left(\frac{(S(t - \Delta t) + \beta * \Delta t)}{T_S} \right) \tag{6}$$

Hard failure process. A hard failure process contains the shock process and degradation damage, that is, $H(t) = W(t) + D(t)$. $D(t)$ is $D_D(t)$ when the resistance to hard failure is measurable at different degradation state levels in equation (5) or is $D_S(t)$ when that is not measurable in equation (6). If hard failure state exceeds hard failure threshold, the system fails and we set $H(t) = T_H$. By using equation (4) and the definition of the hard failure, $H(t)$ is the hard failure process state, which is derived as

System process

The system is subjected to two dependent competing failure processes: a soft failure process and a hard failure process. Any compound process results in failure. They are dependent on the same degradation process and the shock process. Therefore, the system reliability is defined as

$$R(t) = \Pr\{S(t) < T_S \text{ and } H(t) < T_H\} \tag{8}$$

Since the soft failure process state is measurable, the system state indicator is represented by the soft failure process, derived as

$$I(t) = \begin{cases} S(t) & \text{if } I(t - \Delta t) < T_I, \text{ and } S(t) < T_S \text{ and } H(t) < T_H \\ T_I & \text{others} \end{cases} \tag{9}$$

where $T_I = T_S$.

For a critical system, the predicted RUL is defined by the first hitting time since the last estimation, when it exceeds a pre-set critical threshold in the soft failure process or in the hard failure process. If the last updated estimated time is t_{es}, the RUL for prediction is defined as

$$RUL(t_{es}) = \{t - t_{es} | \inf\{t : (S(t) \geqslant T_S \text{ or } H(t) \geqslant T_H)|$$
$$(S(t_{es}) < T_S \text{ and } H(t_{es}) < T_H)\}\} \tag{10}$$

Prognostics method

The prognostics method is based on dependent competing model. The proposed model and in situ data are integrated to predict future behavior by the particle filter. It contains the state transition function $I(t)$ and measurement function

$$Z(t) = h(I(t), \Omega(t)) \tag{11}$$

where $Z(t)$ is the measured data and $\Omega(t)$ is the measurement noise, which is taken as the Gaussian noise $\Omega(t) \sim N(0, \omega^2)$. There are two basic types of prognostic methods, classified with and without the consideration of measurement noise.

Method without measurement noise

If the measurement noise is not considered in prognostics, posterior distribution of system state indicator is only based on prediction model, and it can be expressed by an integral as follows

$$H(t) = \begin{cases} D(t) & \text{if } N(t) - N(t - \Delta t) = 0, \text{ and } H(t - \Delta t) = 0 \\ D(t) & \text{if } N(t) - N(t - \Delta t) = 1, \text{ and } H(t - \Delta t) = 0 \text{ and } W(t) + D(t) < T_H(t) \\ T_H & \text{else} \end{cases} \tag{7}$$

$$p(I(t_{es} + l)|Z(1 : t_{es}))$$
$$= \int \cdots \int \prod_{j = t_{es} + l}^{t_{es} + l} p(I(j)|I(j - 1))p(I(t_{es})|Z(1 : t_{es})) \prod_{j = k}^{k + l - 1} dI(j)$$
$$(12)$$

where $Z(1 : t_{es})$ is the measured system state indicator and l is the predicted time since t_{es}.

Method with measurement noise

In field, measurement noise is unavoidable with the change in environment and measure tools. The measured system state indicator is $Ih(t_{es} + l)$, and the probability distribution is derived as

$$p(Ih(t_{es} + l)|Z(1 : k))$$
$$= \int \cdots \int \prod_{j = t_{es} + 1}^{t_{es} + l} p(Ih(j)|I(j))p(I(j)|I(j - 1))$$
$$p(I(t_{es})|Z(1 : k)) \prod_{j = t_{es}}^{t_{es} + l - 1} dIh(j) \qquad (13)$$

In the prediction process with measurement noise, we assume that when the measured system state reaches the pre-set threshold, the system will also be shut down.

Numerical example

The numerical data of a micro-engine from Sandia National Laboratories and Jiang et al.[7] are listed in Table 1. In this case, the soft failure is usually the broken pin, which results from pin wear of rubbing surfaces as the degradation process with the debris of the pin as the shock damage. Meanwhile, the hard failure is usually the hub fracture, which results from external shocks to the gear hub as the shock process with the wear of the hub as the degradation damage. Therefore, the object is subjected to a dependent competing failure process. The simulation data for observation state are used for the entire process to compare the filtered result and true state.

The RUL estimation framework with degradation damage model is shown in Figure 2. First, the whole process is divided by the estimated time into two parts: the estimation process and the prognostic process. Second, in the estimation process, the system transition with equation (9) is applied with the soft failure process by equation (3) and the hard failure process by equation (7). From Figure 2, it is seen that the application of the proposed model allows jumps in state transition with good estimation result. Third, at the last updated estimated time shown as the line of dashes, the last estimated system state is an essential connection between the estimation process and the prognostic process. In the particle filter, the particles with weights represent the distribution of the system state. Fourth, in the prognostics process, the prognostic system states are predicted by equation (12) or (13). Finally, the PDF of

Table 1. Degradation and shock processes' example values.

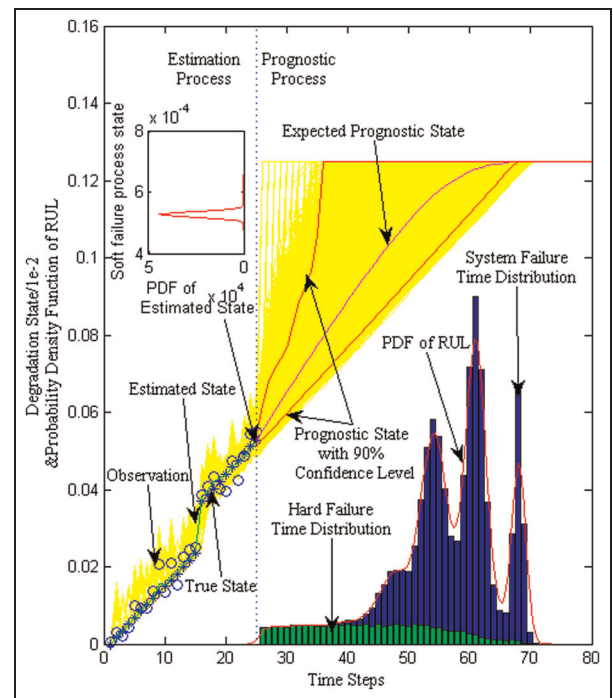| Parameters	Value	Sources
T_S	$0.00125\,\mu m^3$	Tanner and Dugger[52]
T_{HS}	$1.4\,GPa$	Assumption
T_H	$1.5\,GPa$	Tanner and Dugger[52]
ϕ	$0\,\mu m^3$	Tanner and Dugger[52]
β	$N(8.4823e - 9,$ $6.0016e - 10)\,\mu m^3$	Tanner and Dugger[52]
λ	$5e-5$	Assumption
Y	$N(1.2e - 4, 2e - 5)\,\mu m^3$	Assumption
W	$N(1.2, 0.2)\,GPa$	Assumption
a	$1e - 4\,\mu m^3/GPa$	Assumption

Figure 2. Framework of RUL prognostics with degradation damage model.

RUL is plotted by kernel smoothing function estimation, which reflects the competing result of the hard failure process and the soft failure process. Therefore, this whole framework enables RUL estimation for dependent competing failure process.

Meanwhile, PDF of system state at each step is shown in Figure 3. At first, when the measured system state is provided, distributions of system state are concentrated on the estimated system state. After the last estimation point, they are separated into two main paths with the influence of the hard failure. In prognostics without observations, the distribution expands with time steps. Finally, because system states stop increasing when they cross the system state threshold, the distribution of system states is concentered on the threshold with time steps.

Sensitivity analysis of $\Pr(RUL)$ on T_{HD} is performed and plotted in Figure 4 for degradation damage model without noise. T_{HD} has a significant impact on

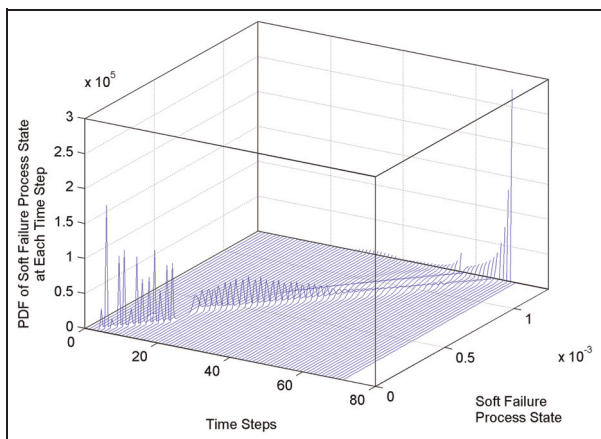**Figure 3.** Soft failure degradation state distribution with degradation damage model.
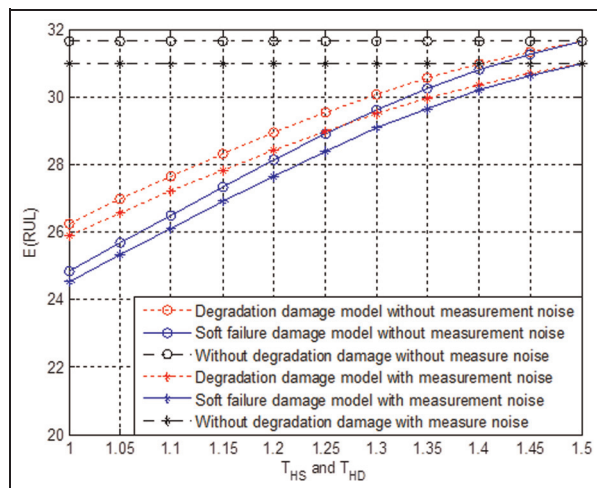

Figure 5. Sensitivity analysis of E(RUL) on T_{HD} and T_{HS}.
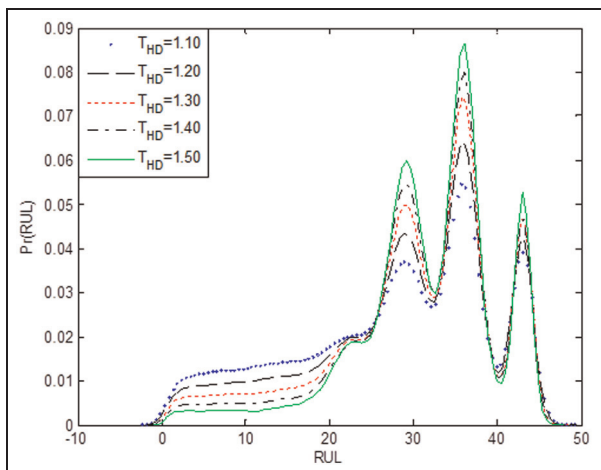

Figure 4. Sensitivity analysis of Pr (RUL) with degradation damage model on T_{HD} without measurement noise.
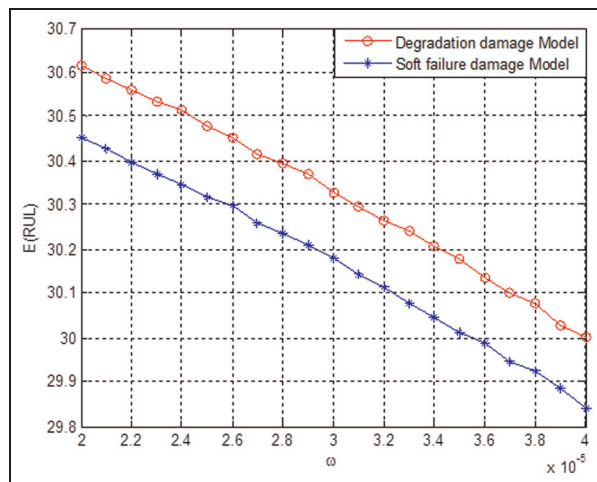

Figure 6. Sensitivity analysis of E(RUL) on ω.

Pr (RUL) in this case. When T_{HD} increases, Pr (RUL) gets smaller first and larger after a critical point. In particular, when $T_{HD} = 1.5$, the degradation damage is always 0. In the early stages of failures, there are more hard failures than soft failure, as shown in Figure 2. Therefore, the results in Figure 4 agree with the cases that if there are two systems with the same shock loads, the one with higher degradation damage is more susceptive to hard failure. It implies that the improvement of resistance to degradation damage will reduce hard failures.

In Figure 5, sensitivity analysis of mean RUL E(RUL) on T_{HD} and T_{HS} is performed for degradation damage model and soft failure damage model, companied with measurement noise cases. E(RUL) in degradation damage model is larger than that in soft failure damage model and tends to be the same with the increase in T_{HD} and T_{HS}. In both the proposed models, E(RUL) grows with T_{HD} and T_{HS}, respectively. And compared with the model without damage to hard

failure, the presented models have a great impact on E(RUL). It implies that reducing the damage to hard failure extends the RUL in systems that experience dependent competing processes. At the same time, the measurement noise will reduce the E(RUL) in each damage model. It indicates that the measurement noise is responsible for the prediction error of RUL.

In Figure 6, sensitivity analysis of E(RUL) on ω with failure threshold model is performed and plotted. It appears that the E(RUL) increases with measurement noise. However, as a matter of fact, increment of E(RUL) raises the estimated RUL instead of the actual RUL. It implies that larger measurement noise will reduce the ability of fault diagnostics.

Conclusion and future directions

In this article, we establish a system model with compound processes that consisted of elemental processes. Specially, the compound processes contain a soft failure process and a hard failure process, and the

elemental processes contain a degradation process and a shock process. First, the PHM framework for dependent competing failure processes is illustrated. Second, the system model for dependent competing is built with the shock damage and the proposed degradation damage. Third, RUL estimation contains information both from off-line database and on-line observations, by the particle filter for nonlinear processes with sequentially updated system health condition. Finally, the example from field experiment for off-line data and simulation data for the real-time process shows that the proposed system model plays an important role in estimating RUL. Sensitivity analysis indicates that (1) degradation damage to the hard failure process reduces the RUL in systems with dependent competing processes and (2) measurement error results in imprecision of the predicted distribution of RUL and reduces the ability of fault diagnostics. In future, we will perform RUL estimation by combining shock damage and degradation damage with PoF approaches. This will potentially improve the development of decision-making strategies for the accuracy and precision of prediction in industry. Especially, the elemental processes will be replaced by physical processes of failures to be applied for particular products to obtain more accurate results.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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Appendix I

Notation

$D(t)$	degradation damage to a hard failure process by a degradation process
$H(t)$	hard failure process state
$I(t)$	system state indicator
$N(t)$	number of shock loads up to time t
$S(t)$	soft failure process state
t_{es}	time to estimate the system state with the last observation
T_H	threshold level for a hard failure
T_{HD}	resistance to a hard failure with the highest level of degradation damage
T_{HS}	resistance to a hard failure with the highest level of soft failure damage
T_I	threshold level for a system failure
T_S	threshold level for a soft failure
$W(t)$	shock process state
$X(t)$	degradation process state
$Y(t)$	shock damage to a soft failure process by a shock process
λ	arrival rate of random shocks
ω	deviation of the measurement noise
Δt	time interval