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RELIABILITY AND RISK ASSESSMENT OF AIRCRAFT ELECTRIC SYSTEMS

NIEZAWODNOŚĆ I OCENA RYZYKA UKŁADU ELEKTRYCZNEGO SAMOLOTU

It is rather difficult in identifying the fault location and performing risk assessment for complex electronic systems. In this paper a reliability assessment method based on the interval analytic hierarchy process (IAHP) and Bayesian network is proposed to facilitate reliability and risk assessment. After considering the major fault factors, the interval eigenvector method (IEM) is also presented to assess the reliability and comprehensive weights of subsystems. The conditional probability matrices for the factors of risk are defined using an inference rule. Then an updating model of information fusion in the context of Bayesian network is established to assess the risk of system. The proposed method is demonstrated through the risk assessment of an aircraft electric system. The result of the illustrative example shows that the proposed method can not only incorporate the evidence information, but also synthesize all the historical information. A further dynamic adjustment in the safety and risk priority of control measures is quite effective to improve the reliability while mitigating the risk of the electric system.

Keywords: *interval analytic hierarchy process, interval eigenvector method, Bayesian network, information fusion, risk assessment.*

Lokalizacja uszkodzeń oraz ocena bezpieczeństwa i ryzyka w przypadku złożonych systemów elektronicznych jest zadaniem dość trudnym. W niniejszej pracy zaproponowano metodę prognozowania niezawodności opartą na procesie przedziałowej hierarchii analitycznej (IAHP), która ma na celu ułatwienie diagnozy uszkodzeń i kontroli ryzyka. Po rozważeniu głównych czynników wywołujących uszkodzenia, zaprezentowano metodę przedziałowych wektorów własnych oraz zdefiniowano, przy użyciu reguły wnioskowania, macierze prawdopodobieństwa dla czynników wpływających na bezpieczeństwo i ryzyko. Następnie, stworzono odnawialny model fuzji informacji w kontekście wnioskowania bayesowskiego służący do oceny stanu zagrożenia Udowodniono, iż włączenie wiedzy eksperckiej do dynamicznej symulacji ułatwia lokalizację uszkodzeń oraz pozwala uzyskać informacje dotyczące diagnozy uszkodzeń. Studium przypadku pokazuje, że dynamiczne dostosowanie priorytetowości związanej z bezpieczeństwem i ryzykiem stosowanych środków kontroli w sposób dość skuteczny zwiększa niezawodność przy jednoczesnym zminimalizowaniu ryzyka w złożonym systemie elektronicznym.

Słowa kluczowe: *proces przedziałowej hierarchii analitycznej, metoda przedziałowych wektorów własnych, sieć bayesowska, fuzja informacji, ocena ryzyka.*

1. Introduction

Electronic systems have been widely used in the modern industrial products and engineering systems, such as the machine tools, airplanes, nuclear power plants and so on. They play vital roles in the normal running and safe operation of these systems. The fast development and extensive application of electronic systems often gives rise to critical problems concerning about the reliability and safety of these systems. This is because that the electronic systems are getting more complex and they are deeply integrated and coupled with other systems in modern products and engineering systems. It is common for an electronic system which is composed by thousands of components to experience a sudden failure. However, it is time consuming for reliability engineers to identified the specific fault location among these thousands components which are deeply embedded in an engineering system. Moreover, the failure of the electronic system often leads to a destructive failure or a horrible accident of the engineering system, such as the failure of the flight control system of an airplanes and the malfunction of the safety control system of a nuclear power plant. Besides, it is relatively difficult to monitor and control the safety level of these systems. Therefore, how to identify the fault location and further mitigate the operation risk is of great importance [12, 7].

Technically, the key point of fault location lies in a comprehensive reliability analysis and decision-making process with multiple attribute decision. The analytic hierarchy process (AHP) is a method that is capable for the implementation of fault diagnostic with multiple attribute decision. Shen and Cheng [15] introduced a fuzzy analytic hierarchy process for the fault diagnostic of lithography process. It can deal with the quantification of managerial causes and subjective judgment encountered in the diagnostic process of lithography process. Wu et al. [16] investigated the sensor deployment for diagnostic of manufacturing system. In their paper, the AHP was used to implement the quantitative determination of the sensors' detectability to fault and further to facilitate the decision-making about the sensor deployment. This method is demonstrated more effective for diagnosis performance improvement than the results obtained from signed directed graph based sensor deployment. Recently, Liu et al. [8] introduced a method for fault diagnosis where the incomplete and unknown information encountered in fault diagnosis was modeled using a fuzzy evidential reasoning approach and analyzed through dynamic adaptive fuzzy Petri nets. Among these works, lots of attentions are laid on the implementation of AHP to the fault diagnosis. One critical issue concerning the implementation of the AHP is the handling of subjective information derived from the experience and judgment of experts. It is a problem related to the quantification and integration of epistemic uncertainty. This epistemic uncertainty existing in the implementation of AHP will lead to a subjective decision-making which is hard to optimize. Moreover, it will give rise to a biased result of reliability and risk assessment if this epistemic uncertainty is not well handled. Therefore, it is necessary to develop an appropriate method for fault diagnostic by handling this epistemic uncertainty properly, and at the meantime to improve the accuracy of reliability and risk assessment when adopting the experts' experience [1,13].

For the evaluation of system reliability, due to the less available of subsystem reliability information, it's particularly difficult to assess the reliability of subsystems. As a result, a precise system reliability assessment can hardly be obtained based on the assessment of subsystem reliability. To solve this problem, we extend the idea from the methods of reliability allocation, where we take a full consideration about the various factors affecting the reliability of subsystems. To the various factors, we turn the impact of these factors into sub-system's reliability allocation weights based on the AHP. The AHP is used to determine how the reliability of system may be controlled by appropriately assigning weights to its components. Here, a proper handling of the epistemic uncertainty introduced by the incorporation of subjective information is needed as discussed above. Nowadays, most engineers adopt the fuzzy comprehensive evaluation (FCE) or AHP to ensure the weights allocation. There are two shortages about the FCE and AHP. On one hand, the impact factors that affect the subsystem reliability often contain some uncertainty. For example, it's difficult to use a precise value to describe the influence level of an impact factor. The engineer usually ignores this kind of uncertainty and adopts single values to construct a judgment matrix for the impact factors. It often causes the estimation error because the single values are not only too restrictive to describe the subjective information but also too deterministic to avoid estimation bias. A improper dealing with the epistemic uncertainty in judgment can cause incorrect assessment results of reliability. On the other hand, the judgment matrix adopted in the classical approaches must verified through the consistency check. If not, great effort will be spend on the tedious reconstruction of the judgment matrix until it meets the check.

For this reason, we adopt a interval analytic hierarchy process (IAHP) as the quantitative method to perform reliability and risk assessment of electronics system. The IAHP is a comprehensive criteria decision making tool that combines interval analysis with AHP as well as the interval eigenvector method (IEM) to calculate weight vector of relative importance (section 2). Compared to the traditonal AHP, it is more flexible for the dealing with uncertainties by including various influence factors. In addition, the utilizing of intervals instead of deterministic values can further facilitate the estimation of system reliability according to the relative weights [3, 9]. After the conditional probability distribution matrix concerning subsystem reliability is derived based on the IAHP, we set up an updating model for information fusion using Bayesian network. It is further used to carry out the risk assessment (section 3). In addition, a simulation model is introduced to assess the risk level by taking into account three critical indexes: weight, reliability, and the risk level. These indexes together present a new approach to assessing the reliability and risks of aircraft electric systems.

2. Fault control and safety-risk prediction

2.1. IEM

In this paper, we investigate a three levelss system. Namely, with the AHP, the objectives, decision criteria and alternatives are arranged in a hierarchical structure similar to a family tree. A hierarchy has three levels: overall goal of the problem at the top, multiple criteria that define alternatives in the middle, and alternative at the bottom level.

In the application of AHP, the key point is to construct a comparison and judgment matrix between various factors. The values within 1~3, 1~5, 1~7 or 1~9 are generally used in the construction of this judgment matrix based on the fundamental research presented by Saaty [14]. Among these scales, the 1~9 scales can present more levels for comparison matrix construction than the others, and we use 1~9 scales in this paper. Moreover, to take account of the epistemic uncertainty introduced by the subjective judgment of experts, fuzzy set theory is incorporated into classical AHP.

In detail, intervals $a_{ij} = \left[a_{ij}^{-}, a_{ij}^{+} \right]$ are chosen for the construction of the judgment matrix. The interval eigenvector method (IEM) is used to facilitate the calculation of the weights within the judgment matrix. The specific procedure and derivation are presented below progressively. Firstly, the judgment matrix for the comparison between the factors of interest is given defined as follow [2]:

$$A = \left(a_{ij} \right)_{n \times n} = \left[A^{-}, A^{+} \right]$$

The elements within the matrix have the following characteristics as:

(1) $a_{ij} = \left[a_{ij}^{-}, a_{ij}^{+} \right]$ is a ratio that describes the importance of the goal between factors i and j.

(2) $A^{-} = \left(a_{ij}^{-} \right)_{n \times n}, A^{+} = \left(a_{ij}^{+} \right)_{n \times n}$.

(1) $\frac{1}{9} \le a_{ij}^{-} \le a_{ij}^{+} \le 9, a_{ij} = 1/a_{ji}, a_{ji} = \left[1/a_{ij}^{+}, 1/a_{ij}^{-} \right], a_{ii} = 1, (i, j = 1, 2, \cdots, n)$.

Table 1. Weight matrix of sub-factors within U_i

	u_{i1}	u_{i2}	$\cdots$	u_{in}
u_{i1}	$[1.0, 1.0]$	$\left[a_{12}^{-}, a_{12}^{+} \right]$	$\cdots$	$\left[a_{1n}^{-}, a_{1n}^{+} \right]$
u_{i2}	$\left[1/a_{12}^{+}, 1/a_{12}^{-} \right]$	$[1.0, 1.0]$	$\cdots$	$\left[a_{2n}^{-}, a_{2n}^{+} \right]$
$\vdots$	$\vdots$		$[1.0, 1.0]$	$\cdots$
u_{in}	$\left[1/a_{1n}^{+}, 1/a_{1n}^{-} \right]$	$\left[1/a_{2n}^{+}, 1/a_{2n}^{-} \right]$	$\left[1/a_{3n}^{+}, 1/a_{3n}^{-} \right]$	$[1.0, 1.0]$

In the middle level, if there're i kinds of factors, then let $U_1, U_2, \cdots, U_i$ denote them. Accordingly, we have the factors set $U=\{U_1, U_2, \cdots, U_i\}$. At the bottom level, U_i includes several sub-factors, and let $u_{i1}, u_{i2}, \cdots, u_{in}$ denote them. The weight matrix of sub-factors within U_i is then given in Table 1.

The matrix A^- and A^+ is then obtained by rearranging the values in Table 1 as follows:

$$A^- = \begin{bmatrix} 1 & a_{12}^- & \cdots & a_{1n}^- \\ 1/a_{12}^+ & 1 & \cdots & a_{2n}^- \\ \vdots & \vdots & 1 & \vdots \\ 1/a_{1n}^+ & 1/a_{2n}^+ & \cdots & 1 \end{bmatrix}, A^+ = \begin{bmatrix} 1 & a_{12}^+ & \cdots & a_{1n}^+ \\ 1/a_{12}^- & 1 & \cdots & a_{2n}^+ \\ \vdots & \vdots & 1 & \vdots \\ 1/a_{1n}^- & 1/a_{2n}^- & \cdots & 1 \end{bmatrix} \quad (1)$$

We use the IEM to calculate the weight vector of matrix A^- and A^+ based on the interval analysis and fuzzy set theory. Let λ, λ^- and λ^+ separately denote the eigenvalue, the low limit of eigenvalue and the upper limit of eigenvalue; and let x, x^- and x^+ separately denote the eigenvector, the low limit of eigenvector and the upper limit of eigenvector. Some basic theorems of fuzzy set theory used in this paper are listed below:

(1) If $Ax=\lambda x$, then we have $A^-x^-=\lambda^-x^-$, $A^+x^+=\lambda^+x^+$.

(2) If $A=\left[A^-, A^+\right]$, λ^- and λ^+ are the maximum eigenvalues of the matrix A^- and A^+, respectively, then we have:

$\lambda=\left[\lambda^-, \lambda^+\right]$ is the eigenvalue of matrix A.

$x=\left[\alpha x^-, \beta x^+\right]$ is the eigenvector cluster of matrix A corresponding to λ, where $\alpha, \beta \in R^+, 0<\alpha x^- \le \beta x^+$. Note that x^- and x^+ are separately the eigenvector of matrix A^- and A^+ corresponding to λ^- and λ^+.

(3) If $x=(x_1, x_2 \cdots, x_n)$ is the eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$, the normalized eigenvector $\tilde{x}$ is given as:

$$\tilde{x}=\frac{1}{\sum_{i=1}^{n} x_i}(x_1, x_2, \cdots, x_n) \quad (2)$$

Based on equation (2), we can get the weight vector of matrix A^- and A^+ denoted by x^- and $\tilde{x}^+$. Then x^- and $\tilde{x}^+$ is separately the normalized eigenvector of positive component x^- and x^+.

Let ω_i denote the weight of the i^{th} sub-system. It is defined as an interval based on the eigenvector cluster of judgment matrix for the i^{th} sub-system, which is named as the interval-number weight. The interval-number weight component of the system is denoted as $\tilde{\omega}=\left[\omega_1, \omega_2, \cdots, \omega_n\right]^T$. Then we can obtain the $\tilde{\omega}$ as follow.

$$\tilde{\omega}=\left[\alpha\tilde{x}^-, \beta\tilde{x}^+\right]=\left[\omega_1, \omega_2, \cdots \omega_n\right]^T \quad (3)$$

$$\alpha=\left[\sum_{j=1}^{n}\left(1\Big/\sum_{i=1}^{n} a_{ij}^+\right)\right]^{1/2}, \beta=\left[\sum_{j=1}^{n}\left(1\Big/\sum_{i=1}^{n} a_{ij}^-\right)\right]^{1/2}, (\forall i, j \in N) \quad (4)$$

This interval-number weight has one major advantage over the classical one that it needs no consistency check, which releases the tedious reconstruction of the judgment matrix. In other words, if there is a consistency interval-number judgment matrix, then A is always the one that satisfy the consistency check. If it is not the one, we can construct the required interval-number weight $\tilde{\omega}$ based on the normalization method presented above.

2.2. Calculate comprehensive interval-number weight vector

Based on the method for interval-number weight derivation presented above, we get the interval-number weight ω_i in the middle levels and the ω_{ij} at the bottom level. Then we calculate the comprehensive weight C_{ii} of the ith unit for the whole system. The interval value of C_{ii} is expressed as $[g_i^-, g_i^+]$ and obtained as follow.

$$C_{ij}=\omega_i\omega_{ij}=[g_i^-, g_i^+], (i, j=1, 2, \cdots, n). \quad (5)$$

where the calculation between the interval-number weights is defined as follow

$$[a, b][c, d]=[\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \quad (6)$$

After obtaining the comprehensive weight C_{ii}, we can obtained the normalized weight of the subsystem through the defuzzification of this comprehensive weight. It is implemented through the defining of a normalized function based on the membership function of fuzzy number under the framework of fuzzy set theory.

(1) Trapezoidal fuzzy number

A trapezoidal fuzzy number $\bar{q}$ has the following membership function [17]:

$$\mu_q^-(x)=\begin{cases} (x-a)/(b-a) & a \le x < b \\ 1 & b \le x < c \\ (d-x)/(d-c) & c \le x \le d \\ 0 & others \end{cases} \quad (7)$$

where $x, a, b, c, d \in R, a \le b \le c \le d$ .1

A pictorial discription of the menbership function of a trapezoid fuzzy numer $\bar{q}=(a, b, c, d)$ is shown in Fig.1. In the interval $[b, c]$, the fuzzy number has the maximum membership degree according to the membership function $\mu_q^-(x)$. And, a, d are respectively the upper and lower bounds of the fuzzy interval.

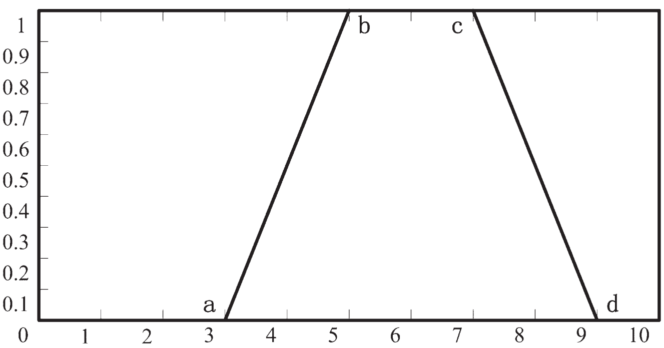

Fig. 1. Membership function of a trapezoid fuzzy number

(2) Calculate the interval-number weight vector and sort it
When A is a trapezoid fuzzy number $Q=(a,b,c,d)$, we define a normalized function $m_u(Q)$ based on the membership function

$$m_u\left(Q\right)=\frac{\left(c^2+d^2+cd-a^2-b^2-ab\right)}{3\left(c+d-a-b\right)} \tag{8}$$

Let ϖ denote the sorting weight, then the sorting weight of an interval-number $g_i=[g_i^-,g_i^+]$ is then obtained as follow:

$$\bar{\omega}=m_u\left(g_i\right)=\frac{1}{2}\left(g_i^-+g_i^+\right) \quad (i=1,2,\cdots,n_m) \tag{9}$$

2.3. Determination of the reliability

Let λ the failure rate of the electronic system. Following the idea of reliability allocation, the failure rate of subsystems are obtained by allocating the system failure rate to each subsystem. These failure rates are calculated based on the comprehensive weight values in index layer derived above. In detail, the relationship between λ_i and the comprehensive weight is given as follow [5]:

$$\lambda_i=\lambda\frac{1}{W_i}\Big/\sum_{i=1}^{n}\frac{1}{W_i} \tag{10}$$

λ_i: the failure rate of the ith subsystem, which is also denoted as the i^{th} attribute in the index layer
W_i: the comprehensive weight of the ith subsystem (attribute).

Following the assumption of exponential distribution, the reliability of the system with the failure rate λ is given as follow:

$$R(t)=e^{-\lambda t} \tag{11}$$

In addition, we further assume that the risk level of a system is inversely proportional to the reliability. Accordingly, we can get the relationships between the risk levels and the reliability $R(t)$. A specific description of the relationship is presented in Table 2.

Table 2. Relationship between the risk levels and the reliability

Risk level	range	Quantified risk level (risk values)
High	$R(t)\in(0,0.3]$	$\left\{1-R(t),\frac{R(t)}{2},\frac{R(t)}{2}\right\}$
middle	$R(t)\in(0.3,0.6]$	$\left\{\frac{R(t)}{2},1-R(t),\frac{R(t)}{2}\right\}$
low	$R(t)\in(0.6,1]$	$\left\{\frac{1-R(t)}{2},\frac{1-R(t)}{2},R(t)\right\}$

By calculating the reliability of each subsystem, we can figure the corresponding risk level that each subsystem experiences. The relevant quantified risk levels (risk values) of these subsystems can also be obtained through the relationship given in Table 2. These risk values are used as the prior information in the following risk assessment, which is implemented through the Bayesian network.

2.4. Bayesian network

Based on the prior information about risk value of each subsystem obtained above, we then carry out the risk assessment using the Bayesian network in this section. We first briefly describe the basic definition and the inference procedure of the Bayesian network.

(1) Definition

Let $X\in\{X_1,X_2,\cdots,X_m\}$ and $Y\in\{Y_1,Y_2,\cdots,Y_m\}$ separately denote two groups of nodes in a Bayesian network. A causal relationship between these two groups is denoted as $X\to Y$. By saying a causal relationship, we mean that the occurrence of the events denoted by the group of nodes Y depend on the occurrence of the events denoted by X. Mathematically, a conditional probability table M which is presented in the form of matrix is defined as follow:

$$M_{y|x}=p\left(y|x\right)=\begin{bmatrix} p(Y_1|X_1) & p(Y_2|X_1) & \cdots & p(Y_n|X_1) \\ p(Y_1|X_2) & p(Y_2|X_2) & \cdots & p(Y_n|X_2) \\ \vdots & \vdots & \ddots & \vdots \\ p(Y_1|X_m) & p(Y_2|X_m) & \cdots & p(Y_n|X_m) \end{bmatrix}$$

Based on the plausible inference method introduced by Pearl [10], the posterior distribution $P\left(X|E\right)$ of any non-evidence node X is defined as a strength of belief $Bel\left(X\right)$, where E denotes the available evidence. The strength of belief is generally calculated by integrating two types of quantified information, that are the quantified prior information and the quantified information provided by the evidence. Let $\delta\left(X\right)$ and $\pi\left(X\right)$ separately denote these two kinds of quantified information, the strength of belief in the node X is given as follows [4]:

$$Bel\left(X\right)=\phi\delta\left(X\right)\pi\left(X\right) \tag{12}$$

where the quantified information described by $\delta\left(X\right)$ and $\pi\left(X\right)$ are separately obtained from the relative conditional matrix of nodes that have causal relationships with the node X, which are given as follow:

$$\begin{cases} \delta\left(X\right)=\prod_j \delta_j\left(X\right) \\ \pi\left(X\right)=\sum_{U_1,U_2,\cdots,U_n} P\left(X|U_1,U_2,\cdots,U_n\right)\prod_i \pi_X\left(U_i\right) \end{cases} \tag{13}$$

a) $\delta_j\left(X\right)$: quantified information of the jth child node
b) $P\left(X|U_1,U_2,\cdots,U_n\right)$: conditional probability of the node variable X in the parent node set $\{U_i\}$
c) $\pi_X\left(U_i\right)$: quantified information of the parent node

Based on the derivation above, when new evidence e is available, we can obtain the updated strength of belief in X as below:

$$\begin{cases} Bel'\left(X\right)=\phi\delta Bel\left(X\right) \\ \phi=\left[\delta Bel(X)-Bel(X)+1\right]^{-1},\delta=P\left(e|X\right)\big/P\left(e|\bar{X}\right) \end{cases} \tag{14}$$

(2) Information fusion

Based on the inference procedure of Bayesian network presented above, we adopt a tree-like Bayesian network to build the risk assessment model as shown in Fig. 2.

As presented in Table 2, for a subsystem, the risk level that this subsystem belongs to is categorized into three categories. As a result, the probabilities that this subsystem belongs to these three risk catego-

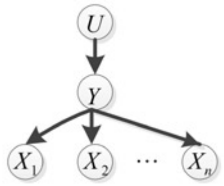

Fig. 2. The tree-like Bayesian network

ries correlate with each other. Let $S=\{a,b,c\}$ denote the risk levels of a subsystem and $\{P_a,P_b,P_c\}$ with $P_a+P_b+P_c=1$ are the corresponding probabilities. The change of the probabilities *b* and *c* will affect the probability of *a*. Since the probabilities of the risk levels affect each other, we have to consider the information fusion problem. We construct an updating model for information fusion based on tree-like Bayesian network for risk assessment which is shown in Fig. 3 [11].

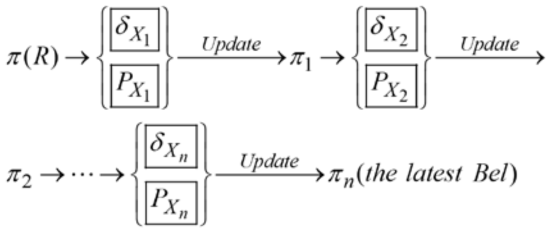

Fig. 3. Updating model for information fusion

3. Reliability and risk assessment of electronic system

3.1. Calculate the weight and reliability of relay system

A large amount of relays are used in the aircraft electronic systems. We can easily get the hierarchical classification of the relays according to different attributes of the relays. In this case study, we establish three hierarchies as shown in Fig. 4 [6].

Based on the procedure presented by Eq. (1) - Eq. (9), we obtain the judgment matrixes and weights for different attributes considered for the risk analysis of the delays. These matrixes and weights are shown in Table 3 – Table 8.

To compare the results between the IAHP and the AHP, the weights of attributes are obtained using the AHP. These weights are obtained using the method introduced by Yuan [18]. We denote these weights as ∇ and presented in Table 9.

A pictorial description of the comparison of the weight between the AHP and IAHP is presented in Figure 5.

Then we separately calculate the standard deviation of these two groups of weight and denote them as σ_i and $\bar{\sigma}_i$. A comparison of the standard deviation of the weights are given in Table 10.

From Table 10, we can figure out that the standard deviation $\bar{\sigma}_i$ of each weight obtained by the IAHP is smaller than the corresponding σ_i using AHP.

Table 3. Judgment matrix and weight of attribute U

	U_1	U_2	U_3	U_4	U_5
U_1	[1.0,1.0]	[2.2,2.7]	[1.2,1.8]	[2.0,6.7]	[1.0,1.7]
U_2		[1.0,1.0]	[0.6,1.1]	[0.5,0.9]	[1.0,1.5]
U_3			[1.0,1.0]	[0.3,0.7]	[0.9,1.6]
U_4				[1.0,1.0]	[1.7,5.0]
U_5					[1.0,1.0]
$\tilde{x}^-=(0.3400,0.1470,0.1607,0.2135,0.1386)$					$\alpha=0.8349$
$\tilde{x}^+=(0.3783,0.1158,0.1393,0.2402,0.1255)$					$\beta=1.0862$
$\omega_1=[0.2838,0.4109],\omega_2=[0.1227,0.1257],\omega_3=[0.1341,0.1513],\omega_4=[0.1782,0.2609],\omega_5=[0.1157,0.1363]$.					
$\bar{\omega}_1=0.3473,\bar{\omega}_2=0.1242,\bar{\omega}_3=0.1427,\bar{\omega}_4=0.2195,\bar{\omega}_5=0.1260$.					

Table 4. Judgment matrix and weight of attribute U_1

	u_{11}	u_{12}	u_{13}
u_{11}	[1.0,1.0]	[1.4,3.3]	[2.0,6.7]
u_{12}	[0.3,0.7]	[1.0,1.0]	[1.0,1.7]
u_{13}	[0.15,0.5]	[0.6,1.0]	[1.0,1.0]
$\tilde{x}_1^-=(0.5562,0.2625,0.1813)$			$\alpha=0.8658$
$\tilde{x}_1^+=(0.6047,0.2259,0.1692)$			$\beta=1.1283$
$\omega_{11}=[0.4815,0.6822],\omega_{12}=[0.2272,0.2548],\omega_{13}=[0.1569,0.1909]$			
$\bar{\omega}_{11}=0.5818,\bar{\omega}_{12}=0.2410,\bar{\omega}_{13}=0.1739$.			

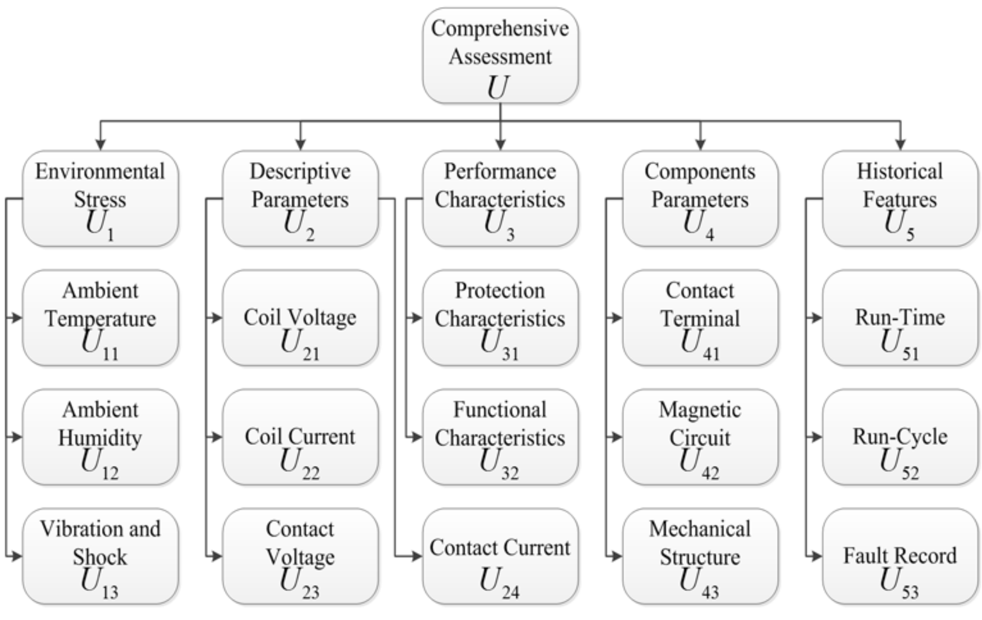

Fig. 4. Attributes hierarchy of a relay

Table 5. Judgment matrix and weight of attribute U_2

	u_{21}	u_{22}	u_{23}	u_{24}
u_{21}	[1.0,1.0]	[0.6,1.2]	[0.6,1.2]	[1.0,1.5]
u_{22}		[1.0,1.0]	[0.6,1.2]	[1.0,1.7]
u_{23}			[1.0,1.0]	[0.6,1.2]
u_{24}				[1.0,1.0]
$\tilde{x}_2^- = (0.2448, 0.2649, 0.2533, 0.2369)$				$\alpha = 0.8806$
$\tilde{x}_2^+ = (0.2370, 0.2661, 0.2680, 0.2287)$				$\beta = 1.1128$
$\omega_{21} = [0.2155, 0.2637], \omega_{22} = [0.2332, 0.2961], \omega_{23} = [0.2230, 0.2982], \omega_{24} = [0.2086, 0.2544].$				
$\varpi_{21} = 0.2396, \varpi_{22} = 0.2646, \varpi_{23} = 0.2606, \varpi_{24} = 0.2315.$				

Table 6. Judgment matrix and weight of attribute U_3

	u_{31}	u_{32}
u_{31}	[1.0,1.0]	[2.2,2.7]
u_{32}		[1.0,1.0]
$\tilde{x}_3^- = (0.7090, 0.2909)$		$\alpha = 0.9787$
$\tilde{x}_3^+ = (0.7090, 0.2909)$		$\beta = 1.0209$
$\omega_{31} = [0.6938, 0.7238], \omega_{32} = [0.2847, 0.2969].$		
$\varpi_{31} = 0.7088, \varpi_{32} = 0.2908.$		

Table 7. Judgment matrix and weight of attribute U_4

	u_{41}	u_{42}	u_{43}
u_{41}	[1.0,1.0]	[0.6,1.0]	[0.5,0.8]
u_{42}		[1.0,1.0]	[0.8,1.7]
u_{43}			[1.0,1.0]
$\tilde{x}_4^- = (0.2668, 0.3690, 0.3641)$			$\alpha = 0.8987$
$\tilde{x}_4^+ = (0.2508, 0.3847, 0.3643)$			$\beta = 1.0952$
$\omega_{41} = [0.2497, 0.2846], \omega_{42} = [0.3316, 0.4213], \omega_{43} = [0.3272, 0.3989].$			
$\varpi_{41} = 0.2706, \varpi_{42} = 0.3664, \varpi_{43} = 0.3630.$			

Table 8. Judgment matrix and weight of attribute U_5

	u_{51}	u_{52}	u_{53}
u_{51}	[1.0,1.0]	[2.0,2.9]	[1.7,5.0]
u_{52}		[1.0,1.0]	[1.2,1.8]
u_{53}			[1.0,1.0]
$\tilde{x}_5^- = (0.5572, 0.2684, 0.1742)$			$\alpha = 0.9046$
$\tilde{x}_5^+ = (0.5809, 0.2279, 0.1910)$			$\beta = 1.0886$
$\omega_{51} = [0.5040, 0.6323], \omega_{52} = [0.2427, 0.2480], \omega_{53} = [0.1575, 0.2079].$			
$\varpi_{51} = 0.5683, \varpi_{52} = 0.2453, \varpi_{53} = 0.1827.$			

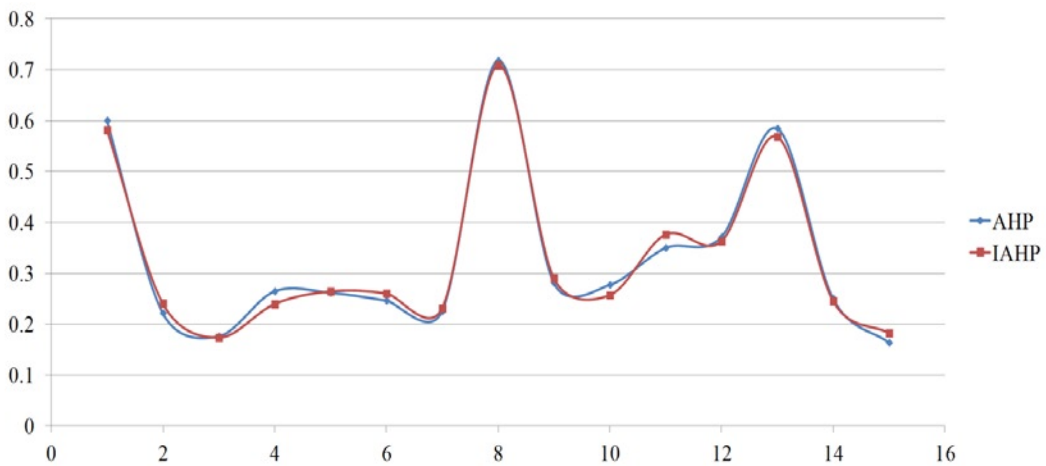

Fig. 5. Comparison of weights obtained by AHP and IAHP

Table 9. Weights obtained using traditional AHP and IAHP

AHP	∇_1	∇_2	∇_3	∇_4	∇_5
	0.3255	0.1362	0.1389	0.2698	0.1296
	$\nabla_{11}=0.6008$ $\nabla_{12}=0.2223$ $\nabla_{13}=0.1769$	$\nabla_{21}=0.2651$ $\nabla_{22}=0.2623$ $\nabla_{23}=0.2467$ $\nabla_{24}=0.2259$	$\nabla_{31}=0.7183$ $\nabla_{32}=0.2817$	$\nabla_{41}=0.2779$ $\nabla_{42}=0.3503$ $\nabla_{43}=0.3718$	$\nabla_{51}=0.5851$ $\nabla_{52}=0.2508$ $\nabla_{53}=0.1641$
IAHP	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_4$	$\bar{\omega}_5$
	0.3473	0.1242	0.1427	0.2195	0.1260
	$\bar{\omega}_{11}=0.5718$ $\bar{\omega}_{12}=0.2410$ $\bar{\omega}_{13}=0.1739$	$\bar{\omega}_{21}=0.2396$ $\bar{\omega}_{22}=0.2646$ $\bar{\omega}_{23}=0.2606$ $\bar{\omega}_{24}=0.2315$	$\bar{\omega}_{31}=0.7088$ $\bar{\omega}_{32}=0.2908$	$\bar{\omega}_{41}=0.2706$ $\bar{\omega}_{42}=0.3664$ $\bar{\omega}_{43}=0.3630$	$\bar{\omega}_{51}=0.5683$ $\bar{\omega}_{52}=0.2453$ $\bar{\omega}_{53}=0.1827$

Table 10. The standard deviation of the weight for each attribute using AHP and IAHP

AHP	$\sigma_1=0.2327$	$\sigma_2=0.0180$	$\sigma_3=0.3087$	$\sigma_4=0.0588$	$\sigma_5=0.2223$
IAHP	$\bar{\sigma}_1=0.2187$	$\bar{\sigma}_2=0.0160$	$\bar{\sigma}_3=0.2956$	$\bar{\sigma}_4=0.0544$	$\bar{\sigma}_5=0.2069$

Table 11. The interval-number comprehensive weight C_{ii}

C_{11}	[0.1366,0.2803]	C_{12}	[0.0645,0.1047]	C_{13}	[0.0445,0.0784]
C_{21}	[0.0264,0.0331]	C_{22}	[0.0286,0.0372]	C_{23}	[0.0274,0.0375]
C_{24}	[0.0256,0.0320]	C_{31}	[0.0930,0.1095]	C_{32}	[0.0382,0.0449]
C_{41}	[0.0427,0.0716]	C_{42}	[0.0591,0.1099]	C_{43}	[0.0583,0.1041]
C_{51}	[0.0583,0.0862]	C_{52}	[0.0281,0.0338]	C_{53}	[0.0182,0.0283]

Table 12. The normalized comprehensive weight $\bar{C}_{ii}$

$\bar{C}_{11}$	0.2084	$\bar{C}_{12}$	0.0846	$\bar{C}_{13}$	0.0614
$\bar{C}_{21}$	0.0297	$\bar{C}_{22}$	0.0658	$\bar{C}_{23}$	0.0324
$\bar{C}_{24}$	0.0288	$\bar{C}_{31}$	0.1012	$\bar{C}_{32}$	0.0415
$\bar{C}_{41}$	0.0571	$\bar{C}_{42}$	0.0845	$\bar{C}_{43}$	0.0812
$\bar{C}_{51}$	0.0722	$\bar{C}_{52}$	0.0309	$\bar{C}_{53}$	0.0232

Table 13. The allocated results of subsystem reliability

λ_{11}	0.0029	λ_{12}	0.0072	λ_{13}	0.0099	λ_{21}	0.0052	λ_{22}	0.0047
λ_{23}	0.0048	λ_{24}	0.0054	λ_{31}	0.0058	λ_{32}	0.0142	λ_{41}	0.0084
λ_{42}	0.0057	λ_{43}	0.0059	λ_{51}	0.0031	λ_{52}	0.0072	λ_{53}	0.0097
R_{11}	0.8413	R_{12}	0.6511	R_{13}	0.5543	R_{21}	0.7335	R_{22}	0.7557
R_{23}	0.7512	R_{24}	0.7248	R_{31}	0.7078	R_{32}	0.4290	R_{41}	0.6062
R_{42}	0.7120	R_{43}	0.7036	R_{51}	0.8313	R_{52}	0.6511	R_{53}	0.5610

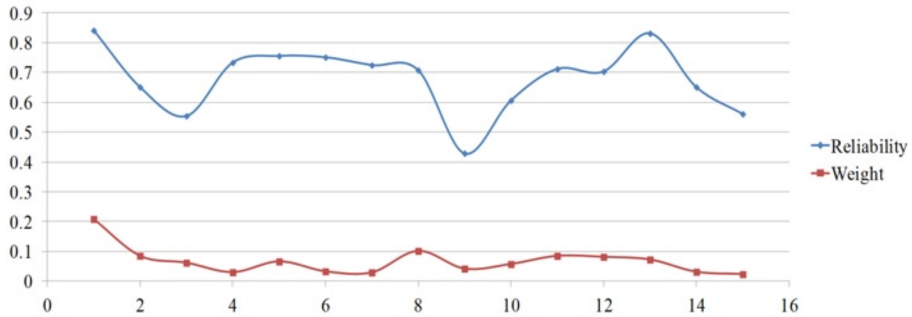

Fig. 6. Comparisons of weight and reliability of subsystems

The IAHP is then demonstrated more credible and better than the AHP for weight derivation. The IAHP is then chosen when we carry out the risk assessment of sub-systems.

According to the Eq. (5) and Eq. (9), we have the interval-number comprehensive weight C_{ii} and the normalized comprehensive weight $\overline{C}_{ii}$ for each unit in the whole system. They are separately given in Table 11 and Table 12.

If the failure rate of the system is $\lambda = 0.02$, we can get the reliability of each subsystem through Eq. (10) and Eq. (11). The results are shown in Table 13.

Based on the weight and reliability of subsystems, we can compare the difference of subsystems through their weight and reliability. A pictorial description of the comparison is given in Figure 6.

3.2. Risk assessment using Bayesian network

In this case study, we consider two aspects and three factors that affect the risk level of the system. The relationships between these aspects and factors for risk level of the system is shown in Fig. 7.

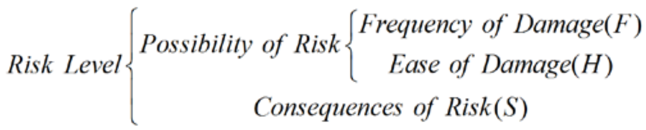

Fig. 7. The relationship of factors for the risk level of the system

And then we present the Bayesian network model for risk assessment. It is shown in Fig. 8.

Fig. 8. The Bayesian network model for risk assessment

We define the following fuzzy subsets and their probabilities in Table 14 for the factors presented in Figure 7.

Then we can approximately get the matrix of conditional probability distributions based on inference rule introduced above and presented in Table 2. The results are shown in Table 15.

Based on the condition probability matrixes, we can get the prior information π_{ii} for each sub-system at the time point t=60. The results are given in Table 16.

By utilizing the updating model for information fusion presented in Figure 3, we can assess the risk level of each sub-system. Take the sub-system u_{13} as an example. We know that the prior information about the risk level of this sub-system u_{13} is identified as $\pi_{13} = (0.3, 0.4, 0.3)$ at the time point $t = 60$ in Table 16. The results of information fusion and risk assessment is obtained and shown in Table 17.

In addition, we can derive the priori information of the sub-system u_{32} when $t = 160$, which is $\pi_{32} = (0.8, 0.1, 0.1)$. Similarly, the results of information fusion and risk assessment at this time point is obtained and shown in Table 18.

According to Fig. 5 and Fig. 6, we find that the failure rate of a susbsystem is inversely proportional to its weight under the corresponding criterion. Accordingly, we can figure out the fault location based on the weights of the subsystems, which is the subsystem with the smallest weight is suspect to give rise to the highest risk.

In addition, by comparing the results given in Table 17 and Table 18, we can conclude that the risk assessment is affected greatly by the prior information of subsystems obtained through the IAHP at the beginning. However, as the progression of the risk assessment, the effect of prior information is gradually mitigated by the incorporation of evidence information. For example, the probabilities of risk levels obtained in the first simulation are close to the probabilities derived from the historical information $\pi(R)$. This is because that an informative prior is incorporated in the first simulation. It is given as that if the probability of high risk level is much larger than other risk levels, the system/subsystem will experience a high risk. As the progression of risk assessment, the results of risk assessment are updating gradually. The probability that the system/subsystem will experience a high risk is gradually decreasing. This is due to the effect that the information provided by the evidences is incorporated gradually. As a result, this assessment method can not only incorporate the evidence information transmitted through the Bayesian network, but also synthesize all the historical results derived through the IAHP.

4. Conclusions

This paper introduces a method for reliability and risk assessment of complex system. The issue of epistemic uncertainty introduced by the incorporation of subject judgment is investigated by adopting the IAHP. The problem of information fusion restulted from the handling of multiple risk factors is studied by utilizing the Bayesian network. The reliability and the weights of subsytems are obtained using the IAHP under the idea of reliability allocation. The reliability and weights are then further incoporated as prior information for the risk assessment of the system. The risk assessment is implemented by combining the prior information and the evidence information through the Bayesian network. Finally, we demonstrate the proposed method through the risk assessment of the aircraft electronic system in various conditions. The results demonstrate that the proposed method can not only incorporate the evidence information, but also synthesize all the historical information.

By utilizing the IAHP and the Bayesian method, the proposed method can take full account of the uncertain factors that affect the fault location. The results obtained using the Bayesian method is consistent with the actual failure data. That is to say, it can accurately

Table 14. Fuzzy subsets and event probabilities

	Set	Probability
Safety and Risk Level	A={high middle low}	[0~0.3,0.3~0.6,0.6~1]
Frequency of Damage	F={few middle many}	[0~0.3,0.3~0.6,0.6~1]
Ease of Damage	H={difficult middle easy}	[0~0.3,0.3~0.6,0.6~1]
Consequence of Risk	S={great middle little}	[0~0.3,0.3~0.6,0.6~1]

Table 15. Conditional probability matrix

Safety and Risk Level	F={few middle many}	H={difficult middle easy}	S={great middle little}
High *Middle* *Low*	$\begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.4 & 0.4 & 0.2 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.4 & 0.3 & 0.3 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.6 & 0.3 & 0.1 \\ 0.1 & 0.45 & 0.45 \end{bmatrix}$

Table 16. The prior information π_{ii} of each sub-system

π_{11}	(0.1,0.1,0.8)	π_{12}	(0.15,0.15,0.7)	π_{13}	(0.3,0.4,0.3)	π_{21}	(0.15,0.15,0.7)	π_{22}	(0.1,0.1,0.8)
π_{23}	(0.1,0.1,0.8)	π_{24}	(0.15,0.15,0.7)	π_{31}	(0.15,0.15,0.7)	π_{32}	(0.2,0.6,0.2)	π_{41}	(0.3,0.4,0.3)
π_{42}	(0.15,0.15,0.7)	π_{43}	(0.15,0.15,0.7)	π_{51}	(0.1,0.1,0.8)	π_{52}	(0.15,0.15,0.7)	π_{53}	(0.3,0.4,0.3)

Table 17. The results of information fusion and risk assessment of sub-system u_{13} at time point $t = 60$

Serial number	δ_F	δ_H	δ_S	Bel
1	[0,0,1]	[0,1,0]	[1,0,0]	[0.8486,0.1454,0.0060]
2	[0.4,0.3,0.3]	[0.8,0.1,0.1]	[1,0,0]	[0.2413,0.6066,0.1221]
3	[0.1,0.8,0.1]	[0,0.2,0.8]	[0,0,1]	[0.2339,0.2650,0.5011]

Table 18. The results of information fusion and risk assessment of sub-system u_{32} when $t = 160$

Serial number	δ_F	δ_H	δ_S	Bel
1	[0,0,1]	[0,1,0]	[1,0,0]	[0.9826,0.0164,0.0010]
2	[0.4,0.3,0.3]	[0.8,0.1,0.1]	[1,0,0]	[0.7902,0.1653,0.0445]
3	[0.1,0.8,0.1]	[0,0.2,0.8]	[0,0,1]	[0.7304,0.0765,0.1931]

identify the fault location and assess the risk level of the subsystems. The accurate fault location and effective risk assessment can greatly facilitate the reliability improvement of the electric systems. In addition, there are some aspects that deserve further investigation. We will continue to investigate the risk assessment by incorporating other scales in the IAHP. In addition, the implication of the proposed method to other engineering system is of interest for further investigation.

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