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Reliability assessment of complex electromechanical systems under epistemic uncertainty

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ABSTRACT

The appearance of macro-engineering and mega-project have led to the increasing complexity of modern electromechanical systems (EMSs). The complexity of the system structure and failure mechanism makes it more difficult for reliability assessment of these systems. Uncertainty, dynamic and nonlinearity characteristics always exist in engineering systems due to the complexity introduced by the changing environments, lack of data and random interference. This paper presents a comprehensive study on the reliability assessment of complex systems. In view of the dynamic characteristics within the system, it makes use of the advantages of the dynamic fault tree (DFT) for characterizing system behaviors. The lifetime of system units can be expressed as bounded closed intervals by incorporating field failures, test data and design expertize. Then the coefficient of variation (COV) method is employed to estimate the parameters of life distributions. An extended probability-box (P-Box) is proposed to convey the present of epistemic uncertainty induced by the incomplete information about the data. By mapping the DFT into an equivalent Bayesian network (BN), relevant reliability parameters and indexes have been calculated. Furthermore, the Monte Carlo (MC) simulation method is utilized to compute the DFT model with consideration of system replacement policy. The results show that this integrated approach is more flexible and effective for assessing the reliability of complex dynamic systems.

1. Introduction

The large complex electromechanical systems (EMSs) have been widely used in aviation and aerospace industry, electric power system, civil machinery, etc. The complexity of system structure and formidable manufacturing cost has limited the system level reliability tests of those EMSs. It also makes the system reliability indexes evaluation become infeasible, since a long time reliability test and large quantities of statistical data are needed. In practice, only a small amount of experimental data, field data and engineering experience information are available, which makes it almost impossible to evaluate the lifetime and reliability of system through data analysis. Furthermore, the uncertainties caused by the lack of data or knowledge, and the dynamic behavior also affect the reliability of the EMS. Therefore, there is a strong requirement to take a series of technological means to evaluate the reliability indexes of EMS, and a comprehensive reliability assessment of the entire system must be performed. Furthermore, the consideration of dynamic uncertainty and maintainability of system or components is a crucial issue to be resolved for the assessment of the reliability of complex EMS.

Reliability assessment [1–3] is implemented through the design, testing, production, storage and usage phases of a product or system, it is a process of analyzing and confirming the reliability of system and its components. It is also a qualitative and quantitative analysis technique to model and predict system reliability throughout the product lifecycle. There are basically four aspects of technical contents of system reliability assessment, including reliability modeling, reliability data collection and processing, unit reliability assessment and system reliability synthesis.

To obtain the reliability of a complex EMS, the reliability model should be built to describe the failure logic relationship between the whole system and its compositions. In recent decades, various reliability modeling methods have been developed for complex systems and the accuracy of the models is improved. Some classical static modeling techniques, including reliability block diagram model, fault tree (FT) model, and binary decision diagrams (BDD) model, have been widely used to model static systems. While considering the complexity of modern EMSs, the dynamic modeling techniques such as Markov model [4], dynamic fault tree (DFT) model [5], and Petri net model [6] have been applied for reliability modeling. DFT analysis method, first proposed by Dugan

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et al. [7,8], is a mature and important method in reliability analysis of dynamic systems. Since then many variations have been published. As such, three criterions are given to rank basic events of DFT, and a transforming method of logic gate is put forward by Hao et al. in [9]. Approximate DFT calculations are presented by Lindhe et al. [10] based on a Markovian approach, which was used for water supply risk modeling and performed by standard Monte Carlo (MC) simulations. Considering the interactive repeated events in different dynamic gates, Merle et al. [11] proposed a new analytic method to solve DFTs with priority dynamic gate and repeated event. An improved sequential BDD method was proposed for quantitative analysis of DFT with interactive repeated events by Ge et al. [12]. Considering the state explosion and computational efficiency problems in DFT model, Mo [13] proposed a multi-value, decision-diagram-based DFT analysis method to analyze the reliability of large dynamic system. A MC-based approach was investigated by Rao et al. [14] to solve dynamic gates, which can be used to alleviate the state space explosion problem. To overcome the limitations caused by the increasing size of FTs in traditional reliability assessment, Chiacchio et al. [15] proposed a Weibull-based composition approach for large DFT to reduce the computational effort. In view of the dynamic characteristic in modern complex EMS, and taking advantage of the dynamic modeling ability of DFT, a DFT model should be built on the basis of system structure and failure behaviors.

As an inheritor of FT, Bayesian network (BN) has a similar state description and reasoning pattern with FT method. It also has the advantage of dealing with multi-state modeling and non-deterministic fault logic representation [16]. BN is a directed acyclic graph (DAG) for system modeling, which is a mathematical model based on probability reasoning [17]. It was first proposed by Pearl [18,19] and has been widely used in reliability and safety analysis. Cai et al. [20] proposed a BN-based approach for reliability evaluation of redundant systems including parallel systems and voting systems by taking account of common cause failures and imperfect coverage. Khakzad et al. [21] presented a new formalism to model cold spare (CSP) gates and sequential enforcing gates with various types of probability distribution functions. Under this formalism, the discrete-time BNs were applied in risk assessment and safety analysis of complex process systems. In [22], BN was used to solve the Pandora temporal FTs, which is a dynamic analysis technique that can capture the sequence-dependent dynamic behavior of system. To overcome the shortcomings of traditional fault tree analysis method, a systemic decision approach was presented in [23] by integrating the predictive, sensitivity and diagnostic analysis techniques in DBN inference. BN has been used for reliability analysis of complex systems with various kinds of uncertainties, and it has an advantage to facilitate the estimation of system reliability by coping with system complexity. Su and Fu [24] presented a causal logic method for qualitative modeling of the BN reliability model of wind turbine when considering the environmental factors and uncertainty.

Except for the nonlinear dynamic characteristic, the uncertainty existing in complex EMS is always another important issue which cannot be ignored. There are basically two types of uncertainties, aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty arises from intrinsic variability and is irreducible, which is also called objective uncertainty. It can be described and propagated by probability theory. Epistemic uncertainty results from incompleteness of knowledge or lack of data, it is called subjective uncertainty [25,26]. Alternative theories and methods have been proposed to represent epistemic uncertainty, such as interval theory [27], evidence theory [28,29], possibility theory [30], info-gap theory [31], random sets [32], fuzzy sets [33], Bayesian approaches [34], probability-box (P-Box) [35–42], etc. P-Box has an advantage for tackling the uncertain parameters without precise probabilistic models in view of epistemic uncertainties. A P-Box is specified by lower and upper (interval-type) bounds on the cumulative distribution functions (CDFs) of uncertain variables [35]. It has been widely used in reliability and risk analysis and is suitable for diverse engineering fields. To build the connection of P-Box with other uncertainty representations, a generalized form of P-Boxes was defined by Destercke et al. [36]. Bayesian P-Boxes are used by Montgomery [37] for risk assessment with multiple parameters distributions. Zhang et al. [38] proposed an interval MC method. An interval importance sampling method [39] and an interval quasi-MC simulation method [40] in structure reliability analysis with the parameter uncertainties modeled by P-Boxes have been investigated also. P-Box was applied as a visual tool by Mehl [41] for cost uncertainty analysis. Furthermore, Yang et al. [42] studied the hybrid reliability analysis under both aleatory and epistemic uncertainty cohere random variables and P-Box variables. To reduce the impact of uncertainties on systems, the representation and quantification of uncertainty need to be addressed first. For the epistemic uncertainty induced by incomplete data, based on the definition of P-Box, an extended parametric P-Box to is proposed to represent the uncertainty in complex EMSs.

Moreover, the maintainability or reparability of components can improve the reliability of whole system. When the replacement policies are considered in EMSs with repairable components, the MC simulation method can be used to compute the reliability model. MC simulation method is a widely used method in reliability analysis of complex systems. By using MC simulation method, the system reliability can be calculated and the effect of a variety of related factors (system reparability, dependency, etc.) to system reliability can also be evaluated. In this regard, Taheriyoun and Moradinejad [43] have combined the FT analysis method with MC simulation method for reliability analysis of waste water treatment plant. Manno et al. [44] presented a MC-based high level modeling framework that is integrated with FT method for reliability assessment of complex system with time dependencies. They also gave a definition of repairable DFT in [45], and a Matlab toolbox named RAATSS was presented based on adaptive transitions system formalism. This can be used to model and evaluate occurrence probability of top event when both repairable and non-repairable subsystems are considered. On the basis of structure function of DFTs, Merle et al. [46] proposed a quantitative analysis method based on MC simulation. The structure function was exploited, and the minimal cut sequences (MCSQs) can also be determined by this MC-based method. A MC DFT method was proposed by Zhang et al. [47] to analyze the reliability indexes of phasor measurement unit.

When considering the dynamic characteristics, uncertainty and maintainability of a system or components, a comprehensive reliability assessment process of complex EMSs has been proposed in this paper. The rest of this paper is organized as follows. In Section 2, a BN reliability modeling process is introduced based on DFT model, and the lifetime of dynamic logic gates is defined. The lifetime distribution analysis has been performed based on the coefficient of variation (COV) method. Section 3 constructs a MC simulation-based framework for lifetime and reliability assessment of complex EMSs with consideration of system reparability. An extended parametric P-Box is defined to present the epistemic uncertainty in Section 4. A case study of an EMS has been presented to demonstrate the effectiveness of the suggested reliability assessment framework in Section 5. Finally, conclusions are made in Section 6.

2. A lifetime evaluation approach based on BN for complex EMS

2.1. BN reliability modeling

Bayesian network is a DAG in which nodes represent random variables and edges between the nodes represent the direct dependency relationships among the variables [18–21]. In this paper, the event-based BN reliability model proposed by Marquez, Neil and Fenton [48–50] is employed to calculate the lifetime of system. In this model, the lifetime of components, basic events and logic gates of FT are represented by continuous nodes. Then the nodes are connected by incoming arcs to next events layer. This kind of BN has been defined as continuous time BN by Hofmann [51] et al., then Boudali and Dugan have given a more accurate definition of continuous time BN [52]. By using the chain rule of probability, the joint probability distribution $f(x)$ of a set of continuous variables $X_1, \cdots, X_N$ will be

$$f(x_1, \cdots, x_N) = \prod_{i=1}^{N} f(x_i \mid pa(X_i)), \tag{1}$$

where $pa(X_i)$ are the parent variables of nodeX_i.

In this paper, BN takes a DFT-like modeling form for a dynamic system. By addressing the issue of traditional FT analysis method which cannot model the sequence correlation of system, Dugan et al. [7,8] proposed a novel DFT analysis method to overcome the limitation. Dynamic logic gates are defined to describe the sequential rules and random failure behaviors of the systems. There are three kinds of commonly used dynamic gates: the priority AND (PAND) gate, the functional dependency (FDEP) gate and the spare (SP) gates [9,13,14]. Based on the definition and failure mechanisms of dynamic gates, the lifetime logic relationships between the inputs and the outputs of the basic static logic gates and commonly used dynamic logic gates of a DFT are given as follows [48–54]:

(1) AND gate: When all the input events or components X_i ($i=1, \ldots, n$) of an AND gate fail, the output of the gate fails, and the lifetime of component X_i is denoted by T_i. Then, the time-to-failure of the AND gate can be defined as a random variable

$$T_{AND} = \max_{T_i}\{T_i\}. \tag{2}$$

The failure probability of the output of the AND gate in time interval (0, t) is

$$\begin{aligned} F_{\mathrm{AND}}(t) &= P(T_{AND} \le t) = P(T_1 \le t, \ldots, T_n \le t) \\ &= P\left(\max_{T_i}\{T_i\} \le t\right). \end{aligned} \tag{3}$$

(2) OR gate: Because the output of an OR gate fails when at least one of the inputs fails, the lifetime of the OR gate can then be defined by

$$T_{OR} = \min_{T_i}\{T_i\}. \tag{4}$$

The failure probability of the output of the OR gate is,

$$\begin{aligned} F_{OR}(t) &= P(T_{OR} \le t) = 1 - P(T_1 > t, \ldots, T_n > t) \\ &= P\left(\min_{T_i}\{T_i\} \le t\right). \end{aligned} \tag{5}$$

(3) PAND gate:The PAND gate is an AND gate, whose output event occurs only when the input events occur sequentially from left to the right. The input of PAND gate could be basic events or output events of other logic gates. It integrates the logical sequence or predefined order of inputs on the basis of AND gate. Therefore, for a PAND gate with two input components X_1 and X_2, the lifetime of PAND gate can be defined as,

$$T_{PAND} = \begin{cases} T_2 & \text{if } T_1 \le T_2 \\ \infty & \text{otherwise} \end{cases}. \tag{6}$$

The lifetime distribution of output of PAND gate is

$$F_{PAND}(t) = P(T_{PAND} \le t) = P(T_1 \le T_2 \le t). \tag{7}$$

(4) SP gate:The SP gates are defined to model the system with spares whose failure criteria cannot be expressed by combinations of system basic events. It is composed of a primary component and several spares with the same function. When the failure rate of primary component is λ, then the failure rate of spare can be denoted as $\alpha\lambda$. Based on the different failure mechanism of spare components, the SP gate can be divided into cold SP ($\alpha=0$), hot SP ($\alpha=1$) and warm SP ($0<\alpha<1$).

1) CSP gate: When ignoring the storage lifetime of the standby units and the switching time of working subsystems, lifetime output of CSP gate T_{CSP} equals to the sum of the primary unit lifetime T_{pr} (for primary unit X_{pr}) and backup units lifetimes T_i (for spare units X_i when in active mode and $i=1,\ldots,n$). Hence, we have $T_{CSP}=T_{pr}+T_1+,\ldots,+T_n$. As such the failure probability for the output of CSP is given as

$$\begin{aligned} F_{CSP}(t) &= P(T_{CSP} \le t) = P\left(T_{pr} + T_1 +, \ldots, + T_n \le t\right) \\ &= F_{pr}(t) * F_1(t) * \cdots * F_n(t). \end{aligned} \tag{8}$$

This formula means the lifetime of CSP gate is equal to the convolution of the lifetime of the primary unit $F_{pr}(t)$ and backup units $F_i(t)$.

2) Warm spare (WSP) gate: For a warm standby system with one primary unit X_{pr} and a spare X_{sp}, the failure rate of spare unit in normal working condition is larger than that of standby mode. Denote T_{pr} as the lifetime of the primary unit, T_{sp} and T'_{sp} are the lifetimes of spare unit in active mode and in standby mode respectively. Therefore, the lifetime of this warm standby system can be obtained by,

$$T_{WSP} = \begin{cases} T_{pr} & \text{if } T'_{sp} \le T_{pr} \\ T_{pr} + T_{sp} & \text{if } T'_{sp} > T_{pr} \end{cases}. \tag{9}$$

This equation illustrates that the lifetime of this system equals to the lifetime of the primary unit T_{pr} when the spare unit fails before the primary unit fails, i.e., $T'_{sp} \le T_{pr}$. And the lifetime of system is $T_{pr}+T_{sp}$ when the primary unit fails before the standby units fail, i.e., $T'_{sp} > T_{pr}$.

3) Hot spare (HSP) gate: Because the HSP has the same failure logic with static AND gate, the lifetime T_{HSP} of a system with one hot spare unit equals to the maximum value of the backup unit and primary unit, and $T_{HSP}=\max(T_{pr}, T_{sp})$.

(5) FDEP gate: A FDEP gate consists of several state dependent basic events and one trigger input. The trigger input can be a basic event or an output event of another gate in the DFT. The basic events are forced to occur by the occurrence of trigger event, which means the basic events are functionally dependent on the trigger event. For non-repairable systems, the FDEP gate can be modeled using OR gates. The lifetime of system with FDEP gate can be computed by Eqs. (4) and (5).

In this logical framework, the marginal lifetime distributions for basic components or events can be defined by any parametric or empirical distribution. After the definitions of the lifetime and the failure probability of the static and dynamic logic gates, the reliability of a system at any mission time t with different kinds of logic gates can be expressed as an analytic closed form solution by the method in [49].

2.2. The lifetime distributions analysis based on COV method

Suppose the lifetimes of a system and components have been both expressed by random continuous variables, which means all the nodes of a BN are continuous time variables. The mean value $E(T)$ of a variable T is used to quantitatively describe the working hours of a system. The corresponding variance $Var(T)$ shows the discrete degree of a random variable. Variance is an absolute value and cannot be used to compare the uncertainty degree of mean lifetime in different situations. Therefore, the COV v is employed to describe the uncertainty degree of lifetime. The COV is defined as follows [55]

$$v(T)=\frac{\sqrt{Var(T)}}{E(T)}. \tag{10}$$

Vast use of this COV method in engineering practice shows that it is easy for engineering applications. When the reliable lifetime of system components or units is given, the COV method can be employed to assess the parameters of their lifetime distributions. For a component whose lifetime T obeys exponential distribution, the distribution function is $F(t;\lambda)$ and $F(t)=1-\exp\{-\lambda t\}$, where λ is the distribution parameter and also called the failure rate of this component. Let $\theta=1/\lambda$, which is the mean time between failure of a component whose lifetime is exponential distributed. $F(t;\lambda)$ is a monotonically increasing function with the mean value $E(T)$ equals to θ, and variance $Var(T)$ is θ^2. Then the COV v_{Exp} of exponential distribution can be defined as

$$v_{Exp}(T)=\frac{\sqrt{Var(T)}}{E(T)}=\frac{\sqrt{\theta^2}}{\theta}=1, \tag{11}$$

which means the uncertainty degree of exponential distribution is fixed. For lifetime variable that follows exponential distribution, the reliable lifetime t_R^{Exp} is given as [56],

$$t_R^{Exp}=\lambda^{-1}\ln R^{-1}, \tag{12}$$

where R $(0\le R\le 1)$ is system reliability. Then, when the reliable lifetime is known, the parameter λ can be computed by Eq. (12).

For two-parameter Weibull distribution $F(t;\beta,\eta)$, the distribution function is $F(t)=1-\exp\{-(t/\eta)^{\beta}\}$, where β is the shape parameter and η is the scale parameter. The mean value of Weibull distribution is $E(T)=\eta\Gamma(1+1/\beta)$ and variance is $Var(T)=\eta^2[\Gamma(1+2/\beta)-\Gamma^2(1+1/\beta)]$, where $\Gamma(\alpha)=\int_0^{\infty}x^{\alpha-1}e^{-x}dx$ $(\alpha>0)$ [57]. The COV v_{Wb} of the Weibull distribution is

$$\begin{aligned}v_{Wb}(T)&=\frac{\sqrt{Var(T)}}{E(T)}=\frac{\sqrt{\eta^2\left[\Gamma\left(1+2\beta^{-1}\right)-\Gamma^2\left(1+\beta^{-1}\right)\right]}}{\eta\Gamma\left(1+2\beta^{-1}\right)}\\&=\sqrt{\frac{\Gamma(1+2\beta^{-1})}{\Gamma^2(1+\beta^{-1})}-1}.\end{aligned} \tag{13}$$

The COV of Weibull distribution is decreasing with the increasing of shape parameter β, which means the uncertainty degree of random variable increases as β becomes smaller. For Weibull distribution, the reliable lifetime t_R, which is the lifetime of a system or a component when system reliability equals to R, is given by

$$t_R^{wb}=\eta(-\ln R(t))^{\frac{1}{\beta}}. \tag{14}$$

For logarithmic normal (lognormal) distribution $F(X;\mu,\sigma)$, μ is the location parameter (logarithmic mean) and σ is the shape parameter (logarithmic variance). They are also the mean value and standard variance of composite variable $X=\ln(t)$. Then the mean value and variance of primitive variable T is $E(T)=\exp\{\mu+\sigma^2/2\}$ and $Var(T)=(\exp\{\sigma^2\}-1)\exp\{2\mu+\sigma^2\}$ [57]. Therefore, the COV v_{Logn} of lognormal distribution can be written as

$$\begin{aligned}v_{Logn}(T)&=\frac{\sqrt{Var(T)}}{E(T)}=\frac{\sqrt{(e^{\sigma^2}-1)e^{2\mu+\sigma^2}}}{e^{\mu+\frac{\sigma^2}{2}}}\\&=\frac{\sqrt{(e^{\sigma^2}-1)e^{2\mu+\sigma^2}}}{\sqrt{e^{2\mu+\sigma^2}}}=\sqrt{e^{\sigma^2}-1}.\end{aligned} \tag{15}$$

This formula indicates that the COV of a random variable, which follows lognormal distribution, is only related to its logarithmic variance and increases with the increasing of logarithmic variance. This makes the evaluation of COV easier, since it is uncorrelated with logarithmic mean.

The relationship between parameters μ and σ with the reliable lifetime t_R can be represented as

$$t_R^{Logn}=\exp\{\mu-\sigma z_R\}, \tag{16}$$

where z_R is the R $(0\le R\le 1)$ quantile of the standardized normal distribution $N(0,1)$. It can be gotten by looking up the quantile table of standardized normal distribution.

In engineering practice, it is often difficult to obtain the accurate lifetime data of complex systems. However, by incorporating the field data, experimental data and the experience of engineers, it is always possible to get an approximate range of key components' lifetime. Accordingly, the service lifetime of a key component can be expressed as a bounded closed interval T^I on real number set $\mathbf{R}$, and

$$T^I=\left[T_L,T^U\right]=\left\{T\in\mathbf{R}:T_L\le T\le T^U\right\}, \tag{17}$$

where T is an interval variable, T_L and T^U are the lower and upper bounds respectively.

The mean value $\overline{T}$ and deviation T^r of interval number T^I are defined as follows [58],

$$\overline{T}=\frac{T_L+T^U}{2}\quad\text{and}\quad T^r=\frac{T^U-T_L}{2}. \tag{18}$$

Then the interval variable T^I can be rewritten as

$$T^I=\left[\overline{T}-T^r,\overline{T}+T^r\right]. \tag{19}$$

The COV of lifetime variable T can be represented as

$$\tilde{v}=\frac{T^r}{\overline{T}}=\frac{T^U-T_L}{T^U+T_L}. \tag{20}$$

Based on the lifetime data of a component, the COV of this component can be calculated by Eqs. (18)–(20). When the lifetime distribution and the reliable lifetime t_R are known, the distribution parameters can be computed by Eqs. (11)–(16).

3. A lifetime and reliability assessment method for repairable EMS based on MC simulation

After the FT modeling of a coherent system with several components and subsystems, a static FT or a DFT model is built. When the lifetime of components follows exponential distribution, Weibull distribution or lognormal distribution, a MC simulation-based method can be applied to simulate the FT model. The time-to-failures of all the Minimal Path Sets (MPSs) of the system FT are sampled using MC simulation method. Then the minimum value of all the lifetime of MPSs is regarded as a sampling lifetime value of the system. The reliability indexes of the system can be estimated through the statistical analysis of sampling records. This MC simulation-based method can dynamically characterize the reliability and failure feature of the system and its components.

3.1. The relationship between mean lifetime and MPSs of system

For a system S with n basic components X_i (i=1, 2,…,n), the system structure can be expressed as

$$S=\{X_1, X_2, \cdots, X_i, \cdots, X_n\}. \quad (21)$$

Suppose the distribution function of each component is $F_i(t)$ for i=1,2,…,n. The structure function is $\Phi[\mathbf{X}(t)]$, where $\mathbf{X}(t)$ is a vector composed by the performance state of basic components $x_i(t)$ for i=1, 2,…, n at time t, and is written as

$$\mathbf{X}(t)=\{x_1(t), x_2(t), \cdots, x_i(t), \cdots, x_n(t)\}, \quad (22)$$

where $x_i(t)=1$ when the ith basic component fails at time t, otherwise $x_i(t)=0$. Then the system performance state $\Phi[\mathbf{X}(t)]$ equals to 1 when the system works well at time t, otherwise $\Phi[\mathbf{X}(t)]$ equals to 0.

Using MC simulation method to sample data for each basic event, the sampling value of normal working time of the ith component is $t_i=F_i^{-1}(\eta)$. For a FT with m MPSs, the ith path set with v basic events is denoted by S_i ($1 \le i \le m$), in which each basic failure event is denoted as X_j ($1 \le j \le v$). Then path set S_i can be expressed as [59,60]

$$S_i=\prod_{j=1}^{v} X_j=\prod_{X_j \in S_i} X_j. \quad (23)$$

The system success event S can be expressed as the sum-of-product of events X_j, which can be represented by the following formula

$$S=S_1+S_2+\cdots+S_m=\sum_{i=1}^{m} S_i=\sum_{i=1}^{m}\left(\prod_{X_j \in S_i} X_j\right). \quad (24)$$

For a system with n basic components, using MC simulation method to sample the lifetime of each component, a set of lifetime sampling value can be obtained and the kth time of sampling for n components can be written as $(t_1^k, \cdots, t_n^k)$. Based on the definition of MPS, we can find that it represents a success state of the whole system, and the normal working of all basic events in this MPS will cause the success of this path set. This means the MPS will be failure only when at least one event in this MPS fails. Therefore, for the kth time of lifetime sampling, the normal working time of MPS S_i can be calculated by

$$T_i^k=\min_{x_j \in S_i}\left(t_j^k\right), \quad (25)$$

where the superscript k represents the ith lifetime sampling, and subscript j is the serial number of basic event or component for $1 \le j \le n$. The whole system will success for at least one MPS appears, then the lifetime of system at ith time sampling can be obtained by

$$T_k=\max_{1 \le i \le m}\left(T_i^k\right). \quad (26)$$

Repeat the previous lifetime sampling process N times, a sample value of the mean lifetime of system is $T_{s\_mean}=1/N\sum_{k=1}^{N} T_k$. Let Y_k be the kth sampling time of a top event. Compare with a given time t_k, if $Y_k<t_k$, it means the system is failed at time t_k, and the failure number of system is cumulated as N_k. Based on the statistical data, the reliability of system R_k at a given time t_k can be calculated as $R_k=N_k/N$.

3.2. Mean lifetime evaluation of system based on MC simulation method

According to the literature investigation and engineering experience, the influence of the replacement of components in reliability analysis of complex system cannot be ignored [61]. In this section, the mean lifetime of the sample system is evaluated by MC simulation method when considering the components replacement, and the result is compared with the situation that components replacement is ignored.

3.2.1. Lifetime evaluation for non-repairable system

When the system is non-repairable, to calculate the lifetime of system we should get the MPSs after the FT modeling of the system. DFT model includes different types of dynamic logic gates such as PAND, FDEP, CSP and HSP gates. So, the occurrence of top event is not only related to the combination of basic events, but also related to the occurrence priority of these basic events. In order to reflect this dynamic characteristic of DFT, the MCS of static FT is extended to minimal cut sequence (MCSQ) [14], which is a minimal failure sequence that causes the occurrence of the top event of DFT. The generating procedure of MCSQ for DFT can refer to [62].

MC simulation can be applied to solve the problem if the parametric distributions of basic events are known. Matlab software can be used to generate the random failure times of each component. By calculating the steps in the former subsection with M simulations, the mean lifetime of the system will be assessed.

3.2.2. Lifetime evaluation for repairable system

Assume that the system state after component replacement is "as good as new", which is a perfect repair model. When considering the component replacement, the mean lifetime of system can be estimated through the mean lifetime of basic components by using MC simulation method.

Suppose a system has l components, the lifetime distributions of different components follow exponential distribution, Weibull distribution and Lognormal distribution with l_1, l_2 and l_3 components respectively, where $l_1+l_2+l_3=l$. Let t_x denote the lifetime of a component and $R_x(t_x)$ denote its corresponding reliability.

For a component with lifetime t_x obeying exponential distribution, according to Eq. (12), the failure rate of component x can be gotten as λ_x, for $x=1, 2, \cdots, l_1$.

For Weibull distribution, the reliable lifetime of component x can be calculated by Eq. (14) and

$$t_x=\eta_x(-\ln R_x(t_x))^{1/\beta_x}, \quad (27)$$

where $x=l_1+1, l_1+2, \cdots, l_2$.

For Lognormal distribution, the reliable lifetime of component x can be calculated by Eq. (16) and

$$t_{xR}=\exp\{\mu_x-\sigma_x z_{xR}\}, \quad (28)$$

where $x=l_2+1, l_2+2, \cdots, l_3$, and z_{xR} is the R quantile of the standardized normal distribution $N(0, 1)$.

Then the flow diagram of service lifetime assessment of sample system using MC simulation method is shown in Fig. 1.

The main steps are as follows:

Step 1: By using the MC simulation method, according to the parameters λ_x, η_x, m_x, $\sigma_x$and μ_x, which can be predicted by Section 3, the pseudo-failure time t_{px}^{Exp} $(x=1, 2, \ldots, l_1)$, t_{px}^{Wb} $(x=l_1+1, l_1+2, \ldots, l_2)$ and t_{px}^{Logn} $(x=l_2+1, l_2+2, \ldots, l)$ can be generated for exponential distribution, Weibull distribution and lognormal distribution, respectively.

Step 2: By simulating the actual replacement situation of system components, the pseudo-failure time of periodic replacement components are determined. For replacement components, if the generated pseudo-failure time t_{px} is less than or equals to replacement time T_{zx}, then this component will fail at time t_{px}. If t_{px} is larger than T_{zx}, it means that this component still survives in this renewal cycle. Then system proceeds to next cycle, a new pseudo-failure time for component is generated and compared

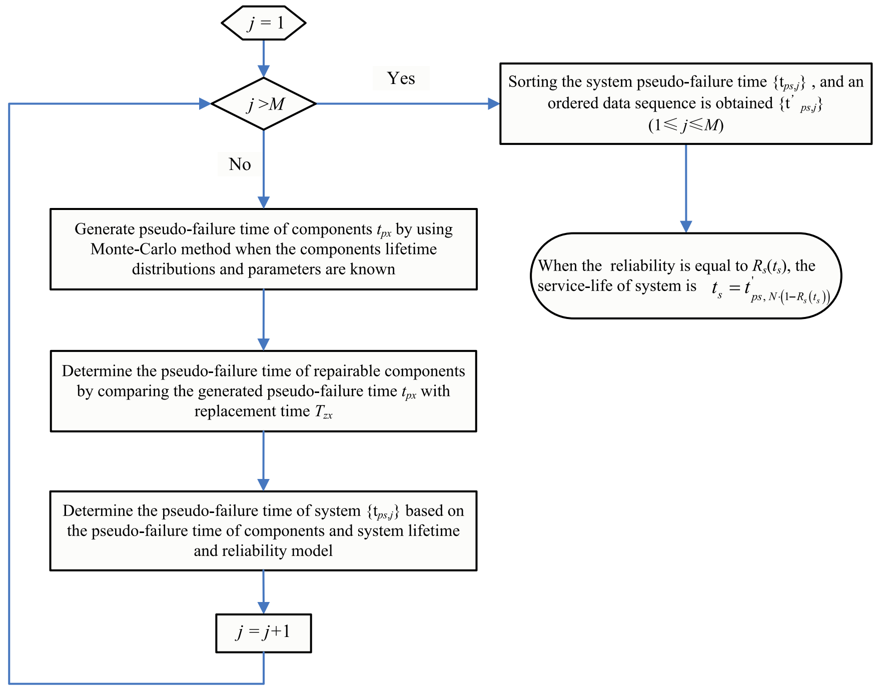

Fig. 1. The flow chart of the service-life assessment based on MC method.

to T_{zx}. Repeating this process until the pseudo-failure time of the component is determined. Therefore, in this simulate process, the pseudo-failure time of a replacement component should be jointly determined by t_{px} and T_{zx}.

*Step 3***:** Based on the relation between system cut sets and system mean lifetime in the former subsection, the pseudo-failure time of system t_{ps} is equal to the maximum pseudo-failure time of cut sets in any simulation process. The pseudo-failure time of a cut set equals to the minimal pseudo-failure time of each basic component. The time-to-failure of repairable components is determined by step 2.

*Step 4***:** Repeating the steps 1 to 3 for M times, we can get the system pseudo-failure time $\{t_{ps,\,j}\}_{j=1}^{M}$ and after sorting these M data, an ordered data sequence can be obtained as $\{t'_{ps,j}\}_{j=1}^{M}$. Then the service lifetime of system with reliability $R_s(t_s)$ is $t_s = t'_{ps,\ N\cdot(1-R_s(t_s))}$

3.3. *Importance sorting by possibility-based NSG ranking method*

FT describes the logic relationship of system failure events and expresses the system structure. On the basis of the system lifetime assessment, the system reliability and the importance of each component can also be evaluated. The importance of components is related to the system structure, the lifetime distribution of each component and the mission time. Therefore, the ranking of component importance will be significant for improving the system design and determining the detection site of system when it is failed. It also can give the guidance for developing a checklist for system diagnosis.

There is no standard consensus on the importance due to the difference of design objects and requirements. The commonly used indexes of importance measure in engineering are probability importance, structural importance and critical importance. In this paper, the probability importance degree is analyzed after which the weak link of the system can be confirmed. When considering the interval uncertainty of system, the probability importance of the ith component of a system with n components is denoted as $[I]_g(i)$, which can be computed by the following equation,

$$[I]_g(i) = \frac{\partial[g(F)]}{\partial[F]_i(t)}, \qquad i = 1, 2, \cdots, n \tag{29}$$

where $[F]_i(t)$ is the interval failure probability or unreliability of component i at mission time t, $[g(F)]$ is the interval probability function of top event and $F_s(t)$ is the unreliability of the system.

When the importance of components is obtained and expressed as a group of interval numbers, the ranking of importance can be performed by interval number ranking methods. To compare the incidence of each component to the whole system, the following possibility-based NSG ranking method [63–65] is used to rank the importance of components represented by interval numbers. For interval numbers $[a] = [a_L, a^U]$ and $[b] = [b_L, b^U]$, the length of the interval numbers $l([a])$ and $l([b])$ are given by

$$l([a]) = a^U - a_L, \quad l([b]) = b^U - b_L. \tag{30}$$

Then the possibility of $[a] \geq [b]$ can be defined as

$$\begin{aligned} p([a] \geq [b]) &= \min\left\{0, 1 - \max\left(\frac{a^U - b_L}{l([a]) + l([b])}, 0\right)\right\} \\ &= \begin{cases} 1 & a_L \geq b^U \\ \frac{a^U - b_L}{l([a]) + l([b])} & a^U > b_L \text{ and } a_L < b^U \\ 0 & a^U \leq b_L, \end{cases} \end{aligned} \tag{31}$$

where $a_L \geq 0, b_L \geq 0$. The possibility degree reflects the degree of interval number $[a]$ is larger than $[b]$, which means when $p([a] \geq [b]) > 0.5$, there is a higher possibility that $[a] \geq [b]$. An interval ranking method can be derived based on this measurement. When the importance of system components are a set of interval numbers $[a_i] = [a_{L,i}, a_i^U]$ for $i = 1,\ldots,n$, by pairwise comparison of the interval numbers using the Eqs. (30) and (31), the corresponding possibility $p_{i,j}$ can be obtained and is denoted as $p_{i,j} = p([a_i] \geq [a_j])$ $(i = 1,2,\ldots,n;\ j = 1,2,\ldots,n)$. Then a possibility matrix $\mathbf{P}$ is built and given by

$$\mathbf{P} = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n,1} & p_{n,2} & \cdots & p_{n,n} \end{pmatrix}. \tag{32}$$

Then denote $\lambda_i = \sum_{j=1}^{n} p_{i,j}$ as the row sum of the possibility matrix $\mathbf{P}$, $\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_n]^{\mathrm{T}}$ as the corresponding row sum vector. The ranking vector $\boldsymbol{\omega} = (\omega_i)$ of matrix $\mathbf{P}$ can be given by

$$\omega_i = \frac{1}{n(n-1)}\left(\lambda_i + \frac{n}{2} - 1\right). \tag{33}$$

The interval numbers $[a_i]$ will be sorted based on the elements of vector $\boldsymbol{\omega}$. This method can be used to assess and compare the importance of each component with respect to the system.

When comparing the introduced probability-based ranking method with the deterministic sorting method, the distinct advantage of the probability-based method is that it not only helps for ranking the interval numbers, but also gives an estimation of the difference degree of two interval numbers. In addition, it can reflect the uncertainty of interval numbers. Therefore, this method is much more adaptive to engineering practice and has great theoretical significance.

4. Extended parametric probability-box for epistemic uncertainty in EMS

Considering the epistemic uncertainty of the system caused by incomplete data, the reliable lifetime of system can be given as interval numbers. From Eqs. (13), (15) and (20), the shape parameter β for Weibull distribution and logarithmic standard variance σ for Lognormal distribution are considered as precise numbers whereas η and μ of Eqs. (14) and (16) are defined as interval parameters, which varies in $[\eta_L, \eta^U]$ and $[\mu_L, \mu^U]$, respectively. Then parametric P-Box is employed to express such parameter uncertainty in system.

P-Boxes have been widely applied to quantify and represent uncertainty in risk analysis [36,37,41]. This has also been used as a tool to solve the statistical problem based on bounding probability distributions. Let a non-negative random variable X to describe the lifetime of a component, $F^U(t)$ and $F_L(t)$ are CDFs for random variable X on real number $\mathbf{R}$, and $F(t)=P\{X \le t\}$. Suppose $\mathbf{F}$ is a set of non-decreasing functions which map $\mathbf{R}$ into [0, 1], where F_L and F^U are the lower and upper bounds of $\mathbf{F}$. Then a P-Box is defined by a probability family which matches the constrains $F_L(t) \le F(t) \le F^U(t)$ and $F \in \mathbf{F}$. In reliability engineering, the survival probability can reflect the performance of a component more intuitively. Analogously, based on the definition of P-Box, an extended P-Box is defined as

$$\mathfrak{R} = \left\{R(t), \forall t \in \mathbf{R} \middle| R_L(t) \le R(t) \le R^U(t)\right\}. \tag{34}$$

where $R(t)=P\{X > t\}=1-F(t)$. An extended P-Box representing by $R(t)$ is frequently unknown for its precise value but for the two bounds R_L and R^U. For example, assume that random variable X_{Wb} follows Weibull distribution with β=2 and η=[50, 70], and X_{Logn} follows lognormal distribution with σ=0.25 and μ=[5, 5.5]. Then the Weibull and lognormal P-Boxes and extended P-Boxes are constructed by taking the envelopes of those distributions. The P-Boxes and extended P-Boxes are shown in Fig. 2(a) and (b) respectively.

The area D between the low bound and upper bound of a distribution can be used to quantify the uncertainty of system, which can be expressed as

$$\begin{aligned} D &= \int_0^{+\infty} (1-F_L(t))dt - \int_0^{+\infty} \left(1-F^U(t)\right)dt \\ &= \int_0^{+\infty} R^U(t)dt - \int_0^{+\infty} R_L(t)dt \\ &= ET^U - ET_L. \end{aligned} \tag{35}$$

This formula shows that the difference of the mean lifetime $(ET^U - ET_L)$ can be used to quantify the uncertainty of a component or system. This provides an alternative optimization goal for uncertainty minimization by using optimization algorithms. Therefore, in this paper classical statistical method has been used to obtain a confidence interval based on observational data [40,42]. Then the unknown distribution parameters can be calculated within interval numbers by Section 4. This subsection provides a practical way to define extended P-Box for distributions with parameter uncertainty.

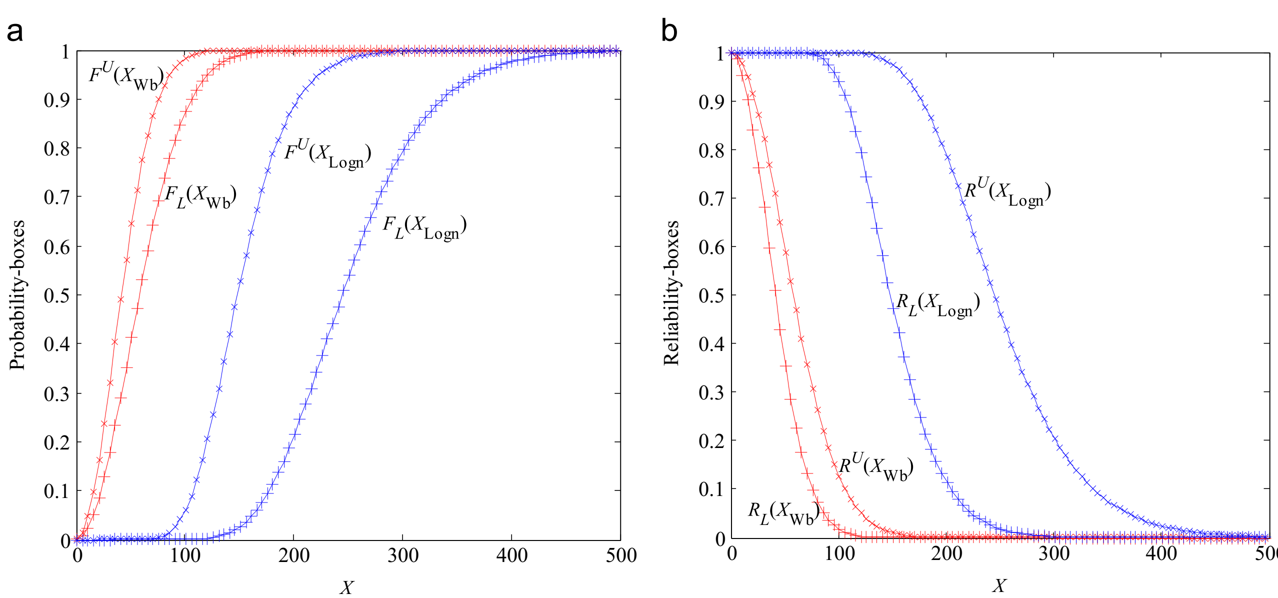

Fig. 2. (a) examples of P-Boxes; (b) examples of extended P-Boxes.

5. Case study

Modern mechanical devices are characterized as multi-functional complex EMSs, which are composed of hundreds or thousands of components. The performance of components can directly influence the operation efficiency of the whole product. Due to the complexity introduced by the environmental impact, lack of data and random interference, uncertainty and nonlinearity characteristics always exist in complex systems [66].

5.1. Description of an electromechanical system

An illustrative complex dynamic EMS is used in this paper as shown in Fig. 3. It is composed of control system, power supply system, powertrain system and hydraulic system. Control system includes two control modules connected in parallel, which are used to perform the start-stop control of the main valve. It also sends signal to hydraulic subsystem and controls the execution. Powertrain system is a key subsystem, including a turbine, a reducer and a pump. The power supply subsystem is composed of two valves in the emergency work mode, but only one main valve in main work mode. This functional relationship of this illustrative system with the subsystems mentioned above is shown in Fig. 3.

To facilitate the method introduced and simplify the calculations, assumptions for the reliability modeling and lifetime prediction of the system are made as follows:

1) Component or subsystem has the same lifetime distribution and the same lifetime with its corresponding assembly component.
2) The components and units, whose failures are rarely happened or their failures do not lead to system failure, are ignored.

5.2. Lifetime evaluation of sample electromechanical system

5.2.1. Dynamic fault tree modeling

In Fig. 3, the emergency work mode is regarded as a cold backup of the main work mode, and the second module of the control system is a hot backup for the first one. For the main work mode, the main valve will be open by the hydraulic system after receiving the signal from the control system. Therefore, in order to ensure normal operation of the main valve, the hydraulic system must be at the normal working condition before the main valve starts to work. As a result, the failure of hydraulic system will force the main valve to enter the failure state. Namely, there is a functional dependency between the hydraulic system and the main valve.

The dynamic logic gates, including CSP, HSP and FDEP, are employed to describe the sequential rules and dynamic behavior of the system. In this paper, based on failure mechanism analysis of this system, the "complex EMS task failure" is chosen as a top event for FT analysis. The DFT of this example system is built as shown in Fig. 4. The meanings of the notations in Fig. 4 are listed

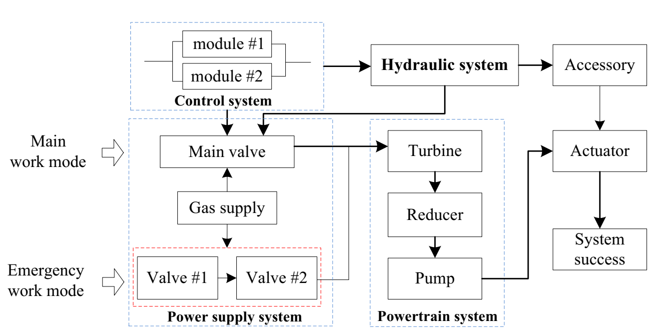

Fig. 3. Function relationship of a complex electromechanical system.

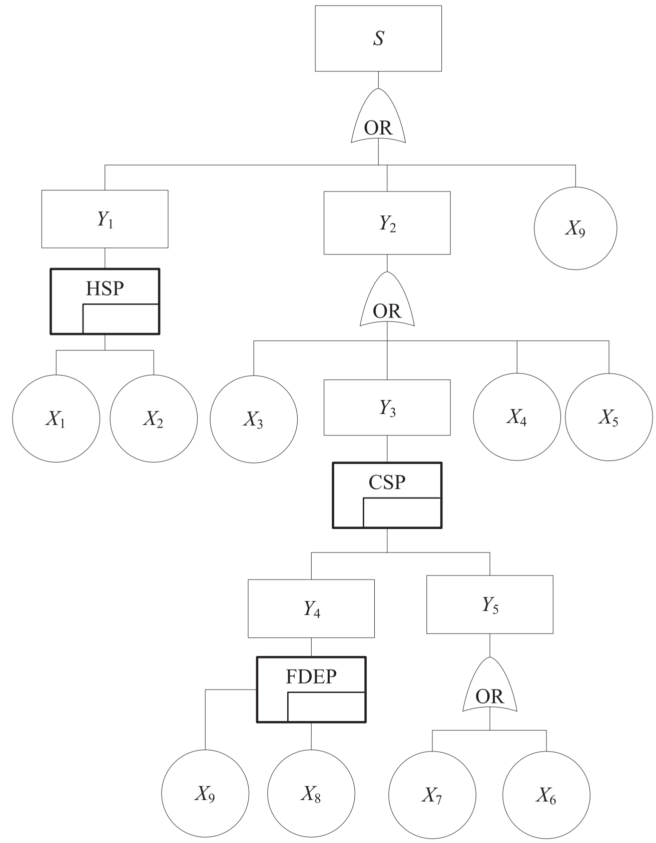

Fig. 4. The DFT model of example system.

Table 1
The number and description of the events in system DFT.

No.	Event description	No.	Event description
S	Complex electromechanical system task failure	X_3	Turbine failure
Y_1	Control system failure	X_4	Reducer failure
Y_2	Powertrain system failure	X_5	Pump failure
Y_3	Power is not transmitted to the subordinate unit	X_6	Valve #1 failure
Y_4	Main work mode failure	X_7	Valve #2 failure
Y_5	Emergency work mode failure	X_8	Main valve failure
X_1	Control module #1 failure	X_9	Hydraulic system failure
X_2	Control module #2 failure		

in Table 1. Finally, by using a similar MCSQ generation method in [60] for DFTs, four MPSQs are obtained for the example system, including $S_1=\{X_1, X_3, X_4, X_5, X_8, X_9\}$, $S_2=\{X_1, X_3, X_4, X_5, X_6, X_7, X_9\}$, $S_3=\{X_2, X_3, X_4, X_5, X_8, X_9\}$ and $S_4=\{X_2, X_3, X_4, X_5, X_6, X_7, X_9\}$.

Based on the mechanical and electrical properties of basic components of the system, lifetime variables of the elementary components are assumed to follow exponential distribution, Weibull distribution and lognormal distribution respectively. According to the accelerated life test and field data analysis, the lifetime distribution and lifetime interval of different subsystems and components of the example system are listed in Table 2.

The lower bound of the lifetime interval in Table 2 is the reliable lifetime t_R of component when its reliability equals to 0.95 under the confidence level of 0.8, and the upper bound is the counterpart of the lower bound when its reliability equals to 0.5. The working frequency of the system is 70 times a year and 2 h each time. According to the engineering experience, for a system composed of several electronic components, its lifetime may not

Table 2
Life distribution and lifetime interval of basic units or subsystem.

No.	Basic component	Life distribution	Lifetime Interval [$t_{R=0.95}$, $t_{R=0.5}$]
X_1	Control module #1	Exp (λ_1); Exp (λ_2)	–
X_2	Control module #2	*Wb* (β_1, η_1); *Wb* (β_2, η_2)	[1841, 4200]
X_3	Turbine	*Wb* (β_3, η_3)	[4733.4, 7000]
X_4	Reducer	*Wb* (β_4, η_4)	[2100, 7000]
X_5	Pump	*Wb* (β_5, η_5)	[4200, 5600]
X_6	Valve #1	Logn (μ_6, σ_6)	[1400, 2100]
X_7	Valve #2	Logn (μ_7, σ_7)	[1400, 2100]
X_8	Main valve	Logn (μ_8, σ_8)	[4576.6, 5600]
X_9	Hydraulic system	Logn (μ_9, σ_9)	[4200, 4900]

Table 3
Lifetime COV and distribution parameters of basic units.

No.	COV	Parameters	No.	COV	Parameters
X_1	1	$\lambda_1=\lambda_2=$1.7e-4	X_6	0.2000	$\mu_6=$[7.2442, 7.5700]; $\sigma_6=0.1980$
X_2	0.3905	$\beta_1=\beta_2=2.769$; $\eta_1=\eta_2=$[4794.4, 5381.5]	X_7	0.2000	$\mu_7=$[7.2442, 7.5700]; $\sigma_7=0.1980$
X_3	0.1932	$\beta_3=6.02$; $\eta_3=$[7439.4, 7752.6]	X_8	0.1006	$\mu_8=$[8.4287, 8.5937]; $\sigma_8=$ 0.1003
X_4	0.5385	$\beta_4=1.935$; $\eta_4=$[8459.8, 9746.6]	X_9	0.0769	$\mu_9=$[8.3428, 8.4692]; $\sigma_9=0.0768$
X_5	0.1429	$\beta_5=8.33$; $\eta_5=$[5851.9, 5999.3]			

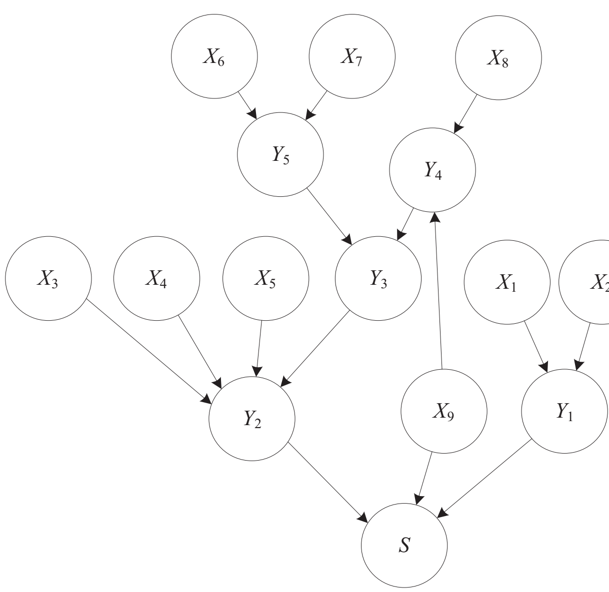

Fig. 5. The BN model of the example system.

follow exponential distribution. Therefore, control modules are given two contrastive assumptions that lifetime of modules obey exponential distribution or two-parameter Weibull distribution.

According to Eqs. (10)–(20) and the data listed in Table 2, the COV of each component can be calculated. Furthermore, the intervals of lifetime distribution parameters can be calculated and listed in Table 3.

5.2.2. Lifetime evaluation of sample EMS based on BN

According to the system structure and the DFT in Fig. 4, a BN model is built for this example system as shown in Fig. 5.

"AgenaRisk" is a software tool which can carry out risk and reliability analysis using BN [48–50]. BN has the advantages for modeling and solving real-world problems, which has been widely used in risk assessment and decision analysis [67]. The AgenaRisk software can perform the BN modeling and information integration for uncertainty related risk and reliability analysis for real engineering complex systems [68]. In this paper, AgenaRisk software is used to carry out BN modeling and data analysis of the example system in Fig. 3. Fig. 6 shows the BN model that is built in "AgenaRisk. 6.2, Revision 2077" and the lifetime distribution and the mean lifetime of each component and subsystem are shown in Table 4.

Table 4 shows that, when X_1 and X_2 follow exponential distributions, the interval of system mean lifetime is [3411.0, 3761.3], and when X_1 and X_2 follow two-parameter Weibull distributions, the interval of system mean lifetime is [3649.1, 4100.8]. From Fig. 6 we can see that the lower bound of system reliability at time $t=3000$ h is 0.81528 when control modules lifetime follows Weibull distributions. By resetting the parameters of the nodes in Fig. 6, the system reliability can be computed by AgenaRisk software easily. Therefore, the system reliability is [0.8153, 0.8665] while control module obeys Weibull distribution and [0.7267, 0.7525] for exponential distribution. Taking the upper percentile of 0.75 as the service lifetime bound of system, the service lifetime interval of the example system can be obtained as [4215.3, 4743.1] under the condition that control module follows Weibull distributions, and as [4180.6, 4697.5] when control module follows exponential distributions.

5.2.3. Lifetime evaluation of sample repairable EMS based on MC simulation

In this section the MC simulation-based method is used to perform the quantitative analysis of the DFT model based on the structure function of sample system by synthesizing the failure mechanism of repairable system. After the sequentialization of the MPSs, the system MPSs are obtained, and $S_1=\{X_1, X_3, X_4, X_5, X_8, X_9\}$, $S_2=\{X_1, X_3, X_4, X_5, X_6, X_7, X_9\}$, $S_3=\{X_2, X_3, X_4, X_5, X_8, X_9\}$ and $S_4=\{X_2, X_3, X_4, X_5, X_6, X_7, X_9\}$. According to Eq. (24), the system success event S of sample system is:

$$\begin{aligned} S &= \sum_{i=1}^{4} S_i = S_1+S_2+S_3+S_4 = \sum_{i=1}^{m}\left(\prod_{X_j\in S_i} X_j\right) \\ &= (X_1X_3X_4X_5X_8X_9)+(X_1X_3X_4X_5X_6X_7X_9) \\ &\quad +(X_2X_3X_4X_5X_8X_9)+(X_2X_3X_4X_5X_6X_7X_9). \end{aligned} \tag{36}$$

The Matlab numerical analysis software is employed to generate the random number with the sampling frequency $M=100{,}000$. Based on the MC simulation method and the simulation process in Section 3.2, the mean lifetime of the sample system is calculated and expressed as an interval number [3412.7, 3779.0] when the lifetime of control modules follows exponential distributions. And when control modules obey Weibull distribution, the mean lifetime of sample system has been obtained as [3616.8, 4084.8]. The results are based on an assumption that the sample system is non-repairable. But in engineering, when considering complexity, economy and maintainability of large-scale complex system, some of the components are always designed as replaceable components. Therefore, in the following part, we have investigated the effect of the replacement of components to the mean lifetime and service lifetime of the whole system.

The lifetime distribution and distribution parameters of each basic component are known from former subsection. Assume that the valve #1 and #2 are replaceable components. From Table 2, we know that the lifetime of valves falls in [1400, 2100](h), so the replacement time is determined as 1400 h. The control modules are assumed to be repairable and replacement time T_z is set as 2100 h. To compare and quantify the impact of the replacement of different components to the lifetime of whole system, the lifetime

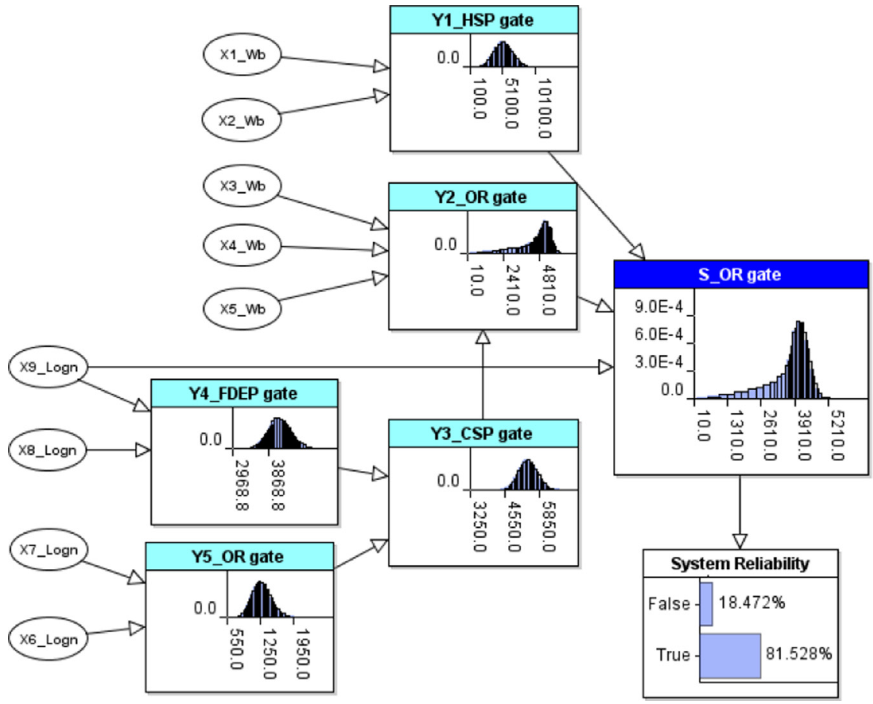

Fig. 6. BN modeling by AgenaRisk software (X_1, X_2 follows Weibull distribution).

Table 4
The mean life intervals of components and subsystems.

No.	$X_1, X_2 \sim$ ***Wb***	$X_1, X_2 \sim$ **Exp.**	No.	$X_1, X_2 \sim$ ***Wb***	$X_1, X_2 \sim$ **Exp.**
X_1, X_2	[4268.5, 4791.3]	5912.8	X_9	[4212.3, 4780.0]	[4212.3, 4780.0]
X_3	[6902.0, 7192.4]	[6902.0, 7192.4]	Y_1	[5213.3, 5850.6]	[8855.8, 8855.8]
X_4	[7509.5, 8648.6]	[7509.5, 8648.6]	Y_2	[4436.6, 4868.7]	[4436.6, 4868.7]
X_5	[5521.1, 5660.2]	[5521.1, 5660.2]	Y_3	[5401.2, 6483.8]	[5401.2, 6483.8]
X_6	[1428.0, 1978.4]	[1428.0, 1978.4]	Y_4	[4132.3, 4726.2]	[4132.3, 4726.2]
X_7	[1428.0, 1978.4]	[1428.0, 1978.4]	Y_5	[1269.0, 1757.7]	[1269.0, 1757.7]
X_8	[4599.9, 5424.9]	[4599.9, 5424.9]	S	[3649.1, 4100.8]	[3411.0, 3761.3]

Table 5
Mean life of sample system with different replace cases.

Mean life Lifetime distribution	**BN method** **Non-repairable**		**MC method** **Repairable** (replacement time T_z)		
X_1, X_2	–	–	X_1, X_2 T_z=2100 (h)	X_6, X_7 T_z=1400 (h)	X_1, X_2, X_6, X_7 T_z=2100/1400 (h)
Exp	[3411.0, 3761.3]	[3412.7, 3779.0]	[3761.6, 4259.3]	[3409.3, 3775.8]	[3765.3, 4257.3]
Wb	[3649.1, 4100.8]	[3616.8, 4084.8]	[3765.5, 4257.3]	[3618.3, 4090.7]	[3762.7, 4260.7]

simulations of system are divided into the following cases: 1) only the control modules are replaceable with T_z=2100 h, 2) only valves #1 and #2 are replaceable for replacement time T_z=1400 h, and 3) both control modules and valves are replaceable. Finally, the simulation results are tabulated in Table 5.

As shown in Table 5, when system is non-repairable, the mean lifetime of system evaluated by BN method almost perfectly matches the MC simulation method results. So the mean lifetime of system is about [3400, 3780] when control modules' lifetime follows exponential distribution, and is about [3600, 4100] when control modules' lifetime follows Weibull distribution. When considering the reparability of system, the replacement of components X_1 and X_2 will makes the mean lifetime of system increase to about [3760, 4260], which means, to some extent the replacement of control modules can improve the lifetime and the reliability of the system. But the replacement of valves X_6 and X_7 almost has no effect on the lifetime of system. This is because valves #1 and #2 are two cold standby of the main valve, only when the main valve fails those two backup valves can be active. Therefore, when the lifetime of the main valve is long enough, the replacement of valves will rarely affect the lifetime of the entire system, even though the lifetime of valves is short.

We can now infer that the importance of control modules X_1 and X_2 are larger than valves X_6 and X_7 in the whole system. Because of the significant difference of system mean lifetime when the control module is assumed to follow different distributions, and through literature research we know that a system which is composed of some exponential distribution components, the lifetime of the system may not obeys exponential distribution. So in the following parts, the assumption that lifetime of control model follows Weibull distribution will be more consistent with engineering practice.

5.3. Reliability analysis and distribution validation of example EMS

5.3.1. Reliability analysis of example system

According to Section 5.2, there are 4 MPSs in the sample system, the occurrence probability of system success event at mission time t_0 is

$$P(T > t_0) = R_S(t_0) = P(S_1 \cup S_2 \cup S_3 \cup S_4)$$

$$
\begin{aligned}
&= \sum_{i=1}^{4} P(S_i) - \sum_{\substack{i=1\\ i\neq j}}^{4} P(S_iS_j) + \sum_{\substack{i=1\\ i\neq j\neq k}}^{4} P(S_iS_jS_k) - P(S_1S_2S_3S_4)\\
&= P(S_1)+P(S_2)+P(S_3)+P(S_4)+P(S_2S_3S_4)\\
&\quad -[P(S_1S_2)+P(S_1S_3)+P(S_2S_4)+P(S_3S_4)]\\
&= 2[P(S_1)+P(S_2)]-[2P(S_1S_2)+P(S_1S_3)+P(S_2S_4)]+P(S_2S_3S_4)\\
&= 2P(x_1)P(x_3)P(x_4)P(x_5)[P(x_8)P(x_9)+P(x_6)P(x_7)P(x_9)]\\
&\quad -P(x_1)P(x_3)P(x_4)P(x_5)[2P(x_6)P(x_7)P(x_8)P(x_9)\\
&\quad +P(x_2)P(x_8)P(x_9)+P(x_2)P(x_6)P(x_7)P(x_9)]\\
&\quad +P(x_1)P(x_2)P(x_3)P(x_4)P(x_5)P(x_6)P(x_7)P(x_8)P(x_9),
\end{aligned}
\tag{37}
$$

where $P(x_i)=P(t_i>t_0)=R(x_i)$, which is the reliability of component x_i at time t_0. The reliable lifetime of exponential distribution, Weibull distribution and lognormal distribution at time t_0 can be calculated by the following equations

$$R_{Exp}^k(t_0)=P(T>t_0)=\exp\{-\lambda_k t_0\},\quad k=1,\cdots,K_1, \tag{38}$$

$$R_{Wb}^k(t_0)=P(T>t_0)=\exp\left\{-\left(\frac{t_0}{\eta_k}\right)^{\beta_k}\right\},\quad k=1,\cdots,K_2, \tag{39}$$

$$R_{Logn}^k(t_0)=P(T>t_0)=\Phi\left(-\frac{\ln t_0-\mu_k}{\sigma_k}\right),\quad k=1,\cdots,K_3, \tag{40}$$

where K_1, K_2 and K_3 are the numbers of component whose lifetime distribution follows exponential distribution, Weibull distribution and lognormal distribution respectively. To compare the effect of repair and maintenance on system reliability, the extended P-Boxes of the sample EMS are gotten based on the lifetime distributions and parameters in Tables 2 and 3, and are shown as Figs. 7 and 8. Then the extended P-Boxes of each component with different lifetime distributions are shown in Figs. 9 and 10.

From Fig. 7, when the lifetime of control modules (X_1 and X_2) are Weibull distributions, the system reliability interval is [0.8159, 0.8668] at service time t=3000 h, which is nearly the same as the result obtained by BN-based method [0.8153, 0.8665]. Similarly, considering the reparability of components, the system reliability increases to [0.8661, 0.8962]. When X_1 and X_2 obey exponential distributions, the system extended P-Boxes are shown in Fig. 8. The system reliability interval at time t=3000 h almost equals to the result by BN-based method, which is [0.7267, 0.7525]. The system reliability will be raised to [0.8484, 0.8779] with consideration of components repair. After analyzing the curve trend of the reliability bounds of the extended P-Boxes in Figs. 7 and 8, we can see that there is a hopping of system reliability at each replacement time, which means that after each replacement of components, system reliability has been improved to a certain extent.

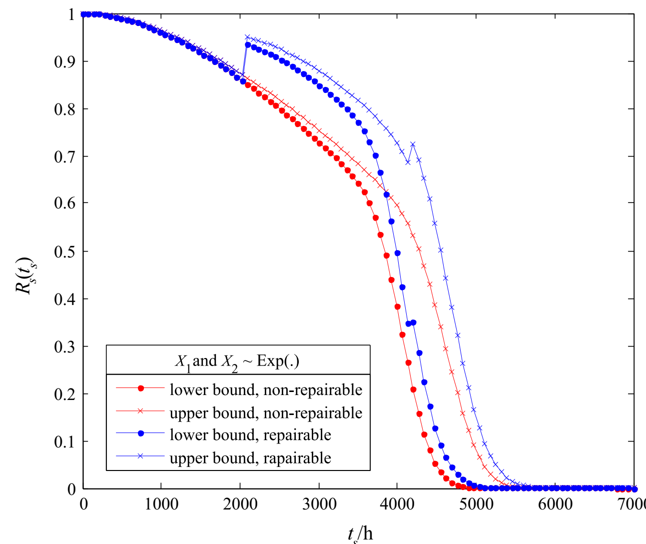

Fig. 8. System extended P-Boxes (X_1 and X_2 follows Exp distribution).

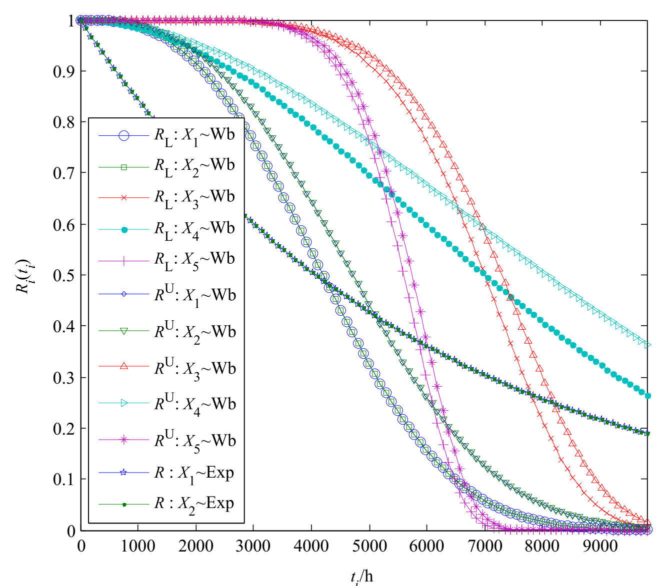

Fig. 9. Components extended P-Boxes (X_1–X_5).

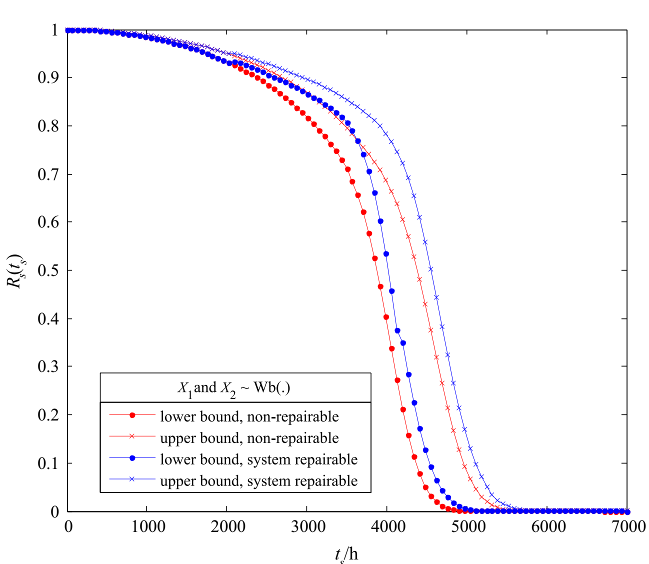

Fig. 7. System extended P-Boxes (X_1 and X_2 follows *Wb* distribution).

5.3.2. *Lifetime distribution analysis of example system*

The system lifetime and reliability extended P-Boxes have been evaluated. As introduced in the former sections, the reliability assessment of system contains not only the qualitative analysis, the lifetime evaluation and system reliability evaluation, as well as the distribution validation and failure rate function investigation are also included. To investigate the reliability function and the lifetime distribution of sample system, the distribution parameters should be estimated first. Then, the probability density function and failure rate function can be determined.

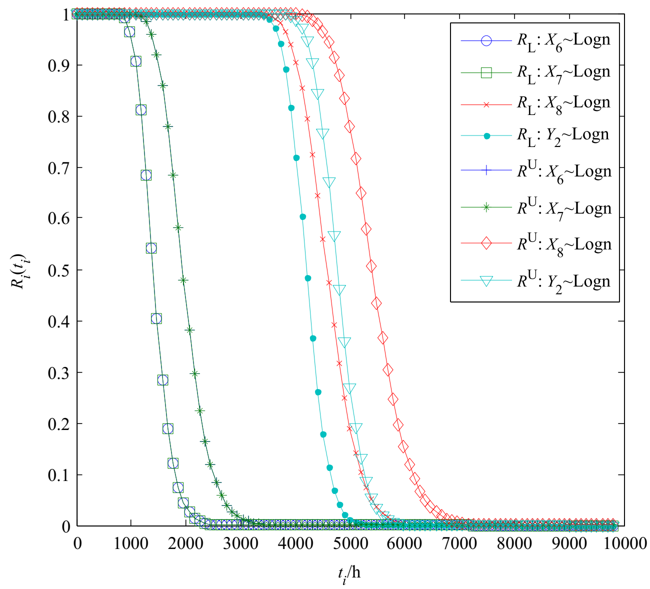

Fig. 10. Components extended P-Boxes (X_6–X_9).

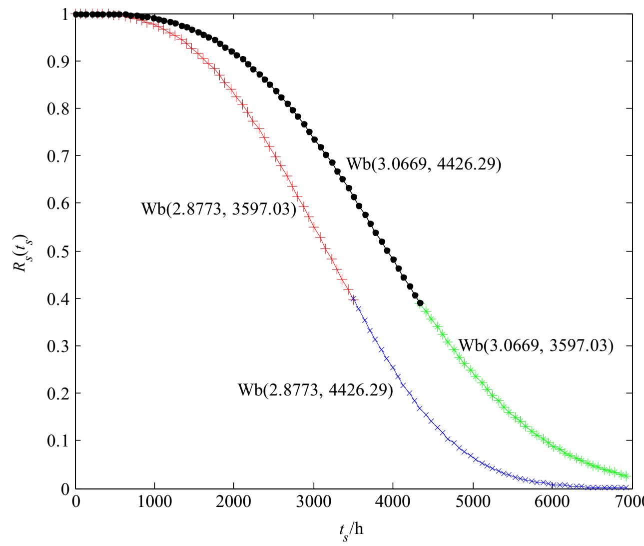

Fig. 11. System extended P-Box.

Assume that the system lifetime follows the Weibull distribution $Wb(\beta_s,\ \eta_s)$. In this section, the least square method is employed to estimate the lifetime distribution of the sample system [60].

For a linear equation,

$$y = a + bx. \tag{41}$$

Based on the least square method, the estimated value of parameters a and b are given by

$$\hat{a} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} {x_i}^2 - \left(\sum_{i=1}^{n} x_i\right)^2}, \tag{42}$$

$$\hat{b} = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{\hat{a}}{n}\sum_{i=1}^{n} x_i. \tag{43}$$

For Weibull distribution, the following equation can be obtained through linear transformation,

$$\ln\ln\frac{1}{R(t)} = -\beta\ln\eta + \beta\ln t. \tag{44}$$

Using the form of Eq. (41) to rewrite the Eq. (44), let $y = \ln\ln(1/R(t))$ and $x = \ln t$, the parameters of Eq. (41) will be $a = -\beta\ln\eta$ and $b = \beta$. It can be seen that the key point of parameter estimation of Weibull distribution is the calculation of x_i and y_i, and the challenge is how to compute the reliability $R(t)$. In this paper, the system reliability with consideration of component repair at different mission time has been calculated and shown in Fig. 7. Above all, the parameter estimation of Weibull distribution by using least square method includes the following steps.

1) According to the system reliability data $R(t_i)$ and the corresponding reliable lifetime t_i by MC simulation, the variables x_i and y_i are calculated, where $x_i = \ln t_i$ and $y_i = \ln\ln(1/R(t_i))$.
2) Calculating the mean value of data as $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$ and $\bar{y} = (1/n)\sum_{i=1}^{n} y_i$.
3) Let $l_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$ and $l_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$, then l_{xx} and l_{xy} can be calculated.
4) Then, the parameters of Weibull distribution can be estimated as $\hat{\beta} = l_{xy}/l_{xx}$ and $\hat{\eta} = \exp(-(\bar{y} - (l_{xy}/l_{xx})\bar{x})/(l_{xy}/l_{xx}))$. The estimated probability density function and failure rate function can be obtained by

$$\hat{f}(t) = \frac{\hat{\beta}}{\hat{\eta}}\left(\frac{t}{\hat{\eta}}\right)^{\hat{\beta}-1}\exp\left[-\left(\frac{t}{\hat{\eta}}\right)\hat{\beta}\right],\quad t \geq 0, \tag{45}$$

$$\hat{R}(t) = \exp\left[-\left(\frac{t}{\hat{\eta}}\right)\hat{\beta}\right],\quad t \geq 0, \tag{46}$$

$$\hat{\lambda}(t) = \frac{\hat{\beta}}{\hat{\eta}}\left(\frac{t}{\hat{\eta}}\right)^{\hat{\beta}-1},\quad t \geq 0. \tag{47}$$

Using the former method and considering the system repair, the lifetime distribution parameters of sample system can be estimated. Then the estimated value of the shape and scale parameters are $\hat{\beta}_s \in [2.8773, 3.0669]$ and $\hat{\eta}_s \in [3597.03, 4426.29]$ respectively. The Weibull extended P-Box can be constructed by taking the envelope of four two-parameter Weibull distributions i.e. $Wb(2.8773,\ 3597.03)$, $Wb(2.8773,\ 4426.29)$, $Wb(3.0669,\ 3597.03)$ and $Wb(3.0669, 4426.29)$, which makes the system extended P-Box as shown in Fig. 11. The failure rate of system at mission time t can also be calculated by Eq. (47) and shown in Fig. 12.

5.3.3. Importance analysis

When the control modules follow Weibull distribution, based on Eq. (29), the probability importance degrees of the components at mission time t=4200 h are calculated and given in Table 6 when the replacement of components are ignored. The importance of each component or subsystem versus mission time t is shown in Fig. 13. When the sample system is repairable, the probability importance degrees of components at t=4200 h are listed in Table 7 and the curve of importance versus time t is shown in Fig. 14.

The NSG ranking method [63–65] is used to sort the interval importance in Tables 6 and 7. According to the possibility-based NSG ranking method in Section 3.3, the possibility matrices $\mathbf{P_1}$ and $\mathbf{P_2}$ for non-repairable and repairable scenarios can both be

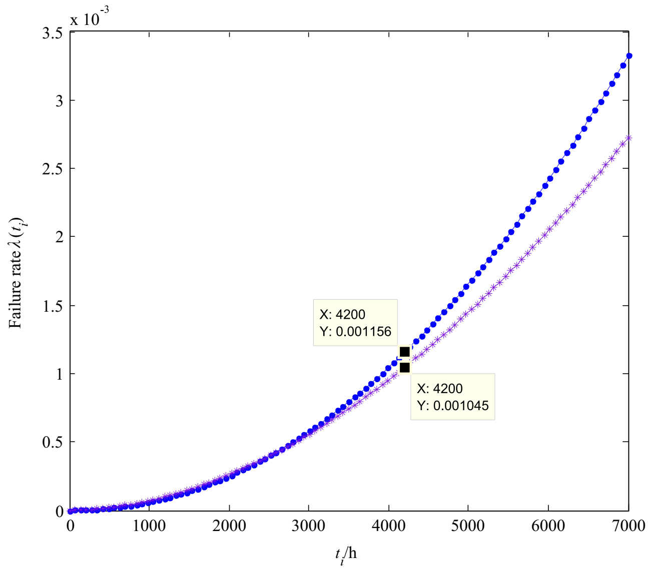

Fig. 12. System failure rate fitting curve.

Table 6
Probability importance of components (non-repairable).

No.	$[I]_g(i)$	No.	$[I]_g(i)$	No.	$[I]_g(i)$
X_1	[0.1412, 0.2844]	X_4	[0.2740, 0.7379]	X_7	0
X_2	[0.1412, 0.2844]	X_5	[0.2255, 0.6384]	X_8	[0.2633, 0.6103]
X_3	[0.2186, 0.6218]	X_6	0	X_9	[0.4236, 0.6384]

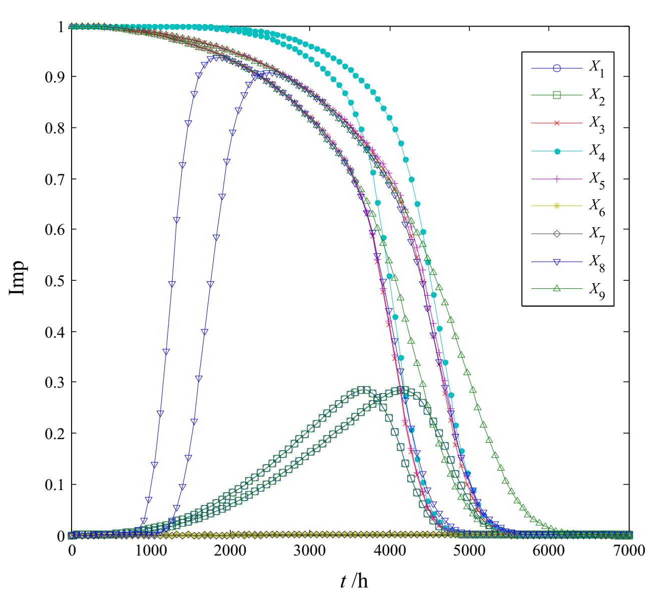

Fig. 13. Components importance when system is non-repairable.

Table 7
Probability importance of components (repairable).

No.	$[I]_g(i)$	No.	$[I]_g(i)$	No.	$[I]_g(i)$
X_1	0	X_4	[0.4544, 0.8802]	X_7	[0.0045, 0.0688]
X_2	0	X_5	[0.3740, 0.7615]	X_8	0
X_3	[0.3625, 0.7417]	X_6	[0.0045, 0.0688]	X_9	[0.7025, 0.7615]

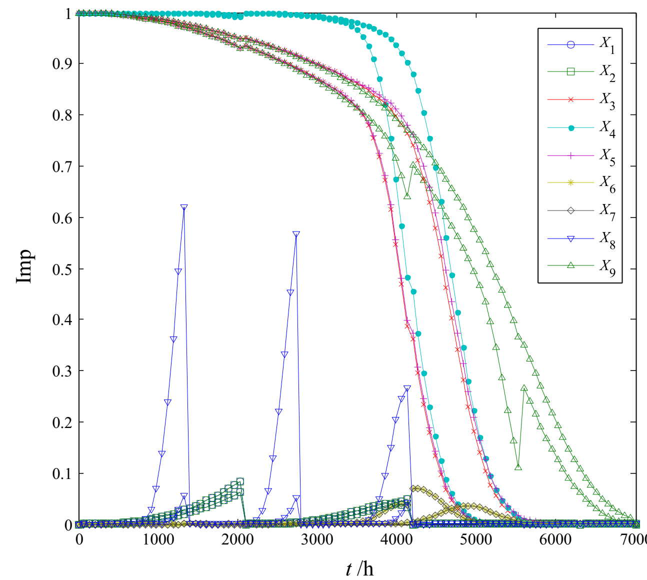

Fig. 14. Components importance when system is repairable.

calculated by Eqs. (30)–(33) and are given as

$$\mathbf{P}_1 = \begin{pmatrix} 0.5000 & 0.5000 & 0.1204 & 0.0171 & 0.1059 & 1.0000 & 1.0000 & 0.0430 & 0 \\ 0.5000 & 0.5000 & 0.1204 & 0.0171 & 0.1059 & 1.0000 & 1.0000 & 0.0430 & 0 \\ 0.8796 & 0.8796 & 0.5000 & 0.4011 & 0.4856 & 1.0000 & 1.0000 & 0.4779 & 0.3207 \\ 0.9829 & 0.9829 & 0.5989 & 0.5000 & 0.5844 & 1.0000 & 1.0000 & 0.5853 & 0.4631 \\ 0.8941 & 0.8941 & 0.5144 & 0.4156 & 0.5000 & 1.0000 & 1.0000 & 0.4936 & 0.3422 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.0000 & 0 & 0 \\ 0.9570 & 0.9570 & 0.5221 & 0.4147 & 0.5064 & 1.0000 & 1.0000 & 0.5000 & 0.3323 \\ 1.0000 & 1.0000 & 0.6793 & 0.5369 & 0.6578 & 1.0000 & 1.0000 & 0.6677 & 0.5000 \end{pmatrix} \tag{48}$$

$$\mathbf{P}_2 = \begin{pmatrix} 1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0.5000 & 0.3569 & 0.4796 & 1.0000 & 1.0000 & 1.0000 & 0.0895 \\ 1.0000 & 1.0000 & 0.6431 & 0.5000 & 0.6224 & 1.0000 & 1.0000 & 1.0000 & 0.3665 \\ 1.0000 & 1.0000 & 0.5204 & 0.3776 & 0.5000 & 1.0000 & 1.0000 & 1.0000 & 0.1321 \\ 1.0000 & 1.0000 & 0 & 0 & 0 & 0.5000 & 0.5000 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0 & 0 & 0 & 0.5000 & 0.5000 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0.9105 & 0.6335 & 0.8679 & 1.0000 & 1.0000 & 1.0000 & 0.5000 \end{pmatrix} \tag{49}$$

Then the ranking vectors $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ of matrices $\mathbf{P_1}$ and $\mathbf{P_2}$ can be computed as,

$$\begin{aligned} \boldsymbol{\omega}_1 &= [\omega_1 \quad \omega_2 \quad \cdots \quad \omega_n]^{\mathrm{T}} \\ &= [0.0943 \quad 0.0943 \quad 0.1312 \quad 0.1416 \quad 0.1327 \quad 0.0764 \\ &\qquad 0.0764 \quad 0.1346 \quad 0.1464]^{\mathrm{T}}, \end{aligned} \tag{50}$$

$$\begin{aligned} \boldsymbol{\omega}_2 &= [\omega_1 \quad \omega_2 \quad \cdots \quad \omega_n]^{\mathrm{T}} \\ &= [0.0903 \quad 0.0903 \quad 0.1379 \quad 0.1477 \quad 0.1393 \quad 0.1042 \\ &\qquad 0.1042 \quad 0.0903 \quad 0.1585]^{\mathrm{T}}. \end{aligned} \tag{51}$$

According to matrices $\mathbf{P_1}$, $\mathbf{P_2}$ and ranking vectors $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$, the ranking result of the components importance $[\boldsymbol{I}]_{\mathbf{g}}(\boldsymbol{i})$ for non-repairable scenario can be represented as

$$\begin{aligned} &[I]_{g1}(X_9) \ge [I]_{g1}(X_4) \ge [I]_{g1}(X_8) \ge [I]_{g1}(X_5) \ge [I]_{g1}(X_3) \\ &\quad \ge [I]_{g1}(X_1) \ge [I]_{g1}(X_2) \ge [I]_{g1}(X_6) \ge [I]_{g1}(X_7). \end{aligned}$$

Denote the symbol "$\succ$" as the optimal order relation of two interval numbers, then the corresponding ranking result will be

$$\begin{aligned} &[I]_{g1}(X_9) \underset{0.5369}{\succ} [I]_{g1}(X_4) \underset{0.5853}{\succ} [I]_{g1}(X_8) \underset{0.5064}{\succ} [I]_{g1}(X_5) \underset{0.5144}{\succ} [I]_{g1}(X_3) \\ &\quad \underset{0.8796}{\succ} [I]_{g1}(X_1) \underset{0.5000}{\succ} [I]_{g1}(X_2) \underset{1.0000}{\succ} [I]_{g1}(X_6) \underset{1.0000}{\succ} [I]_{g1}(X_7). \end{aligned}$$

When the system is repairable, the importance ranking result is

$$[I]_{g2}(X_9) \underset{0.6335}{\succ} [I]_{g2}(X_4) \underset{0.6224}{\succ} [I]_{g2}(X_5) \underset{0.5204}{\succ} [I]_{g2}(X_3) \underset{1.0000}{\succ} [I]_{g2}(X_6) \underset{0.5000}{\succ} [I]_{g2}(X_7) \underset{1.0000}{\succ} [I]_{g2}(X_1) \underset{1.0000}{\succ} [I]_{g2}(X_2) \underset{1.0000}{\succ} [I]_{g2}(X_8).$$

As shown in Fig. 13, the probability importance of valves stays at a low level. This is because they are cold standby of the main valve, which can only be active when the main valve fails.. So the valves #1 and #2 both have a lower importance in the entire system, and the change of their reliability will have also no obvious influence on the reliability of whole system. For the importance of main valve, it has a holding phase at first stage and then it gradually increases and followed by a decreasing process. The existence of the standby components makes a relative low importance of the whole system. But when the reliability of the spare components gradually reduced due to the storage or some other impact factors, the main valve will be assigned a heavier mission. This makes its reliability has an increase process.

From Fig. 14, we can see that when the replacement of control modules and the valves are considered, each replacement of components will lead to an adjustment of the importance of all system components. The importance of the repairable components will decrease after each replacement, and an increasing adjustment process will occur in the importance of other non-replacement components of the entire system. From Figs. 7 and 8, comparing the system reliability, when system is non-repairable and when the components replacement are considered, it is obviously that the regular maintenance or replacement of the short-lived components has significant promotion on system lifetime and system reliability.

6. Conclusion

For the complex systems with limited system-level test data, via comprehensive analysis and processing of test data, field data and design data, the lifetime of system components can be finally represented as bounded closed intervals. Using the COV method, the distribution parameters of basic component lifetime variables are evaluated. It is obviously that parameter uncertainty exists in the complex system, and then the parameters are treated as interval numbers. In view of the epistemic uncertainty caused by the lack of adequate and precise data, based on the definition of P-Box, a similar consideration has been used to define a new concept called extended P-Box to convey the present of epistemic uncertainty in system. It has shown that the extended P-Box can reflect the relevant reliability information more intuitively than P-Box.

The dynamic logic gates are used to describe the dynamic failure behavior of complex EMS. Then a DFT model is built to characterize the failure logic relationships among system components of the complex system. After the definition of lifetime logic relationship for commonly used static and dynamic logic gates, the DFT are mapped into an equivalent BN. By utilizing AgenaRisk software to facilitate the calculation of BN, the mean lifetime and system reliability have been calculated and expressed as interval numbers. For repairable system with components replacement, the MC simulation method is utilized to compute the DFT model by updating the pseudo-failure time of replaceable components. The comparative studies show that the BN-based method tends to be easier to be applied in engineering practices. The MC simulation-based method is more accurate and the repair cases can be considered. The reliability analysis of complex system illustrates that the system reliability can be improved to a certain extent by making reasonable replacement plans for basic components.

Finally, under the assumption that the system lifetime variable is a two-parameter Weibull distribution, the distribution parameters are estimated by least square method, and the system failure rate and probability density function are also calculated. The probability importance analysis of system is utilized to find out weak points of the entire system and to provide guidance for system design, fault diagnosis and maintenance planning. This paper provides an effective and flexible integrated method for reliability assessment of complex EMS, which can be easily implemented in engineering practices. At this point, we have dealt with the reliability assessment of complex EMS; the design improvement and reliability growth for such systems is an avenue for future work.

Acknowledgment

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