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Probabilistic Low Cycle Fatigue Life Prediction Using an Energy-Based Damage Parameter and Accounting for Model Uncertainty

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ABSTRACT: Probabilistic methods have been widely used to account for uncertainty of various sources in predicting fatigue life for components or materials. The Bayesian approach can potentially give more complete estimates by combining test data with technological knowledge available from theoretical analyses and/or previous experimental results, and provides for uncertainty quantification and the ability to update predictions based on new data, which can save time and money. The aim of the present article is to develop a probabilistic methodology for low cycle fatigue life prediction using an energy-based damage parameter with Bayes' theorem and to demonstrate the use of an efficient probabilistic method, moreover, to quantify model uncertainty resulting from creation of different deterministic model parameters. For most high-temperature structures, more than one model was created to represent the complicated behaviors of materials at high temperature. The uncertainty involved in selecting the best model from among all the possible models should not be ignored. Accordingly, a black-box approach is used to quantify the model uncertainty for three damage parameters (the generalized damage parameter, Smith–Watson–Topper and plastic strain energy density) using measured differences between experimental data and model predictions under a Bayesian inference framework. The verification cases were based on experimental data in the literature for the Ni-base superalloy GH4133 tested at various temperatures. Based on the experimentally determined distributions of material properties and model parameters, the predicted distributions of fatigue life agree with the experimental results. The results

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Figures 1, 2 and 4–9 appear in color online: http://ijd.sagepub.com

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show that the uncertainty bounds using the generalized damage parameter for life prediction are tighter than that of Smith–Watson–Topper and plastic strain energy density methods based on the same available knowledge.

KEY WORDS: energy, life prediction, low cycle fatigue, probabilistic, turbine disk, uncertainty.

INTRODUCTION

THE TURBINE DISK is a critical flight safety component of gas turbine aero-engines, for which a failure could lead to catastrophic results. This component is subject to high temperature, corrosive, and oxidative conditions. Low cycle fatigue (LCF) at high temperature is a key failure mode of turbine disks. With an increasing performance and thrust-to-weight ratio of gas turbine aero-engines, higher stress and temperature will be borne by disks, and higher reliability is desirable. In order to reduce weight and improving working life while keeping or increasing reliability (Besson, 2010), an accurate algorithm for probabilistic LCF life prediction of high-temperature structures is essential, which is the main purpose of this contribution.

In physics-based reliability assessment of mechanical components, physical models are used to predict the life of the components that operate under the basic failure mechanisms such as fatigue and creep. Component fatigue data contain significant amounts of scatter (Grell and Laz, 2010). Additional uncertainties in engineering analysis arise due to three types of sources (Zhang and Mahadevan, 2000): (1) physical uncertainty, (2) statistical uncertainty, and (3) model uncertainty. Physical uncertainty refers to the natural variability or fluctuations in the environment, test instruments, observer, and so on. Hence, for the same physical quantity, repeated observations do not yield identical results. Statistical uncertainty is described by the uncertainty in the statistical distribution parameters of the random variables identified in the first source, due to the scarcity in the data. Model uncertainty arises from the fact that models of physical processes generally have many underlying assumptions and often are not valid for all situations. It occurs because models are not perfect and includes uncertainty in both probabilistic and mechanical models. In particular, it is worth mentioning the recent article (Park et al., 2010), where model probability is used to quantify the model uncertainty, suggest that uncertainty in the error of model prediction as well as model uncertainty should be incorporated into a response prediction. In each case, the uncertainty exists in model accuracy as well as model selection. Historically, these uncertainties have been dealt with by applying experience-based safety factors to the fatigue analysis of critical components, driving the likelihood of failure to an acceptable level. Compared with deterministic analyses, probabilistic methods represent the input parameters as distributions and predict distributions of performance. In principle, by formalizing the available knowledge as a prior credibility on model parameters, a probabilistic method via Bayes' theorem will make more accurate inference on the quantities of interest. Recently, numerous studies have focused on probabilistic aspects of fatigue failure, including fatigue cracks propagation (Wang, 2009) and stress-life (S-N) data analysis (Guida and Penta, 2010) using Bayesian inference, as well probabilistic fatigue life prediction using the AFGROW life prediction software (Grell and Laz, 2010). However, very few attempts have been made in the past to use Bayesian approach in the probabilistic LCF life assessment and the model uncertainty analysis.

By combining probabilistic methods with a damage parameter, it is possible to predict LCF life for turbine disks and to evaluate the total uncertainty of those predictions. Until now, various damage parameters have been proposed for assessing the fatigue life of structures or materials, such as Smith–Watson–Topper (SWT; Smith, 1970), plastic strain energy density (PSED; Morrow, 1965), Fatemi-Socie criterion (FSC; Fatemi and Socie, 1988), thermodynamic entropy (TE; Naderi, 2010), viscosity-based damage parameter (VDP; Zhu and Huang, 2010; Zhu et al., 2011a), and generalized damage parameter (GDP; Zhu et al., 2011b). Since these traditional structural life prediction methods are deterministic in nature, they do not explicitly account for the uncertainties associated with their life estimates. In general, it is possible to improve the accuracy of these methods using complex approaches, such as the finite element method; however, it is impossible to completely and formally characterize different uncertainties. Estimations made by the deterministic models for life cycle design and prognosis purposes are useful only in certain cases where a large amount of fatigue test data are available. Uncertainties are mainly due to the lack of data available to build the models. Such data could be gathered from material tests, full-scale fatigue tests and inspections during the service life. To account for the scarcity of materials data and fatigue test data, the uncertainty of the model structure itself, and its predictions must be characterized. Probabilistic methods such as the Bayesian approach provide a formal framework to characterize such uncertainties. This article proposes a probabilistic method to estimate the LCF life of a turbine disk alloy, by considering all available data that contribute to uncertainties associated with the damage parameter.

Accounting for and quantifying the uncertainties associated with a model prediction allows not only for a judgment of confidence in the model prediction but can also result in the estimation of other important qualities such as the probability of exceeding a physical limit. This article will expand previous research capable of characterizing the model uncertainty for applications to LCF life prediction. The studied alloy is a Ni-base Superalloy GH4133, which is used as a turbine disk material in gas turbine engines. Using the experimental results of GH4133 (Wang, 2006), an energy-based damage parameter, and estimates of experimental uncertainty, research performed by Ontiveros et al. (2010) resulted in a probabilistic LCF life prediction framework and an estimation of the model uncertainty, as shown in Figure 1. With regard to the probabilistic LCF life prediction framework showed in Figure 1, σ, ε, and T are the measured stress, strain, and temperature, respectively. $P_i(i = 1, 2, \ldots, N)$ represents the model parameters of the damage parameters.

This research attempts to (1) find a practical way to efficiently incorporate different possible sources of uncertainty into the fatigue life predictions and (2) quantify the total model uncertainty using the black-box approach. This methodology follows the Bayesian inference framework, which was used to determine an estimation of uncertainty associated with the model predictions when compared with experimental results and a black-box approach in

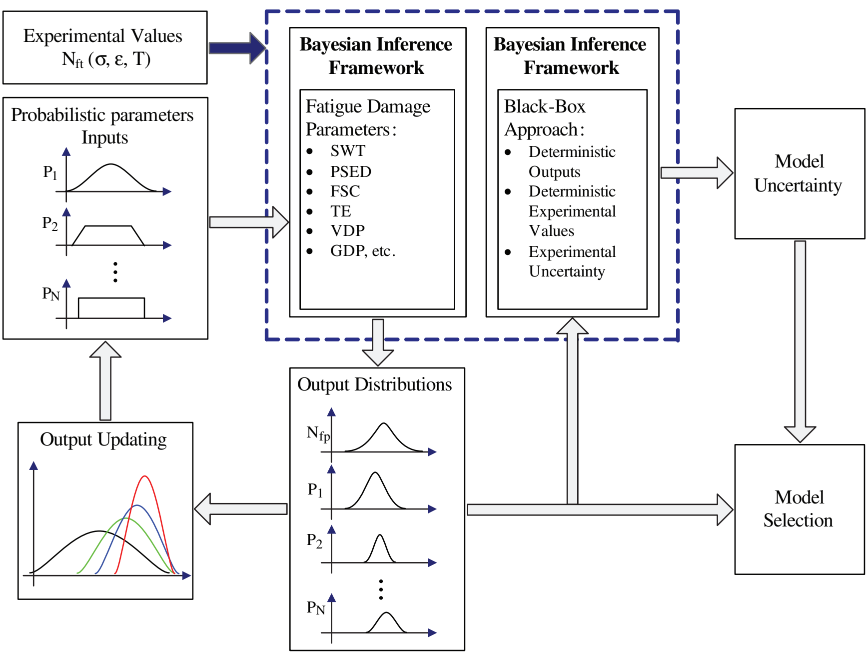

Figure 1. *Probabilistic LCF life prediction framework using Bayes' theorem.*

which no consideration of the uncertainties associated with the inner workings of model was given.

The article is organized as follows: In the next section, the previously developed generalized energy-based damage parameter (Zhu et al., 2011b) capabilities are addressed more extensively to account for the effects of temperature and mean stress on the fatigue life. Then, a probabilistic LCF life prediction framework developed using Bayes' theorem is introduced. In order to account for the model uncertainty, a black-box approach (Ontiveros et al., 2010) is used to describe the probabilistic relationship between the mean model predictions and the experimental values. In the Probabilistic LCF Life Predictions section, this probabilistic life prediction methodology was verified using three different damage parameters with experimental results of GH4133 under different temperatures. The Output Updating Using a Bayesian Inference section is devoted to account for model uncertainty of GDP, SWT, and PSED damage parameters. Finally, the article is concluded in the Conclusion section.

PROBABILISTIC LCF LIFE PREDICTION FRAMEWORK

A Generalized Energy-Based Damage Parameter for LCF Life Prediction

Under cyclic loading conditions, the interactive behavior between the stress and the strain during deformation can be represented by hysteresis loops. Thus, an energy perspective can be used to quantify this interaction. A certain quantity of energy is gradually dissipated by cyclic fatigue and creep during LCF at high temperature. Once the critical energy is reached, fracture will occur. As more damage is accumulated, more energy is dissipated. Following this one-to-one relation, the dissipated energy of a material is used to measure the damage of the material. The accumulated interaction effects in a material result in its eventual failure. Consequently, in terms of the stress–strain hysteresis loop under high temperature, various researchers have developed fatigue life curves by adopting an energy parameter to estimate the life of a material under different loading conditions. These methods are often called energy-based fatigue life prediction models in a broad sense.

A considerable amount of effort has been extended in defining a suitable damage parameter that correlates the life to failure (Fatemi A, and Socie, 1988; Voyiadjis and Kattan, 2009; Naderi, 2010; Zhu and Huang, 2010). Up to now, the damage parameters have been represented by stress, strain, inelastic strain energy, and strain energy density. It was found that the total plastic energy required for fatigue failure is not a constant but increases with a decrease in stress amplitude, and this total energy depends on the stress or plastic strain amplitude via the cyclic stress–strain behavior (Halford, 1961; Morrow, 1967). This means that it is difficult or almost impossible to use this definition to measure fatigue damage directly in the vast majority of cases. Recent research indicates that it is better to use the PSED as a damage parameter rather than the plastic strain range for drastic hardening or softening conditions (Hong, 2005; Lee, 2008). The stable PSED, which is defined as the inner area of the cyclic stress–plastic strain hysteresis loop, is commonly applied as a damage parameter in predicting the fatigue life. Many studies have attempted to establish the fatigue criteria based on the PSED (Koh, 2002; Chiou and Yip, 2006; Lee, 2008). Using the cyclic strain energy density parameter, Koh (2002) investigated fatigue damage and fatigue life of high pressure tube steel under strain-controlled tests. The total cyclic strain energy density provided a good prediction on the fatigue behavior of this steel. According to the mean strain effects in a LCF regime, Chiou and Yip (2006) proposed a modified energy parameter, namely, the stable PSED under tension conditions, for life prediction. Since the magnitude of this damage parameter is half of that of the PSED, there are no intrinsic differences between either of these two parameters for life prediction. Lately, in order to account for the effect of temperature on fatigue life, Lee et al. (2008) developed an energy-based life prediction model using the PSED. Though the lives predicted by this model are within a factor of 2.5, this model exhibits an overestimating tendency as temperature increases.

According to the damage parameters reviewed above (Chiou and Yip, 2006; Lee et al., 2008), degradation mechanisms such as creep and mean stress effects have not been adequately addressed. These effects are considered to be important factors affecting the fatigue resistance at high temperature (Gao et al., 2005). In order to account for the effects of creep and mean stress on the LCF lives of high-temperature structures, a new damage parameter for life prediction was developed based on the PSED method (Zhu et al., 2011b).

In LCF, there is a considerable amount of plastic straining within the material, and the hysteresis energy absorbed during fatigue cycling has been postulated as a basis for failure analysis. Therefore, the fatigue resistance of the material can be characterized in terms of its capacity to absorb and dissipate plastic strain energy. Through the analysis of cyclic plastic strain energy, Morrow (1965) has expressed the relation between the PSED ΔW_p and the fatigue life N_f as

$$\Delta W_p \cdot N_f^{\alpha} = C_1 \tag{1}$$

where α and C_1 are material constants representing the fatigue exponent and the material energy absorption capacity, respectively.

Assuming that the material used satisfies the Masing's hypothesis, the PSED ΔW_p, absorbed during a cycle is the area of the hysteresis loop and can be derived from the cyclic stress–strain curve (Morrow, 1965):

$$\Delta W_p = \frac{1-n'}{1+n'} \cdot \Delta\sigma \cdot \Delta\varepsilon_p \tag{2}$$

The cyclic stress–strain response describes the relationship between flow stress and plastic strain amplitude under cyclic loading. Based on the Ramberg-Osgood relation (1943), the cyclic stress–strain curve can be described by

$$\frac{\Delta\sigma}{2} = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'} \tag{3}$$

where the cyclic strain hardening exponent n' is a measure of work hardening during cycling.

Using Equation (3), the cyclic strength coefficient K' and the cyclic strain hardening exponent n' can be obtained from the log–log linear regression analysis of the cyclic strain amplitudes and the corresponding cyclic stress amplitudes for fully reversed fatigue tests.

Substituting Equation (3) into Equation (2) gives

$$\Delta W_p = 4K'\frac{1-n'}{1+n'} \cdot \left(\frac{\Delta\varepsilon_p}{2}\right)^{1+n'} \tag{4}$$

The LCF life is dependent on test parameters (Zhu et al., 2011c). In case of failure under time-dependent damaging mechanisms such as creep and environmental corrosion, experimental results from Chen et al. (2001) and Sun et al. (2008) showed that both the shape and the size of hysteresis loop are influenced by cyclic frequency, loading waveform, and fatigue hardening/softening. Enlightened by this characteristic, an attempt has been made to deduce a new damage parameter for life prediction, in which the fatigue-creep toughness is used as the control parameter, as explained below.

Similar to the assumption that toughness of a material is a product of its ductility and its cyclic strength (Goswami, 2004), Ostergren proposed the strain energy damage function model (Ostergren, 1967). The strain energy damage function ΔW_s can be expressed approximately by the multiplication of the inelastic strain range $\Delta\varepsilon_{\text{in}}$and maximum tension stress $\sigma_{\max}$, that is,

$$\Delta W_s = \Delta\varepsilon_{\text{in}}\sigma_{\max} \tag{5}$$

where the inelastic strain range $\Delta\varepsilon_{in}$ can be replaced by the plastic strain range $\Delta\varepsilon_p$ under the pure fatigue mode. The relationship between strain energy and fatigue life can be expressed by the power exponent function as

$$\Delta W_s N_f^{\beta} = C_2 \tag{6}$$

where β and C_2 are material constants that are determined experimentally.

Under LCF conditions, $\Delta\varepsilon_{in}$ is approximated by $\Delta\varepsilon_p$. By integrating Equation (4), we can obtain a new expression to describe the process of LCF.

$$N_f = \left(\frac{C_2^{1+n'}}{2^{n'-1}\sigma_{\max}^{1+n'} \frac{1+n'}{(1-n')K'} \Delta W_p} \right)^{\frac{1}{\beta(1+n')}} \tag{7}$$

Then, a new life evaluation relation can be obtained as shown in Equation (8) after rearranging various terms and further simplifying Equation (7).

$$N_f = p\left(\Delta W_p \sigma_{\max}^{1+n'}\right)^{q} \tag{8}$$

$$p = \left(\frac{C_2^{1+n'}(1-n')K'}{2^{n'-1}(1+n')} \right)^{\frac{1}{\beta(1+n')}} \tag{9}$$

where p and q are material parameters that related with the cyclic stress–-strain relationship of material.

Based on Equation (8), it should be noted that $\Delta W_p \sigma_{\max}^{1+n'}$ follows a certain law with LCF life, which includes factors influencing fatigue life and creep life and also takes into account the mean stress effects. This will be referred to as a GDP.

Moreover, the mechanism of cyclic hardening effect has been incorporated into the proposed damage parameter when using it for LCF life prediction. The generalized energy-based damage parameter in Equation (8) is a semi-empirical and physics-based relation, used in many different situations. This equation describes the average behavior, and the life in different tests varies around this average life. In order to describe the variation, a probabilistic life prediction framework is developed in the section on A Probabilistic LCF Life Prediction Framework Using Bayes' Theorem.

A Probabilistic LCF Life Prediction Framework Using Bayes' Theorem

In the current study, the input parameters were modeled as distributions, and perturbed variables for the probabilistic methods were incorporated into the physical or mechanical model (e.g., the generalized energy-based damage parameter). The end result was a predicted distribution of LCF life.

This section focuses on the physical and statistical model updating. A variety of probabilistic methods were implemented and evaluated to consider efficiency and accuracy for the various specimens, such as Monte Carlo simulation, the most probable point methods and a Bayesian approach (Haldar and Mahadevan, 2000).

In general, it is better to make decisions subject to uncertainty using all the knowledge available, old and new, objective and subjective. This is especially true when estimating turbine safety and managing the life cycle of a turbine. A Bayesian approach can potentially provide more accurate estimates by combining evidence such as test data with prior knowledge available from theoretical analyses and/or previous experimental results, which can reduce required testing and save time and resources. The Bayesian inference is a technique used to update a given state of knowledge. The Bayesian framework is presented in Figure 2. In a typical Bayesian analysis, the estimation of a vector of unknown model parameters ξ is updated from its prior probability distribution function (PDF) using information inferred from observed data, $\mathbf{D}$. The prior distribution of the parameters is denoted by $\pi_0(\xi)$ and the information acquired from the data, $\mathbf{D}$ often represented by the likelihood function, $L(\mathbf{D}|\xi)$. Following the Bayesian framework, the result, $\pi(\xi|\mathbf{D})$, is an updated state of knowledge in the form of a posterior PDF. Through the posterior distribution, the current state of knowledge about the total uncertain quantities can be summarized. The posterior of ξ is obtained via Bayes' rule

$$\pi(\xi|\mathbf{D}) = \frac{\pi_0(\xi)L(\mathbf{D}|\xi)}{\int_\xi \pi_0(\xi)L(\mathbf{D}|\xi)\mathrm{d}\xi} \tag{10}$$

where $\xi = \{p, n', q, s\}$ is the vector of the parameters p, n', q, and s, which will be defined in more detail later. $L(\mathbf{D}|\xi)$ is the likelihood function or probability of the observed data $\mathbf{D}$ with given parameters ξ. And $\pi(\xi|\mathbf{D})$ is the posterior joint distribution of ξ after combining observed evidence and prior knowledge of fatigue tests.

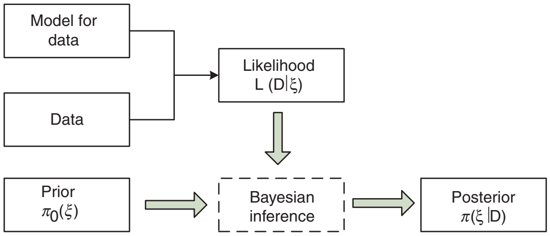

Figure 2. *Bayesian inference framework (Azarkhail and Modarres, 2007b).*

The Bayesian estimation approach treats parameters of the distribution model in Equation (8) as random, not fixed, quantities. Before modeling the current sample data, old information and/or even subjective judgments are often used to construct a prior distribution model for these parameters. Then the likelihood of observed data is used to revise this prior judgment into a so-called posterior distribution model for the model parameters. Thus, Bayes' theorem is a method of mathematically expressing a decrease in uncertainty gained by an increase in knowledge.

It should be noted that the factor $L(\mathbf{D}|\xi)/\int_{\xi}\pi_0(\xi)L(\mathbf{D}|\xi)\mathrm{d}\xi$ represents the impact of the evidence on the belief in the PDF of the parameters. Multiplying the prior PDF of the parameters by this factor provides a theoretical mechanism to update the prior knowledge of parameters with new data. Thus, Bayes' theorem measures to what degree new evidence should alter a belief in the PDF of the parameters.

Many fatigue test results indicated that the lognormal distribution is suitable for describing the statistical scattering of the fatigue lives (Nelson, 1982). Thus, the assumption of a lognormal distribution in the scatter of experimental results is a common assumption made, but given expert judgments, other PDFs may be used. The data are represented in the form of a likelihood function based on a lognormal distribution and are combined with a subjective prior distribution. The general form of the lognormal likelihood function used in the model parameter uncertainty analysis is shown as follows:

$$L\big(\mathbf{D}\big|\{p,q,s\}\big) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}sN_f}\exp\left(-\frac{1}{2}\left[\frac{\ln\big(N_f\big)-\ln(p)-q\ln\big(\Delta W_p\sigma_{\max}^{1+n'}\big)}{s}\right]^2\right) \tag{11}$$

where s is a model parameter equal to the natural logarithm of the standard deviation of the number of cycles to failure.

The variability of cyclic strength coefficient K' and cyclic strain hardening exponent n' resulting from the scatter of historical data are assumed to follow lognormal distributions. Based on the stress–strain relationship in Equation (3), the conditional PDF of K' and n' at a given stress and strain can then be obtained as

$$L\big(\mathbf{D}\big|\{K',n',s'\}\big)=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}s'(\Delta\sigma/2)}\exp\left(-\frac{1}{2}\left[\frac{\ln(\Delta\sigma/2)-\ln(K')-n'\ln\big(\Delta\varepsilon_p/2\big)}{s'}\right]^2\right) \tag{12}$$

where s' is a model parameter equal to the natural logarithm of the standard deviation of stress range.

The intercept parameters of the GDP (i.e., p, n', q, and s) automatically take into account any possible non-zero mean for error such that the model remains insensitive to a new parameter if being introduced as mean. This likelihood is combined with the PDFs developed to represent the prior state of knowledge resulting in an estimation of the posterior as described by Equation (10). As turbine disk test results can be prohibitively expensive and time consuming to acquire, model parameters p, n', q, and s in Equations (11) and (12) are better determined using Bayesian estimation rather than the maximum likelihood estimation for instances in which only small samples set are available. In a Bayesian approach, the uncertainty bounds are derived using the whole posterior joint distribution of parameters. The joint distribution of model parameters is the most efficient way to incorporate different possible sources of uncertainty into model predictions. Within the Bayesian framework, in order to make a fatigue life prediction at a given stress, the mean prediction life $\tilde{N}_f$ can be estimated as illustrated in Equation (13). This is basically the expected value of the model prediction for all possible parameters p, n', q, and s as presented with the posterior distributions.

$$\tilde{N}_f = \int\limits_{p,n',q,s} \pi\left(\{p, n', q, s\}|\mathbf{D}\right)\left(p\left(\Delta W_p \sigma_{\max}^{1+n'}\right)^q\right)\mathrm{d}p\mathrm{d}n'\mathrm{d}q\mathrm{d}s \qquad (13)$$

In the Bayesian approach, the posterior joint distribution of parameters represents the most up to date state of knowledge given the present evidence. Moreover, the available information is stored as a distribution of the model parameters. This distribution may be considered as prior information to be updated when new data become available. In practice, the statistical inferences in these equations involve high-dimensional integrations that usually are very computationally intensive. Therefore, most of Bayesian analyses are performed using Markov Chain Monte Carlo (MCMC) simulation. As an efficient technique for Bayesian inference problems, MCMC simulation is a general simulation technique based on drawing samples iteratively from proposed distributions and then correcting those draws in each step of the process to better approximate the target posterior distribution when this target distribution cannot be directly sampled. MCMC process starts with an initial guess for the parameter vector ξ and calculates the new set of parameters utilizing the features of the target posterior distribution. It generates a sample set $\xi = \{\xi_1, \xi_2, \xi_3, \ldots, \xi_m\}$, $\xi_i = \{p_i, n'_i, q_i, s_i\}$, $i = 1, 2, 3, \ldots, m$, representing the posterior density of parameters p, n', q, and s. For a given ξ, the PDF of LCF life can be readily predicted.

Clearly, any dependence among the parameters represented by the vector ξ is taken into account by the MCMC simulation.

For this study, a Bayesian updating procedure has been constructed to estimate the parameters p, n', q, and s in Equations (11) and (12). The Bayesian inference is solved using the WinBUGS (Lunn et al., 2000) platform to run the necessary MCMC simulation. Due to its convenience to use, it has been recently used in reliability engineering for accelerated life test data analysis (Azarkhail and Modarres, 2007a) and parameter estimation research (Azarkhail and Modarres, 2007b). For detailed information on WinBUGS, readers are referred to Cowles (2004).

Model Uncertainty Using a Black-Box Approach

Models are essential to understanding physical behaviors and predicting the responses of physical systems. Generating a life prediction model is the process of idealizing the complicated load conditions into a relatively simple form through making a set of assumptions. Moreover, apart from the simplifying assumption, models may also vary depending on the decisions made during modeling process with regard to the modeler's preference and requirements of model user. Given two or more life prediction models assessing a mechanical component, the problem of choosing a single approximation model that best assesses its fatigue life with the highest fidelity is often difficult. As model uncertainty derives from our lack of knowledge, it is categorized as epistemic uncertainty. As aforementioned, the third main source of uncertainties in engineering analysis, model uncertainty, should be incorporated into LCF life prediction. An interesting question to be answered is how precise are the obtained predictions expected to be.

In this study, the Bayesian inference is used a second time to characterize the total model uncertainty when compared to experimental results, similar to the work presented in Azarkhail et al. (2009). Output updating with independent experimental results (results not used to update model parameters) helps account for uncertainties not captured in the distributions developed for inputs and model parameters. In a black-box approach, the uncertainty is quantified with no knowledge of the model's inner workings or their respective contributions to the output uncertainty. The black-box approach is presented graphically in Figure 3.

From the black-box viewpoint, the error associated with the experimental measurements is assumed to be independent of that of the error resulting in model predictions (Azarkhail et al., 2009). For the cases that experimental uncertainties are not directly provided, experimental uncertainties must be estimated. Very rarely is information pertaining to experiential uncertainties available. When unavailable, uncertainties will need to be developed using

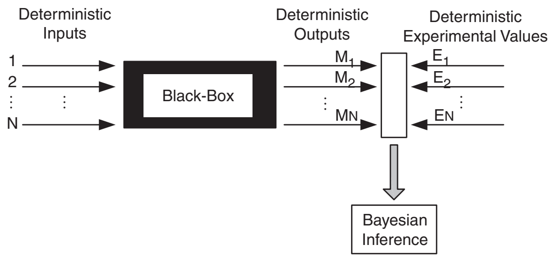

Figure 3. *Black-box approach to model uncertainty (Ontiveros et al., 2010).*

expert judgment or technological knowledge available from theoretical analyses and/or previous experimental results.

According to the black-box methodology, both the model prediction and the experimental result are considered to be independent representations of the physical reality being predicted. In order to compare these representations to the real value, the concept of a 'multiplicative error' (Azarkhail et al., 2009) for each test $i = (1, 2, \ldots, k)$ is used to characterize the experimental error $F_{t,i}$ and the model prediction error $F_{p,i}$. It is assumed that the ratio of real life and model prediction or experimental results is a random variable with lognormal distribution, which can be defined and represented as shown in Equations (14) and (15):

$$\frac{N_{\mathrm{real},\,i}}{N_{ft,i}} = F_{t,i} \quad F_t \ \sim LN(b_t, s_t) \tag{14}$$

and

$$\frac{N_{\mathrm{real},\,i}}{N_{fp,i}} = F_{p,i} \quad F_p \ \sim LN\big(b_p, s_p\big) \tag{15}$$

Combining Equations (14) and (15) leads to the relationship between the experimental uncertainty and model uncertainty,

$$F_{t,i} N_{ft,i} = F_{p,i} N_{fp,i} \tag{16}$$

and

$$\frac{N_{ft,i}}{N_{fp,i}} = \frac{F_{p,i}}{F_{t,i}} = F_{pt,i} \tag{17}$$

Assuming F_p and F_t are independent gives:

$$F_{pt} \ \sim LN\Big(b_p - b_t, \sqrt{s_p^2 + s_t^2}\Big) \tag{18}$$

After having observed a random sample of number of cycles to failure $\mathbf{N_{ft}} = \{N_{ft,1}, N_{ft,2}, \ldots, N_{ft,n}\}$ and model prediction $\mathbf{N_{fp}} = \{N_{fp,1}, N_{fp,2}, \ldots, N_{fp,n}\}$, one can combine the prior $\pi_0(b_p, s_p)$ with the likelihood

$$L(N_{ft,i}, N_{fp,i}, b_t, s_t | b_p, s_p) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\left(\frac{N_{ft,i}}{N_{fp,i}}\right)\sqrt{s_p^2 + s_t^2}} \exp\left(-\frac{1}{2} \times \frac{\left[\ln\left(\frac{N_{ft,i}}{N_{fp,i}}\right) - (b_p - b_t)\right]^2}{s_p^2 + s_t^2}\right) \tag{19}$$

Thus, the resulting posterior joint distribution using a Bayesian inference is

$$\pi(b_p, s_p | N_{ft,i}, N_{fp,i}, b_t, s_t) = \frac{\pi_0(b_p, s_p) \times L(N_{ft,i}, N_{fp,i}, b_t, s_t | b_p, s_p)}{\int_{s_p} \int_{b_p} \pi_0(b_p, s_p) \times L(N_{ft,i}, N_{fp,i}, b_t, s_t | b_p, s_p) \mathrm{d}b_p \mathrm{d}s_p} \tag{20}$$

where $\pi_0(b_p, s_p)$ is the prior joint distribution of parameters, $\pi(b_p, s_p | N_{ft,i}, N_{fp,i}, b_t, s_t)$ is the posterior joint distribution of parameters. The aim of this posterior joint distribution is to update the distributions of parameters b_p and s_p as new available data are incorporated.

Based on the above assumptions, the real-life distribution N_{real} can be readily obtained using MCMC samples of $\pi_0(b_p, s_p)$. By given a model prediction N_{fp}, using Equation (15) yields:

$$\left.\begin{array}{l} N_{fp} \text{ given as a model prediction} \\ F_p \sim LN(b_p, s_p) \\ N_{\text{real}} = F_p N_{fp} \end{array}\right\} \Rightarrow N_{\text{real}} \sim LN(\ln(N_{fp}) + b_p, s_p) \tag{21}$$

In this section, a black-box approach to evaluate model likelihood using experimental and model predictions under a Bayesian inference framework is developed so as to make an informed estimation of model uncertainty. This methodology is demonstrated with the engineering problem of a LCF life assessment process in the sections Probabilistic LCF Life Predictions and Output Updating Using a Bayesian Inference.

PROBABILISTIC LCF LIFE PREDICTIONS

The proposed probabilistic life prediction methodology was applied to experimental results of the turbine disk material GH4133 (Study on the Material Properties of turbine Disk and Case of an Aeroengine Series, 1996; Wang, 2006) to verify its feasibility and prediction capability. The heat treatment conditions of this alloy are austenitization (8 h at 1080°C, air-cooled) and tempering (16 h at 750°C, air-cooled). Details of mechanical properties of the materials, test conditions, and strain-life data are reported in Study on the Material Properties of turbine Disk and Case of an Aeroengine Series (1996) and Wang (2006). The tests were performed under axial total strain control with a triangular fully reversed waveform, using an axial extensometer placed on the specimen. Numerous tests were carried out with various conditions: mechanical strain range of 0.5–1.4% for isothermal LCF at temperature 400°C and 500°C under strain ratio $R_\varepsilon = -1$. To start the Bayesian updating, prior distributions are needed for the parameters. Knowledge about the distributions of the parameters is very limited due to limited theoretical analysis and experimental results. Therefore, we should subjectively capture as many uncertainties as possible in the priori estimate of the model parameters.

In the probabilistic analyses, input parameter variability was considered for coefficient K' and exponent n'. Once the prior distributions of the parameters $K'$and n' were chosen based on the theoretical analysis in Zhang (2007), new knowledge from the test results can be used to update these priors of K' and n' using Equation (10). Distributions for the input parameters were developed from available literature data for GH4133. To simulate specimen-to-specimen material property variability, each probabilistic variable was perturbed from its mean according to its distribution. Based on Equation (3), the material property of parameters can be obtained from the Bayesian inference approach using the prior likelihood in Equation (12) for the experimental data provided in Wang (2006). The result of Bayesian analysis for the input material parameters is listed in Table 1.

Using Bayesian approach, the available data can be incorporated into a distribution over the model parameters. The non-informative prior

Table 1. Summary statistics of input material parameters.

Parameter	Mean	Standard deviation	2.50%	Median	97.5%
$\ln(K')$	7.403	0.04954	7.306	7.403	7.501
n'	0.1005	0.006093	0.0885	0.1005	0.1125

distributions for the model input parameters are chosen to be uniform. The prior and posterior distributions of each input material parameter are presented in Figure 4.

Similarly, based on the experimental results of GH4133, the marginal posterior distributions of model input parameters can be obtained using the prior likelihood in Equation (13) and the generalized energy-based damage parameter in Equation (10). The result of Bayesian analysis for the model input parameters is listed in Table 2 and graphically in Figure 5.

After updating with the new test results, the posterior distributions of the parameters are more accurate than the prior distributions. Thus, probabilistic LCF life predictions utilizing the energy-based damage parameter can be performed based on the Bayesian analysis for the model input parameters. Combining Equation (13) with the summary statistics of model parameters and the material properties in Tables 1 and 2, the probabilistic LCF life for high-temperature structures or materials can be predicted.

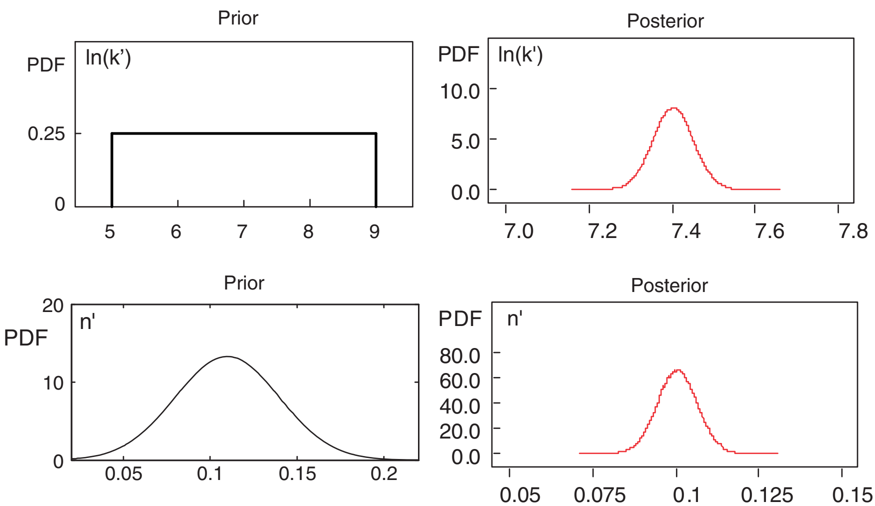

Figure 4. *Marginal distributions of model input material parameters using MCMC in WinBUGS. MCMC: Markov Chain Monte Carlo.*

Table 2. Summary statistics of model input parameters.

Parameter	Mean	Standard deviation	2.50%	Median	97.5%
$\ln(p)$	27.99	1.114	25.79	28	30.16
q	−0.5362	0.03084	−0.5963	−0.5365	−0.4753
s	0.3777	0.05799	0.2844	0.3708	0.5107

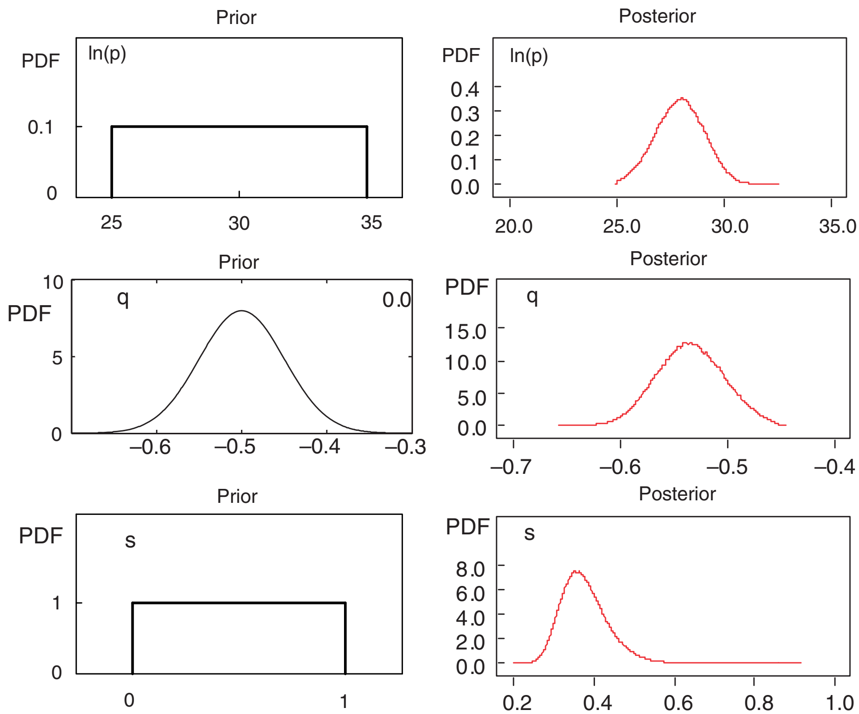

Figure 5. *Marginal distributions of model input parameters using MCMC in WinBUGS. MCMC: Markov Chain Monte Carlo.*

OUTPUT UPDATING USING A BAYESIAN INFERENCE

For this study, a Bayesian inference approach has been used for two purposes. At the first stage, it is constructed to develop PDFs over model parameters using the historical data, K', n', p, q, and s in Equations (11) and (12). Using Equation (13), the mean prediction life $\tilde{N}_f$ can be estimated based on the posterior distributions of p, n', q, and s. At the second stage, the Bayesian inference is used to compare model predictions with experimental results for the total model uncertainty.

The black-box approach for output updating requires the experimental uncertainty for the LCF life to be quantified as shown in Figure 1. Using the LCF life data available from other tests, the coefficient of variation calculated was used to place an uncertainty on the experimental lives. Figure 6 shows the PDF of the coefficient of variation calculated by the ReliaSoft Weibull++ 7 software. With the coefficient of variation calculated from 10

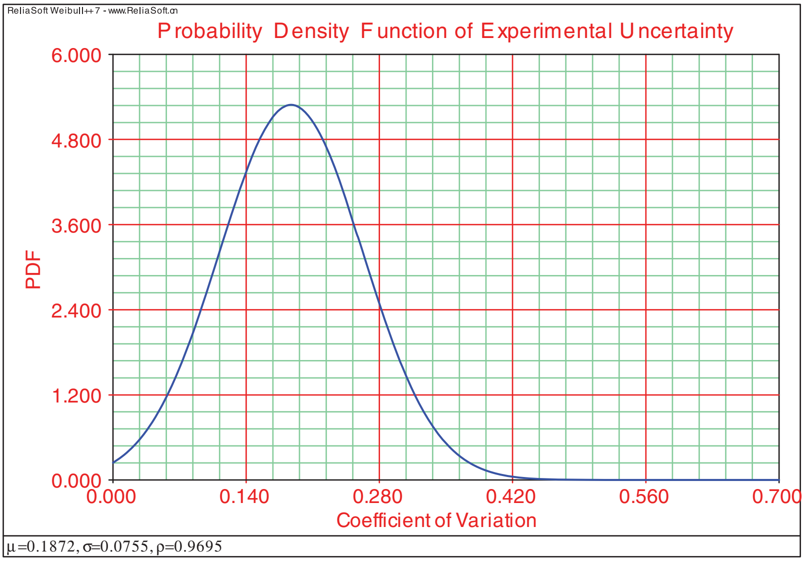

Figure 6. *Probability density function of experimental uncertainty.*

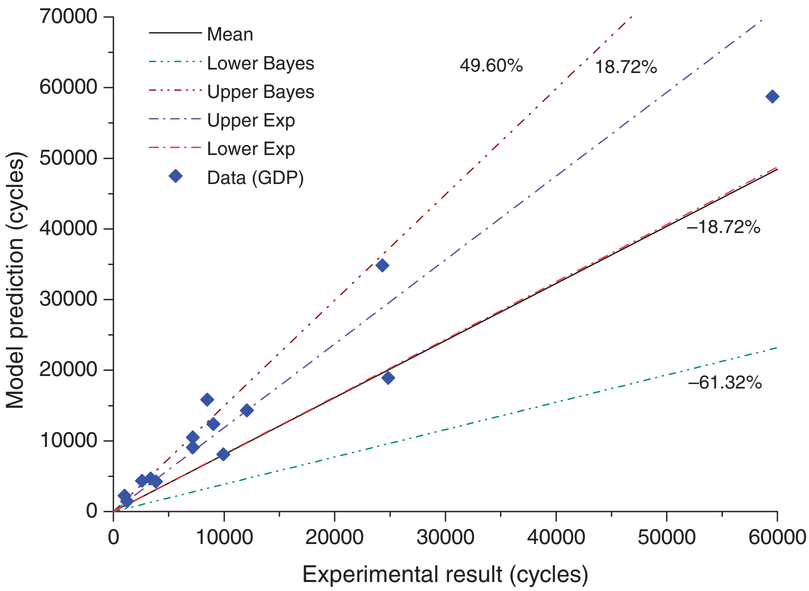

Figure 7. *Black-box results for LCF life prediction using the generalized damage parameter. LCF: low cycle fatigue.*

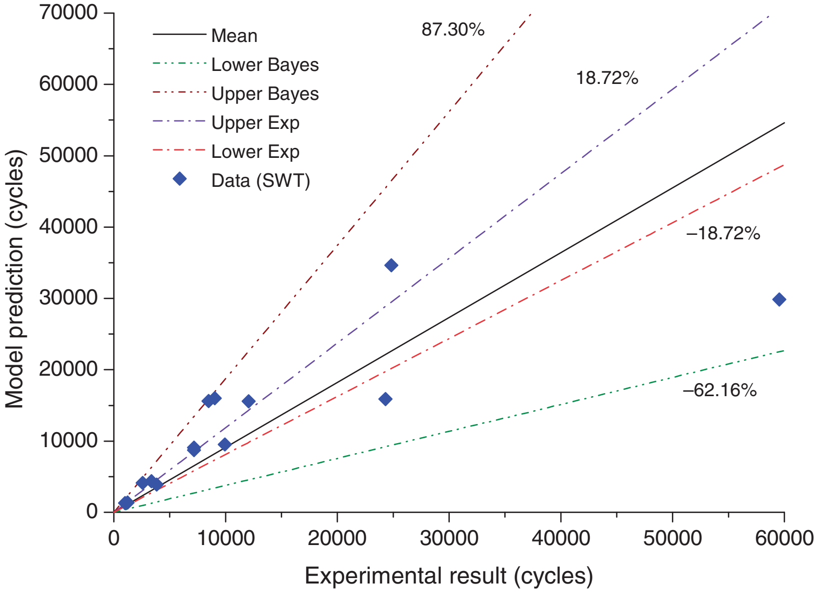

Figure 8. *Black-box results for LCF life prediction using the SWT damage parameter. LCF: low cycle fatigue; SWT: Smith-Watson-Topper.*

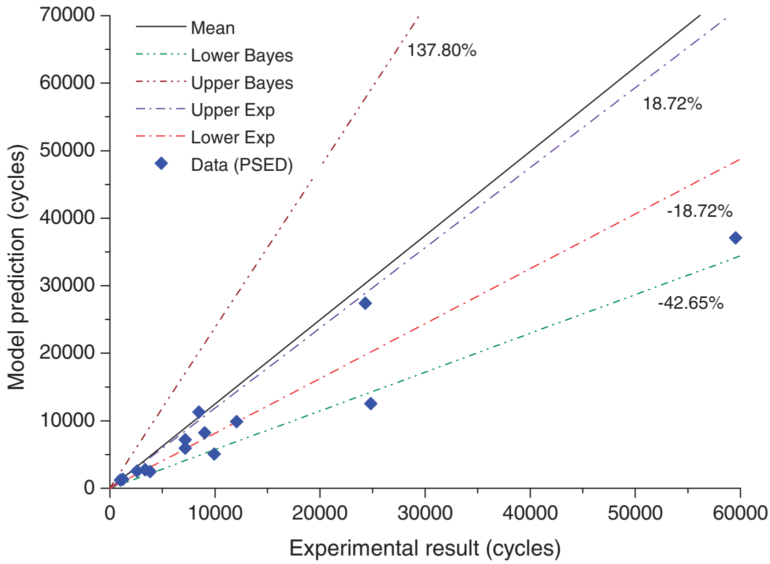

Figure 9. *Black-box results for LCF life prediction using the PSED damage parameter. LCF: low cycle fatigue; PSED: plastic strain energy density.*

group tests, a normal distribution was fit to the coefficient of variation with the following parameters: mean = 0.1872 and standard deviation = 0.0755 as presented in Figure 6. From the above analysis, the experimental uncertainty for use in the black-box approach was determined to be approximately, 18.72%, as the 'Upper/Lower Exp' lines shown in Figures 7–9. This experimental uncertainty is a combination of the measurement uncertainty and model input uncertainty.

The mean of the model prediction $\tilde{N}_f$ was compared with the experimental result in the black-box approach. The summary statistics for the marginal posterior PDFs of parameters (b_p, s_p) and the multiplicative error factor F_p using GDP are shown in Table 3.

The resulting estimated model uncertainty using Bayes' theorem has an upper bound of +49.6% and lower bound of −61.32% as shown in Figure 7, which correspond to the upper and lower bounds of the multiplicative error factor F_p.

The capability of this new model was evaluated and compared with two other popular parameters, the SWT and PSED ones. Smith et al. (1970) proposed that fatigue damage can be evaluated as the product of maximum stress and strain amplitude. The product of $\sigma_{\max}\varepsilon_a$ is conventionally referred to as a SWT parameter, which can be applied for LCF life prediction under strain controlled tests and simplified as

$$(\sigma_{\max}\varepsilon_a)N_f^{\gamma} = C \tag{22}$$

Similarly, the summary statistics of the black-box results using the SWT and PSED parameters are shown in Table 4. Based on SWT method in Equation (22) and PSED method in Equation (1), the black-box results for LCF life prediction are given in Figures 8 and 9, respectively.

Figures 7–9 show that all the predicted cyclic lives by these three damage parameters are in a factor of ±2 to the test ones. Probabilistic life prediction using the GDP method has a tighter uncertainty bounds than the SWT and PSED methods based on the same available knowledge. The tighter bounds are attributed to the GDP, which considers the mean stress effect and the

Table 3. Black-box summary statistics using GDP for experimental results.

Parameter	Mean	Standard deviation	2.50%	Median	97.5%
b_p	−0.2706	0.09179	−0.4527	−0.2707	−0.08733
s_p	0.3176	0.0788	0.2009	0.3051	0.506
F_p	0.8074	0.2916	0.3868	0.7624	1.496

GDP: generalized damage parameter.

Table 4. Black-box summary statistics using the SWT and PSED for experimental results.

Model	Parameter	Mean	Standard deviation	2.50%	Median	97.5%
SWT	b_p	−0.1737	0.1019	−0.3757	−0.1734	0.02868
	s_p	0.3803	0.0828	0.2579	0.3672	0.578
	F_p	0.9105	0.3935	0.3784	0.8399	1.873
PSED	b_p	0.1563	0.09428	−0.03168	0.157	0.3424
	s_p	0.3381	0.07813	0.225	0.3254	0.5262
	F_p	1.247	0.4781	0.5735	1.168	2.378

PSED: plastic strain energy density, SWT: Smith–Watson–Topper.

mechanism of cyclic hardening effect on fatigue life. Through the control damage parameter ($\Delta W_p \sigma_{\max}^{1+n'}$), it was shown that the probabilistic life prediction methodology developed could transform the complex correlation between N_f and $\sigma_{\max}$, σ_m, $\Delta\sigma$, $\Delta\varepsilon_p$, material properties (K', n') into a rational relation. The stochastic nature of fatigue performance was simulated by treating material properties (K', n') and the total model uncertainty as variables with distributions. The probabilistic predictions using GDP showed good agreement with the experiment results by mean and bounds (e.g., 2.5% and 97.5%). Based on the physical basis for both stress-life and fracture-mechanics analysis, $\Delta W_p \sigma_{\max}^{1+n'}$ can predict the LCF life with tighter uncertainty bounds than the others, as [+49.6 %, − 61.32 %]for GDP, [+87.3 %, − 62.16 %]for SWT, and [+137.8 %, − 42.65 %]for PSED, which leads to better decision making and model selection based on the same available knowledge.

According to both the GDP and SWT methods, the value of F_p for the model predictions is less than 1, which suggests a bias in this model to overpredict the real fatigue life. For the PSED method, the value of F_p for the model predictions is larger than 1, it shows a bias to underpredict the real fatigue life. Moreover, the uncertainty bounds will shift to represent the error associated with the model when the model has a tendency to overpredict or underpredict the fatigue life. It appears that, in Figures 7 and 9, the uncertainty bounds do not fully capture the scatter of data. This stems from the fact that the scatter of data in this analysis is assumed to be a result of the error in the experimental results and model predictions. Furthermore, the results presented for the model uncertainty are those of the model prediction corrected by the multiplicative error of model to the real fatigue life F_p. Therefore, the uncertainty bounds presented are those of the model estimation of reality. It also means that the GDP parameter has a tendency to over predict the real fatigue life as shown in Figure 7 and the PSED parameter to under predict the real fatigue life as shown in Figure 9. In the overpredict condition, the estimation of reality given the model prediction is expected to be lower.

As previously discussed, one of the advantages of Bayesian inference is that the previous analysis can be updated with new data. Using the WinBUGS platform to solve the Bayesian inference in probabilistic life prediction, it will reduce computational intensity and characterize the uncertainties associated with model input parameters. In engineering, the GDP can be used to evaluate the LCF damage with adequate test data and the developed probabilistic framework can be used to predict the LCF life by considering uncertainties due to lack of data. Besides, the application of this probabilistic methodology to other cases such as random loading spectrum and updating with more knowledge will be further evaluated.

CONCLUSIONS

In this article, a probabilistic LCF life prediction framework using different damage parameters is developed to quantify the input uncertainty of material properties and model uncertainty resulting from creation of different deterministic model parameters. To check the feasibility and validity of this methodology, the LCF test data of GH4133 were compared with the predicted results by the GDP, SWT, and PSED parameters. Some conclusions can be drawn from the present investigation.

1. Using different damage parameters, the proposed framework successfully implemented a Bayesian inference to the LCF life prediction, which incorporates the uncertainty of model parameters and material properties into the prediction. By updating the input parameters with new data, this probabilistic methodology can provide more valuable information for assessing the life of structural components.
2. When using a model to obtain a better predictive capability of fatigue damage in the selection, design, and safety assessments of engineering components, the uncertainty due to models should be accounted for. The black-box approach gives a quick estimation of uncertainty and describes the probabilistic relationship by comparing the mean model predictions to the experimental values.
3. It should be pointed out that the GDP developed yields more satisfactory probabilistic life prediction results for GH4133 under different temperatures than both the SWT and the PSED ones. It has a tighter uncertainty bounds than the SWT and PSED methods based on the same available knowledge. The proposed probabilistic methodology appears to be an

interesting alternative to the deterministic methods for LCF life prediction and estimating lower bounds of design quantiles.

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NOMENCLATURE

b_p = mean, error of model to the real value
b_t = mean, error of experiment to the real value
C, C_1, C_2 = material constants representing the material energy absorption capacity
$\mathbf{D}$ = vector of Data
E = elastic modulus
F_p = multiplicative error of model to the real value
F_t = multiplicative error of experiment to the real value
F_{pt} = multiplicative error of experiment to model prediction
K' = cyclic strength coefficient
$L(.)$ = likelihood function
$LN(.)$ = lognormal distribution function
N_f = number of cycles to failure
$\tilde{N}_f$ = mean prediction life
N_{fp} = model prediction
N_{ft} = experimental result
N_{real} = real fatigue life
n' = cyclic strain hardening exponent
s = model parameter, natural logarithm of the standard deviation of the number of cycles to failure
s' = model parameter, natural logarithm of the standard deviation of stress range
s_p = standard deviation, error of model to the real value
s_t = standard deviation, error of experiment to the real value
R_ε = strain ratio
α, β, γ = material constants representing the fatigue exponent
ε_a = strain amplitude
$\Delta\varepsilon_t$ = total strain range

$\Delta\varepsilon_e, \Delta\varepsilon_p$ = elastic strain range and plastic strain range
$\Delta\varepsilon_{\text{in}}$ = inelastic strain range
ΔW_p = plastic strain energy density (PSED)
ΔW_s = strain energy
$\sigma_{\max}$ = maximum stress
σ_m = mean stress
$\Delta\sigma$ = stress range
$\sigma_{\max}\varepsilon_a$ = SWT parameter
ξ = vector of parameters
$\pi(\xi|\mathbf{D})$ = posterior joint distribution of parameters
$\pi_0(\xi)$ = prior joint distribution of parameters

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