Original Article

Institution of MECHANICAL ENGINEERS
Journal of RAIL AND RAPID TRANSIT

Probabilistic modeling of damage accumulation for time-dependent fatigue reliability analysis of railway axle steels

Proc IMechE Part F:
J Rail and Rapid Transit
2015, Vol. 229(1) 23–33

DOI: 10.1177/0954409713496772
pif.sagepub.com
SAGE

Shun-Peng Zhu, Hong-Zhong Huang, Yanfeng Li, Yu Liu and Yuanjian Yang

Abstract
From the viewpoint of engineering applications, the prediction of the failure of railway axles plays an important role in preventing the occurrence of fatigue fractures. Combining a nonlinear damage accumulation model, a probabilistic *S-N* curve, and a one-to-one probability density functions transformation technique, a general probabilistic methodology for modeling damage accumulation is developed to analyze the time-dependent fatigue reliability of railway axle steels. The damage accumulation is characterized as a distribution in a general degradation path, which captures a nonlinear damage accumulation phenomenon under variable-amplitude loading conditions; its mean and variability change with time. Moreover, a framework for fatigue reliability assessments and service life prediction is presented based on the estimation of the evolution and probabilistic distribution of fatigue damage over time. The proposed methodology is then validated by experimental data obtained for a railway axle (45 steel and LZ50 steel). The time-dependent reliability is analyzed and demonstrated through probabilistic modeling of cumulative fatigue damage, and good agreement between the predicted results and the experimental measurements under different variable amplitude loadings is obtained.

Keywords
Fatigue, probabilistic, life prediction, reliability, damage accumulation, railway axle

Date received: 28 July 2012; accepted: 3 July 2013

Introduction

As one of the critical safety components of rail vehicles, the railway axle transmits the weight of the vehicle to the wheels and carries the driving and braking forces under complex load conditions.[1,2] Materials aging, through the evolution and accumulation of fatigue damage, is one of the major factors for a reduction in the reliability and safety of railway axles/components, whose failure often results in derailments, deaths and injuries.[3] Treatment of cumulative fatigue damage has been received considerable attention in the past decades.[4–8] Existing models focus on the deterministic fatigue process, however, the fatigue damage accumulation in railway axles/components under service operation is stochastic in nature. This stochasticity is caused by the inherent variability in fatigue resistance of materials as well as the statistical nature of service loads experienced by the components.[9] Life prediction and reliability assessment is a challenging task in the analysis and design of critical components.[5,10–15] There is considerable interest in developing a new approach to predict the lifetime of railway axles/components through probabilistic modeling of damage accumulation, particularly for the components, such as railway axle and bogie frame, operating in harsh environments.

From both deterministic and probabilistic points of view, determination of the relationship between engineering components' fatigue life and applied stress is important for reliability-based design processes. For fatigue damage analysis under stochastic loading, a conventional method is to define an equivalent stress range based on a linear damage rule (also called the Palmgren–Miner rule) and deterministic *S-N* curves. Its main shortcomings are the lack of

School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, People's Republic of China

Corresponding author:
Hong-Zhong Huang, School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu, Sichuan 611731, People's Republic of China.
Email: hzhuang@uestc.edu.cn

an assessment of variability, loading sequence effects, load interaction effects and uncertainty. Due to the uncertainty in the statistical analysis of experimental data, it is still difficult to accurately estimate the probability distribution of fatigue life under variable-amplitude loading. There are numerous stochastic methods reported in the literature for the calculation of cumulative damage.[4,9,16–20] Recently, by considering the randomness of loading process and fatigue resistance of materials, Shen et al.[9] proposed a probabilistic distribution model of fatigue damage under narrow band random loading. Combining a nonlinear fatigue damage accumulation rule and a stochastic *S-N* curve representation technique, Liu and Mahadevan[4,17] developed a general method for stochastic fatigue life prediction and two methods for time-dependent fatigue reliability analysis. However, these methods require additional experimental data compared with the traditional prediction methods and the uncertainties associated with external loading are ignored. Based on the Weibull model via compatibility and functional equations techniques, Castillo et al.[18] suggested a generalized model to predict fatigue behavior for a given level of stress. A series of papers have been published in which damage tolerance options have been applied to railway axles and the factors that influence the residual lifetime as well as the required inspection interval investigated.[21–23] These studies show the potential for moving towards a light-weight design of railway components. Another approach reported in the literature was create an amplitude crack growth algorithm based on the probabilistic approach inherent in the NASGRO crack growth equation to study the effects of corrosion on the fatigue behavior of A1N steel.[6,20,24,25] Rathod et al.[19] developed a method for probabilistic modeling of fatigue damage accumulation for multi-level stress loading conditions using the Palmgren–Miner rule and a probabilistic *S-N* curves. The major limitations of this method is that it assumes that the damage accumulation process is a linear phenomenon whereas for real engineering components the damage process is generally a nonlinear phenomenon (the Palmgren–Miner rule is not sufficient to describe the fatigue failure mechanism), and also it is prone to uncertainty.

For mechanical engineers, the estimation of damage state during an inspection is important for the updating of the reliability analysis of an existing railway axle/component. There are two aspects that need to be considered during the probabilistic modeling of fatigue damage accumulation:[4,26] first, an accurate physics-based damage accumulation model; and second, an appropriate uncertainty modeling technique. A review of the research reported in the literature highlighted that the used uncertainty modeling process usually involves complex mathematical calculations. In this paper, a simple approach is developed to characterize the stochastic nature of fatigue damage accumulation modeling of railway axle steels. This approach attempts to minimize the computational complexity and can be explained by well-known physical laws. Moreover, it can be effectively used for fatigue reliability analysis of railway axles/components under variable loading.

The aim of the present paper is to develop a general framework to describe the relationship between fatigue life and applied stress and then to verify and validate the framework using railway axles/component data. The rest of this paper is organized as follows. The section 'Probabilistic modeling of fatigue damage accumulation' presents details about the new approach for probabilistic modeling of damage accumulation in order to capture the evolution of fatigue damage and its variability. The primary focus is to discuss how damage accumulation can be measured in terms of fatigue life distributions using the probability distribution function (PDF) transformation technique. The section 'A framework for time-dependent reliability analysis' presents a framework for fatigue reliability analysis by estimating the probabilistic distribution of fatigue damage over time. The section 'Experimental validation' presents a validation of the proposed methodology using the experimental data obtained for railway axle steels. Conclusions are drawn in the final section.

Probabilistic modeling of fatigue damage accumulation

Varying loads on railway axles/components lead to a cumulative failure mechanism. Cumulative damage is an irreversible degradation process that occurs throughout the life of mechanical components which ultimately causes failure.[5,27,28] There are many types of cumulative damage in engineering (including fatigue, creep, corrosion, wear and erosion), which are commonly used as a measure of degradation. Degradation analysis is a probabilistic modeling process of a failure mechanism degradation path based on the comparison of a projected distribution to a predefined failure threshold.[29,30] Degradation paths evolve in the space of a degradation measure (indicator) which evolves toward the failure threshold. Based on this analysis, the behavior under cumulative damage can be predicted for structural safety analysis, planning of inspection times and design criteria. The component reliability manifested as the degradation measure is deteriorating probabilistically with time, which is graphically shown in Figure 1.

In Figure 1, D_0 is the initial damage, D_C is the critical threshold value which varies appreciably among components/specimens in practice. Thus, a probabilistic threshold value rather than a deterministic threshold value is more appropriate, such as the critical cumulative damage at the fatigue failure point for railway axles/components.

Using Figure 1, Wang and Coit[29] explained that the variability of a degradation measure increases

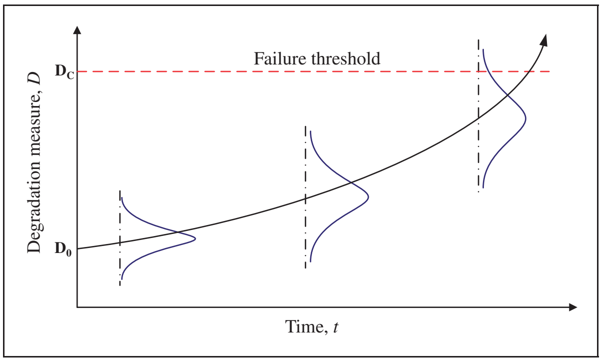

Figure 1. Degradation path example.

with time. As a measure of degradation, the cumulative fatigue damage at any given measurement point can be viewed as a random variable, which follows a probability distribution where the mean value and variability increase with time. As a large amount of scatter is observed in fatigue data, the number of cycles to failure at any given stress level can be considered to be a random variable distributed normally or lognormally.[31,32] The cumulative fatigue damage is also a random variable, which is a function of loading time and can be deduced from the distribution of fatigue life. In the following sections, a new probabilistic approach for modeling fatigue damage accumulation is proposed to improve the deficiencies from a physics-based perspective as it minimizes the computational complexity.

Modeling the mean value of cumulative fatigue damage

Consider a railway axle/component subjected to constant amplitude loading. According to the basic physics of fatigue failure, the cumulative fatigue damage $D(n)$ at loading cycles n is initially equal to D_0, and then increases monotonically. If environmental and frequency-based effects on $D(n)$ are ignored, the rate of damage accumulation should depend on D_0, the actual state of damage, and the loading stress amplitude S. In most engineering problems, the dependency on the actual state of damage can be characterized in terms of the loading cycles n. Although the linear relationship between the damage accumulation and the number of loading cycles is applicable in many cases, there are situations where a nonlinear description is more appropriate; based on the nature of the fatigue failure. Applying the presented definitions, the general form of the cumulative fatigue damage curve in Figure 1 can be expressed as

$$D(n) = D_0 + f(S, D_0)n^a \tag{1}$$

where $f(S, D_0)$ describes the rate of damage accumulation associated with cyclic loading and a is a yet-to-be-determined "damage accumulation exponent" which depends on the amplitude of alternating stress. The function $f(S, D_0)$ is determined based on the boundary conditions and failure criterion. It is assumed that failure occurs when the cumulative damage $D(n)$ equals the critical threshold damage D_C, and the number of loading cycles n equals the constant amplitude fatigue life N_f. Substituting these conditions into equation (1) gives

$$f(S, D_0) = \frac{D_C - D_0}{N_f^a} \tag{2}$$

Substituting equation (2) into equation (1) leads to

$$D(n) = D_0 + (D_C - D_0)\left(\frac{n}{N_f}\right)^a \tag{3}$$

The S-N curve equation which described the relationship between fatigue life N_f and stress level S can be given as

$$N_f S^m = C \tag{4}$$

where C is a fatigue strength constant and m is the slope of the S-N curve.

Combining with equation (4), equation (3) can be rewritten as

$$D(n) = D_0 + (D_C - D_0)\left(\frac{S^m}{C}\right)^a n^a \tag{5}$$

It should be noted that equation (5) satisfies the two basic physical conditions discussed in Ye and Wang.[33] Figure 1 illustrates possible damage accumulation curves as a function of fatigue loading cycles as described in equation (5). Moreover, it is noticed that the damage accumulation curve begins at the initial value D_0 and passes through the location experiencing the failure criterion, i.e. $D(N_f) = D_C$. It is worth noting that equation (3) reduces to the Marco–Starkey model[34], the Corten–Dolan cumulative damage theory[35] and the Palmgren–Miner rule[36] when $a = 1$ and $D_C = 1$. Thus, by considering both the S-N curve model and the physics-based perspective, equation (5) is a general damage accumulation model under single-stress level loading. Similarly, assuming that no initial damage has occurred and damage evolves nonlinearly in a power relationship by using damage accumulation exponent a_i at stress level S_i and failure occurs when $D_C = 1$, equation (5) can be extended to the multi-level stress condition as

$$D(n) = \sum_{i=1}^{j} D(n_i) = \sum_{i=1}^{j} \left(\frac{S_i^m}{C}\right)^{a_i} n_i^{a_i} \tag{6}$$

Equation (6) represents a load-dependent damage model with nonlinear evolution (when $a_i \neq 1$). Using equation (5) and equation (6), the mean value of the cumulative fatigue damage at any given loading cycle can be calculated for constant and variable amplitude loading conditions, respectively. However, fatigue cyclic loading is a stochastic process in nature. Thus, it is extremely important to treat fatigue damage accumulation as a random process and to calculate the distribution of damage accumulation, which is explored based on the PDF of fatigue life N_f in the following sections.

Distribution of cumulative fatigue damage

Fatigue damage increases with the number of applied loading cycles under constant/variable amplitude loading. Numerous models to describe the average or typical fatigue damage accumulation behavior have been reported in the literature. However, individual fatigue damage accumulation paths may diverge significantly from the mean. Therefore, the distributions of cumulative fatigue damage depicted in Figure 1 need to be remodeled and recalculated. Treating the fatigue failure life as a random variable which follows a certain distribution allows the distribution of damage accumulation to be established using the one-to-one PDF transformation technique.[37] A probabilistic interpretation of the general damage accumulation curve is shown in Figure 2, which describes how to obtain the PDF of the damage accumulation based on the known PDF for fatigue life at any given stress level.

In Figure 2, curve c_1 is the trend line of the mean cumulative damage as given by equation (5) at a given stress level S, and it depicts a nonlinear relationship between the cumulative damage and number of loading cycles. Note that the initial variability of loading cycles is zero and increases with the number of loading cycles. Considering that cumulative damage at a given stress level is S and no initial damage, equation (5) can be simplified to

$$D(n) = kn^a \tag{7}$$

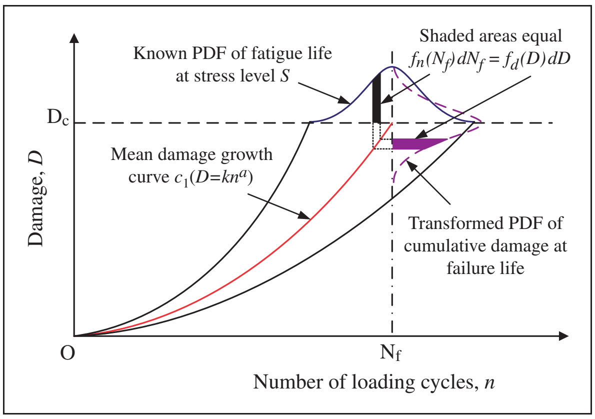

Figure 2. Graphical interpretation of one-to-one PDF transformation under cyclic loading.

where $k = D_C(S^m/C)^a$. Similarly, the cumulative damage $D(n_i)$ under a multi-level stress can be expressed as the sum of $k_i n_i^{a_i}$ with $k_i = D_C(S_i^m/C)^{a_i}$.

According to the one-to-one PDF transformation methodology developed in Benjamin and Cornell[37], when a random variable is directly or functionally related to another random variable as shown graphically in Figure 2, the unknown PDF of that variable can be derived using this transformation technique based on the known PDF of another variable. Thus, two aspects are needed to derive the distribution of cumulative damage $D(n)$: first, a clearly defined relation between cumulative damage and loading cycles, such as equation (7) in this work; and second, the known PDF of the number of loading cycles. As mentioned earlier, the variability of the fatigue failure life N_f can be described by a lognormal distribution with a mean of μ_{N_f} and standard deviation of σ_{N_f}. The PDF of a lognormally distributed N_f is defined as

$$f_n(N_f) = \frac{1}{N_f \sigma_{N_f}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln N_f - \mu_{N_f}}{\sigma_{N_f}}\right)^2\right),$$
$$N_f \sim LN(\mu_{N_f}, \sigma_{N_f}) \tag{8}$$

To obtain the PDF of cumulative damage $D(n)$, the cumulative distribution function must be obtained and then differentiated. As previously mentioned, since the cumulative damage is a function of the fatigue failure life as shown in equation (3), the relationship between the PDF of the cumulative damage, $f_d(D)$, and that of the fatigue failure life can be derived as[37]

$$f_d(D)dD = f_n(N_f)dN_f \tag{9}$$

Equation (9) can be graphically derived from the equal-shaded areas shown in Figure 2. Substituting equation (7) and equation (8) into equation (9) gives the PDF of $D(N_f)$ as follows

$$f_d(D) = \frac{1}{Da\sigma_{N_f}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln D - \ln k - a\mu_{N_f}}{a\sigma_{N_f}}\right)^2\right) \tag{10}$$

It should be noted from equation (10) that the cumulative damage also follows a similar distribution, which can be represented as

$$D(N_f) \sim LN(a\mu_{N_f} + \ln k, a\sigma_{N_f}) \tag{11}$$

where the standard deviation of $D(N_f)$ after transformation is $\sigma_D = a\sigma_{N_f}$.

Modeling the trend curve of the variance

As shown in equation (7), the cumulative damage increases nonlinearly with the number of loading cycles at any given stress level. Many researchers have demonstrated that the variability or standard deviation of cumulative damage increases monotonically with an increasing number of loading cycles, while the variability of fatigue lives increases with decreasing stress levels.[19,29,38,39] Based on the previous discussions, the variability of the cumulative damage can be derived as a function of the variability in the fatigue lives.

Assuming that the variability in loading cycles is equal to zero at the initial stage (i.e. at $n = 0$ where the initial cumulative damage equals zero), it continuously increases to a certain value at the fatigue failure life. Using the geometric reasoning technique proposed in Rathod et al.,[19] the rate of change in variability as a function of the number of loading cycles can be interpreted as shown in Figure 3.

In Figure 3, c_1 represents the mean cumulative damage trend line and c_2 is the 1-σ curve of fatigue life distribution. Similarly, by converting the coordinate system in Figure 3 into a double logarithmic coordinate system, the rate of change of the standard deviation r_σ of loading cycles can be derived as

$$r_\sigma = \frac{\sigma_{N_f}}{\ln N_f} \tag{12}$$

Furthermore, the standard deviation of the number of loading cycles n can be obtained as

$$\sigma_n = \left(\frac{\sigma_{N_f}}{\ln N_f}\right) \ln n \tag{13}$$

Considering equations (11) and (13), the standard deviation of the cumulative damage D is given by

$$\sigma_D = a\left(\frac{\sigma_{N_f}}{\ln N_f}\right) \ln n \tag{14}$$

It should be noted that equation (14) can be used to capture the variability in the cumulative damage

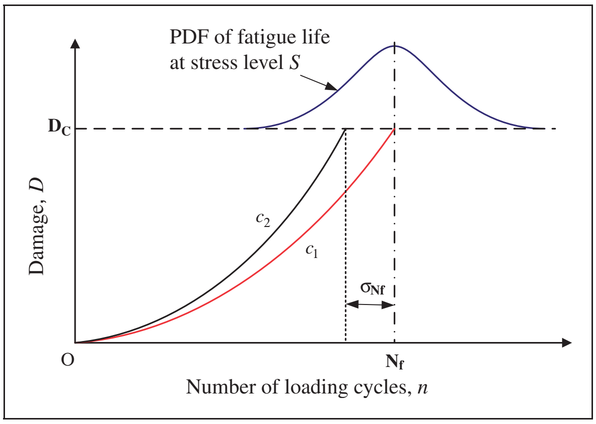

Figure 3. Graphical interpretation of the rate of change in variability as a function of the number of loading cycles.

under constant amplitude loading. Assuming that damage is accumulated stochastically and independently under each level stress loading, equation (14) can be extended to obtain the total variability in the cumulative damage $D(n)$ at the time of fatigue failure for components under multi-level stress loading

$$\sigma_D = \sqrt{\sum_{i=1}^{j} \left(a_i \left(\frac{\sigma_{N_{fi}}}{\ln N_{fi}}\right) \ln n_i \right)^2} \tag{15}$$

where i represents the level of stress under multi-level stress loading.

In the next section, the methodology developed for probabilistic analysis of cumulative fatigue damage is used to predict fatigue life and assess the reliability of railway axles/components.

A framework for time-dependent reliability analysis

Fatigue failure of a railway axle/component can be reflected specifically in the evolution and accumulation of damage, which compromises reliability and safety. Based on the current state of a railway axle/component, ensuring reliability and continued safety require a quantitative assessment of the component in its projected future state. For this assessment, the proposed probabilistic modeling approach can be used to estimate the distribution of damage accumulation over its projected operation time. The framework for fatigue reliability analysis is depicted in Figure 4, with its outcome being the basis for decisions on its suitability for continued service using the labels "Reliable," "Conditional Reliability" and "Unreliable." The process labeled as probabilistic estimation of damage accumulation in Figure 4 consists of both physics-based and data-driven-based techniques and is the key element of this framework. It requires the development of methods that are predictive and can provide accurate estimates of the evolution and probabilistic distribution of fatigue damage over time for reliability and safety assessments and service life prediction.

The need to assess the future state of railway axles/components arises because of their safety-critical nature. At the current damage state, a physics-based predictive model combined with a data-driven based probabilistic model is implemented in this paper to estimate the distribution of damage accumulation. It aims to assess the remaining useful life and reliability in a projected future state. The deterministic physics-based predictive model is based on a nonlinear fracture mechanics and/or continuum damage mechanics model. However, the data-driven probabilistic model is based on a stochastic approach; it attempts to identify and quantify the uncertainty that arises from physical variability, statistical uncertainty and model structure uncertainty. The data-driven probabilistic model complements the physics-based predictive

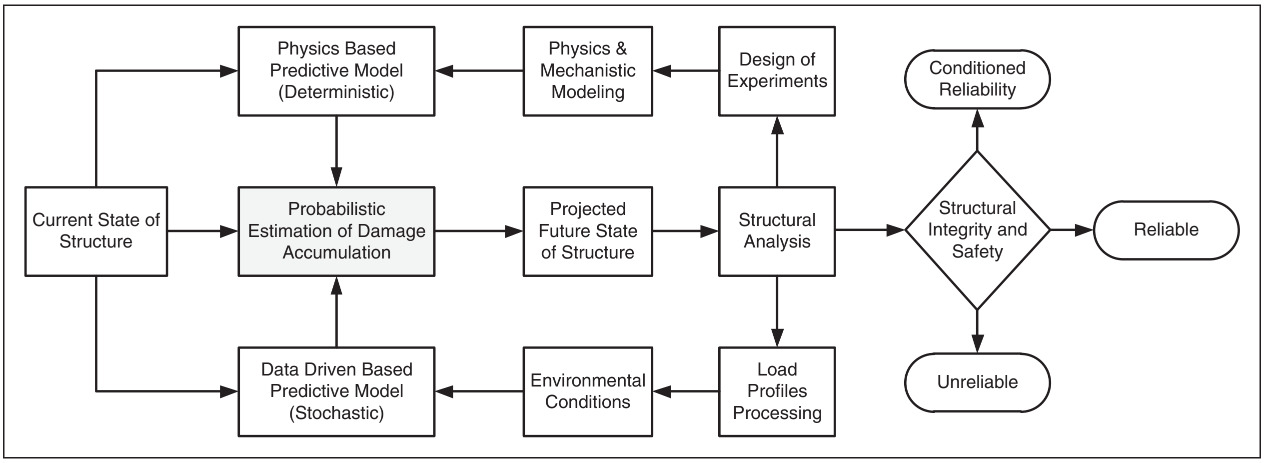

Figure 4. A simplified flow diagram for life prediction and reliability assessment of engineering components.

model by accounting for unmeasured and/or unmodeled exogenous influences. Based on the damage estimation of the current state of a structure, the damage state of that structure at any instance follows a distribution that is not deterministic due to the uncertainty in fatigue damage accumulation. Moreover, its distribution in the projected future state can be estimated by the data-driven probabilistic model if physical parameters such as initial damage size, loading spectrum, environmental conditions, are known. The present paper discusses only the reliability assessment of engineering components. Based on the developed probabilistic approach for modeling damage accumulation, the objective of this analysis is to compute the reliability for a given service time, which is now briefly discussed.

According to the boundary conditions of fatigue process, fatigue failure occurs when the cumulative damage $D(n)$ reaches the critical threshold damage D_C, where $E(D_C) = 1$. The following two assumptions are used in this paper.

1. The critical threshold damage has the same distribution as the cumulative damage measure.
2. At the fatigue failure point ($n = N_f$), the variability $\sigma^2_{D_C}$ of D_C is equal to that of cumulative damage measure σ^2_D which continuously increases with loading life.

Since the damage accumulation is treated as a dynamic random variable with a lognormal distribution, its mean and variance are dependent on the usage life given by equation (5) and equation (13).

For a given D_C, the critical threshold damage of the component under consideration, failure occurs when the random cumulative damage D is larger than D_C. The limit state function $G(n)$ for fatigue reliability analysis can be written as

$$G(n) = D_C - D(n) \tag{16}$$

where $G(n) = 0$, referred to as the limit state, is the boundary between the safe domain (defined by $G(n) > 0$) and the failure domain (defined by $G(n) < 0$).

Following the lognormal assumption of cumulative fatigue damage, given the model for $D(n)$, one is able to derive the reliability of a component in terms of the general damage accumulation curve

$$\begin{aligned} R &= \Pr[G(n) > 0] \\ &= 1 - \Phi\left(-\frac{\mu_{D_C} - \mu_D}{\sqrt{\sigma^2_{D_C} + \sigma^2_D}} \right) \end{aligned} \tag{17}$$

Substituting equation (6) and equation (15) into equation (17) gives

$$R = 1 - \Phi\left(-\frac{\mu_{D_C} - \sum_{i=1}^{j} D_C \left(S_i^m / C\right)^{a_i} n_i^{a_i}}{\sqrt{\sigma^2_{D_C} + \sum_{i=1}^{j} \left(a_i\left(\sigma_{N_{fi}} / \ln N_{fi}\right) \ln n_i\right)^2}} \right) \tag{18}$$

It is possible to assess and predict the structural reliability using the scheme described in Figure 4, which is based on the probabilistic assessment of damage accumulation for a given time period and provides a dynamic reliability prediction that considers continuous degradation phenomenon of the railway axles/components with usage cycles. In the next section, the effectiveness of the proposed probabilistic modeling method and framework is validated using the experimental data.

Experimental validation

In this section, results obtained by applying the proposed probabilistic modeling method are compared with experimental data available in the literature. The objective is to demonstrate the applicability of the proposed probabilistic framework in managing the condition railway axles/components.

Table 1. Statistics of constant amplitude *S-N* curve data.

Material	Reference	Stress amplitude S_i (MPa)	Statistics of fatigue life ($\ln N_f$) Mean	Standard deviation
45 steel-1	Zheng and Wei [40]	525	5.33	0.32
		500	5.50	0.34
		475	5.59	0.29
		450	5.82	0.35
		400	6.15	0.26
45 steel-2	Yan et al.[41]	750	4.49	0.15
		650	5.00	0.14
		630	5.04	0.12
		590	5.24	0.10
		520	5.65	0.24
LZ50 steel	Wang et al.[42]	320	10.48	0.36
	Yan et al.[43]	305	11.62	0.26
		290	12.40	0.37
		280	13.08	0.36
		275	13.44	0.83

Experimental values of fatigue properties of materials

The statistics of collected experimental data of 45 steel and LZ50 steel are listed in Table 1, including material name, reference, loading stress amplitude, mean number of cycles to failure and corresponding standard deviation at different stress levels. The fatigue life under constant amplitude loading is assumed to follow a lognormal distribution.

Validation of the fatigue reliability estimation

The objective of time-dependent fatigue reliability analysis is to predict the reliability variation under different loading conditions. The variability $\sigma_{D_C}^2$ of threshold damage at fatigue failure life and the variability of cumulative damage at any given usage cycle are estimated for this purpose. Using equation (14) and the values listed in Table 1, the variability in threshold damage at fatigue failure life is computed under the assumption $\sigma_{D_C}{=}\sigma_D$ at N_f

$$\sigma_{D_C}{=}a\left(\frac{\sigma_{N_f}}{\ln N_f}\right)\ln n \tag{19}$$

The general damage accumulation path theory states that a nonlinear relationship exists between accumulated damage level and number of loading cycles. Similarly, the variability of cumulative damage at any given usage cycle (n) can be obtained. In order to assess the reliability of railway axles/components subjected to different variable loading conditions, the variability of threshold damage at fatigue failure life needs to be estimated. The values of σ_{D_C} can be calculated for different loading stress levels using equation (19). The damage accumulation exponent a is an experimentally fitted function of the stress amplitude, which accounts for loading sequence effects.

Using the data in Table 1, the model parameters for 45 steel were obtained by fitting the *S-N* curve model in equation (4) as

$$m = 2.43604,\ C = 9.85123 \times 10^8 \tag{20}$$

Using the proposed probabilistic damage accumulation model, the reliability can be calculated based on a random process representation of the material's *S-N* curve (see Table 1). Once this variability is estimated, the time-dependent reliability of 45 steel for any given time period can be assessed using equation (18) as

$$R{=}1-\Phi\left(-\frac{1-\left(1.0151\times10^{-9}S^{2.43604}\right)^a n^a}{\sqrt{\sigma_{D_C}^2+\left(a\left(\sigma_{N_f}/\ln N_f\right)\ln n\right)^2}}\right) \tag{21}$$

The reliability function equation (21) expresses the reliability as a function of usage cycles (n) and thus is time dependent. The reliability for 45 steel at different stress level is plotted in Figure 5.

Similarly, using equations (18) and(19), the time-dependent reliability curves for LZ50 steel under different stress level are plotted in Figure 6.

From Figures 5 and 6, it is clear that reliability lessens with an increase in time or number of loading cycles. Moreover, initially the reliability is relatively high and decreases as the number of usage cycles

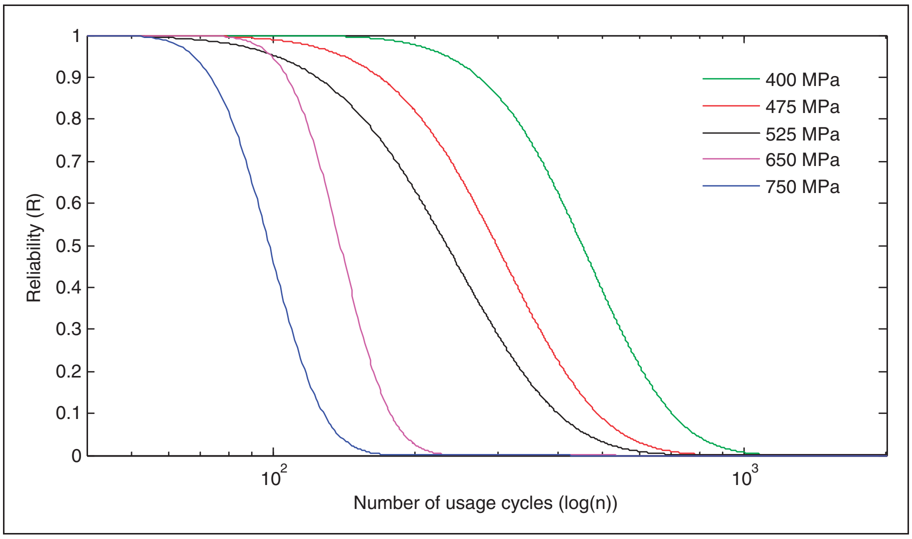

Figure 5. Time-dependent reliability plot for 45 steel at different stress levels.

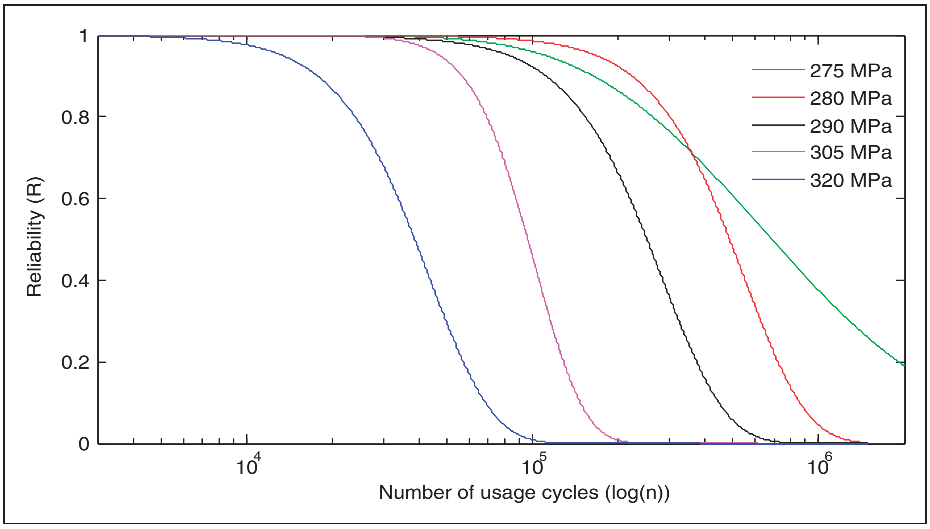

Figure 6. Time-dependent reliability plot for LZ50 steel at different stress level.

increases, which explains the periods of crack initiation and propagation. The high and stable reliability phase represents the crack initiation period as it relates to stress levels. The reliability loss phase corresponds to the crack propagation period. It should be noted that higher loading stress levels could result in smaller crack initiation periods. The faster loss of reliability during the crack propagation phase indicates a faster degradation or higher rate of damage accumulation occurred in these materials.

The fact that railway axles are subjected to random amplitude loadings should be taken into account. For the probabilistic fatigue modeling and reliability analysis, the final objective is to predict the reliability variation at any given usage cycle under different loadings. Thus, to assess and predict the reliability of railway axles/components subjected to multi-level stress loading, the variability of the threshold damage at N_f needs to be estimated first based on the experimental descriptions listed in Table 2 and equation (15). Similarly, the variability in damage accumulation at a given usage cycle can be obtained. The fatigue tests were performed according to the programmed block loading listed in Table 2. After estimating the variability at threshold damage and variability at any given usage cycle, the reliability of 45 steel subjected to specific amplitude loadings can be estimated using equation (18). The variation of predicted reliability is compared with those from experimental data. The comparisons are shown in Figure 7 in which the predicted and experimental variations are plotted together. The curves are the prediction of the proposed method and the points are experimental results.

It should be noted from Figure 7 that the predicted results (prediction 1 & 2) closely agree with

Table 2. Experimental description of 45 steel under variable amplitude loadings.

Material	Loading	Variable amplitude loadings[a]
45 steel-1	1	Multi-block loading: $240(10^5) \rightarrow 350(8 \times 10^4) \rightarrow 400(2.5 \times 10^4) \rightarrow 500(10^4) \rightarrow 400(2.5 \times 10^4) \rightarrow 350(8 \times 10^4) \rightarrow 240(10^5)$
45 steel-2	2	Multi-block loading: $500(1.5 \times 10^4) \rightarrow 590(4 \times 10^3) \rightarrow 626.6(5 \times 10^3) \rightarrow 590(4 \times 10^3) \rightarrow 500(1.5 \times 10^4)$

[a]The stress level S_i (MPa) and the applied number of cycles n_i.

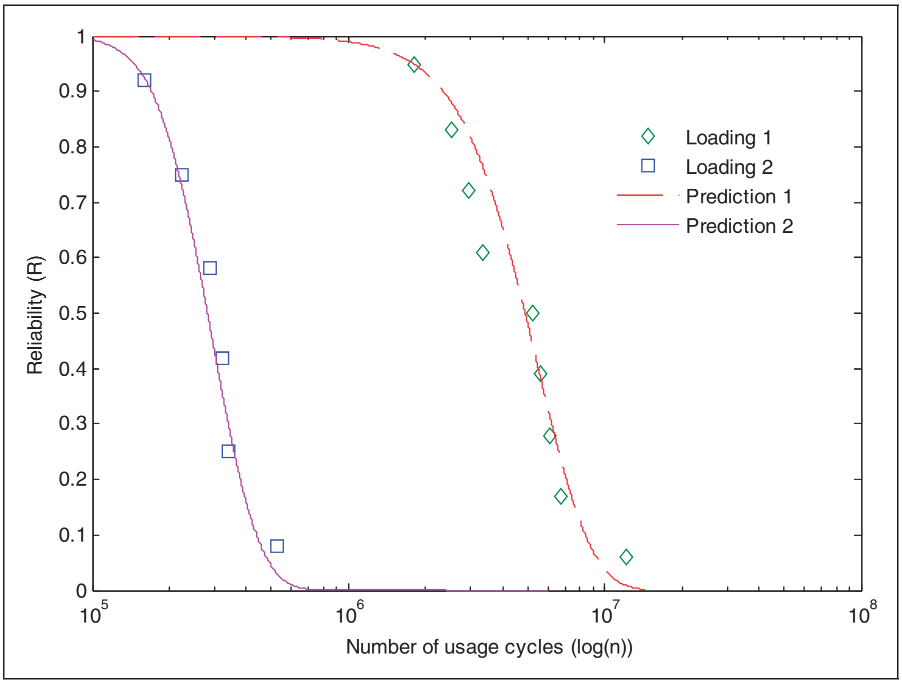

Figure 7. Fatigue reliability variation in predicted and experimental results for 45 steel.

the experimental data (loading 1 & 2) for 45 steel under different loading conditions. Thus, the proposed methodology is efficient in estimating the time-dependent reliability variation under cyclic fatigue loading and it is appropriate for preliminary analysis at the design stage with respect to fatigue for most metallic materials. This provides a simple way to model the probabilistic distribution of damage accumulation and capture the real life behavior of a railway axle/component. Moreover, it can be extended to model the probabilistic damage accumulation under other fatigue failure distributions, such as normal or Weibull distributions. It has been validated using experimental data under variable amplitude loading, however, railway axles/components are subjected to much more complex loading conditions, such as multi-axial fatigue loading. In particular, the application of the proposed method for time-dependent reliability analysis and life assessment of full-scale railway axles under variable loading conditions needs to be further evaluated.

Conclusions

In this paper, a probabilistic modeling approach to describe the evolution and accumulation of fatigue damage in railway axles/components is developed. The study is essential for the proper characterization of a railway axle/component's response during its service. There are three key contributions reported in this paper. First, a general probabilistic method for modeling damage accumulation is developed to analyze the time-dependent fatigue reliability. The model is able to capture the real-life behavior of railway axle steels. It combines a nonlinear fatigue damage accumulation rule and the one-to-one PDF transformation technique. Second, the main advantage of the proposed method is that it treats damage accumulation as a nonlinear phenomenon in a general degradation path, which can be extended to damage accumulation analysis under non-fatigue degradation behaviors. Existing models in the literature are shown to be special cases of the proposed method. Third, a framework for fatigue reliability analysis is presented through probabilistic modeling of cumulative fatigue damage. It emphasizes the accurate estimates of the evolution and probabilistic distribution of fatigue damage over time for reliability assessments and service life prediction. The proposed method is suitable for, and validated for, deterministic variable-amplitude loading of railway axle steels, which contributes to a balance of the cost-effectiveness and safety for railway axles under a stochastic operation environment. Further validation is required for small-scale specimens under multi-axial fatigue loading conditions. Moreover, extensive work on probabilistic analysis theory and tools is still needed when dealing with different types of uncertainties during structural health management of railway axle/components.

Funding

This research was partially supported by the Open Project Program of Traction Power State Key Laboratory of Southwest Jiaotong University under the contract number TPL1101, and the National Natural Science Foundation of China under the contract number 11272082.

Acknowledgements

The authors wish to express their thanks to Dr Tongdan Jin of Texas State University, Dr Guoliang Huang of the University of Arkansas and Dr Gary Paradee of the University of Maryland for their helpful discussions and correcting the use of English in this paper.

References

1. Bayraktar M, Tahrali N and Guclu R. Reliability and fatigue life evaluation of railway axles. *J Mech Sci Technol* 2010; 24(3): 671–679.
2. Beretta S, Ghidini A and Lombardo F. Fracture mechanics and scale effects in the fatigue of railway axles. *Engng Fract Mech* 2005; 72(2): 195–208.
3. Smith R and Hillmansen S. A brief historical overview of the fatigue of railway axles. *Proc IMechE, Part F: J Rail Rapid Transit* 2004; 218(4): 267–277.
4. Liu Y and Mahadevan S. Stochastic fatigue damage modeling under variable amplitude loading. *Int J Fatigue* 2007; 29(6): 1149–1161.
5. Fatemi A and Yang L. Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials. *Int J Fatigue* 1998; 20(1): 9–34.
6. Beretta S and Carboni M. Variable amplitude fatigue crack growth in a mild steel for railway axles: experiments and predictive models. *Engng Fract Mech* 2011; 78(5): 848–862.
7. Alihosseini H and Dehghani K. Modeling and failure analysis of a broken railway axle: effects of surface defects and inclusions. *J Fail Anal Prevent* 2010; 10(3): 233–239.
8. Zerbst U, Lundén R, Edel KO, et al. Introduction to the damage tolerance behaviour of railway rails-a review. *Engng Fract Mech* 2009; 76(17): 2563–2601.
9. Shen H, Lin J and Mu E. Probabilistic model on stochastic fatigue damage. *Int J Fatigue* 2000; 22(7): 569–572.
10. Zhu SP, Huang HZ and Wang ZL. Fatigue life estimation considering damaging and strengthening of low amplitude loads under different load sequences using fuzzy sets approach. *Int J Damage Mech* 2011; 20(6): 876–899.
11. Zhu SP and Huang HZ. A generalized frequency separation–strain energy damage function model for low cycle fatigue–creep life prediction. *Fatigue Fract Engng Mater Struct* 2010; 33(4): 227–237.
12. Zhu SP, Huang HZ, Li HQ, et al. A new ductility exhaustion model for high temperature low cycle fatigue life prediction of turbine disk alloys. *Int J Turbo Jet Engines* 2011; 28(2): 119–131.
13. Kim J, Yi J, Zi G, et al. Fatigue life prediction methodology using entropy index of stress interaction and crack severity index of effective stress. *Int J Damage Mech* 2013; 22(3): 375–392.
14. Kwofie S and Rahbar N. A fatigue driving stress approach to damage and life prediction under variable amplitude loading. *Int J Damage Mech* 2013; 22(3): 393–404.
15. Zerbst U, Beretta S, Köhler G, et al. Safe life and damage tolerance aspects of railway axles-a review. *Engng Fract Mech* 2013; 98: 214–271.
16. Nagode M and Fajdiga M. On a new method for prediction of the scatter of loading spectra. *Int J Fatigue* 1998; 20(4): 271–277.
17. Liu Y and Mahadevan S. Efficient methods for time-dependent fatigue reliability analysis. *AIAA J* 2009; 47(3): 494–504.
18. Castillo E, Fernández-Canteli A and Ruiz-Ripoll ML. A general model for fatigue damage due to any stress history. *Int J Fatigue* 2008; 30(1): 150–164.
19. Rathod V, Yadav OP, Rathore A, et al. Probabilistic modeling of fatigue damage accumulation for reliability prediction. *Int J Qual Stat Reliab* 2011; doi:10.1155/2011/718901.
20. Beretta S and Carboni M. Experiments and stochastic model for propagation lifetime of railway axles. *Engng Fract Mech* 2006; 73(17): 2627–2641.
21. Zerbst U, Schödel M and Beier HT. Parameters affecting the damage tolerance behaviour of railway axles. *Engng Fract Mech* 2011; 78(5): 793–809.
22. Beretta S, Carboni M and Cervello S. Design review of a freight railway axle: fatigue damage versus damage tolerance. *Materwissen Werkstftec* 2011; 42(12): 1099–1104.
23. Zerbst U and Beretta S. Failure and damage tolerance aspects of railway components. *Eng Fail Anal* 2011; 18(2): 534–542.
24. Beretta S, Carboni M, Fiore G, et al. Corrosion–fatigue of A1N railway axle steel exposed to rainwater. *Int J Fatigue* 2010; 32(6): 952–961.
25. Beretta S, Carboni M, Lo Conte A, et al. An investigation of the effects of corrosion on the fatigue strength of AlN axle steel. *Proc IMechE, Part F: J Rail Rapid Transit* 2008; 222(2): 129–143.
26. Zhu SP, Huang HZ, Ontiveros V, et al. Probabilistic low cycle fatigue life prediction using an energy-based damage parameter and accounting for model uncertainty. *Int J Damage Mech* 2012; 21(8): 1128–1153.
27. Zhu SP, Huang HZ, Liu Y, et al. A practical method for determining the Corten–Dolan exponent and its

application to fatigue life prediction. *Int J Turbo Jet Engines* 2012; 29(2): 79–87.
28. Zhu SP, Huang HZ, He LP, et al. A generalized energy-based fatigue-creep damage parameter for life prediction of turbine disk alloys. *Engng Fract Mech* 2012; 90: 89–100.
29. Wang P and Coit DW. Reliability and degradation modeling with random or uncertain failure threshold. In: *The annual reliability and maintainability symposium*, Institute of Electrical and Electronics Engineers, Orlando, FL, 22–25 January 2007, pp.392–397.
30. Usynin A, Hines JW and Urmanov A. Uncertain failure thresholds in cumulative damage models. In: *The annual reliability and maintainablity symposium*, Institute of Electrical and Electronics Engineers, Las Vegas, NV, 28–31 January 2008, pp.334–340.
31. Wu W, Liou H and Tse H. Estimation of fatigue damage and fatigue life of components under random loading. *Int J Press Vessels Pip* 1997; 72(3): 243–249.
32. Yao J, Kozin F, Wen YK, et al. Stochastic fatigue, fracture and damage analysis. *Struct Saf* 1986; 3(3–4): 231–267.
33. Ye DY and Wang ZL. A new approach to low-cycle fatigue damage based on exhaustion of static toughness and dissipation of cyclic plastic strain energy during fatigue. *Int J Fatigue* 2001; 23(8): 679–687.
34. Marco S and Starkey W. A concept of fatigue damage. *Trans ASME* 1954; 76(4): 627–632.
35. Corten HT and Dolan TJ. Cumulative fatigue damage. In: *The international conference on atigue of metals*, Institution of Mechanical Engineers, London, UK, 10–14 September 1956, pp.235–246.
36. Miner MA. Cumulative damage in fatigue. *J Appl Mech* 1945; 12(3): A159–A164.
37. Benjamin JR and Cornell CA. *Probability, statistics, and decision for civil engineers*. New York, NY: McGraw-Hill, 1970.
38. Coit DW, Evans JL, Vogt NT, et al. A method for correlating field life degradation with reliability prediction for electronic modules. *Qual Reliab Engng Int* 2005; 21(7): 715–726.
39. Pascual FG and Meeker WQ. Estimating fatigue curves with the random fatigue-limit model. *Technometrics* 1999; 41(4): 277–290.
40. Zheng XL and Wei JF. On the prediction of *P-S-N* curves of 45 steel notched elements and probability distribution of fatigue life under variable amplitude loading from tensile properties. *Int J Fatigue* 2005; 27(6): 601–609.
41. Yan JH, Zheng XL and Zhao K. Prediction of fatigue life and its probability distribution of notched friction welded joints under variable-amplitude loading. *Int J Fatigue* 2000; 22(6): 481–494.
42. Wang H, Gao Q and Yan GL. Fatigue behaviors of medium carbon steel in low stress and ultra-long life regime. *J Mech Strength* 2004; 26: 267–270.
43. Yan GL, Wang H and Gao Q. On ultra-long life fatigue behavior of 50 axle steel under ultrasonic frequency. *Chin Railw Sci* 2004; 25(2): 78–81 (In Chinese).