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Life cycle reliability assessment of new products—A Bayesian model updating approach

Weiwen Peng [a], Hong-Zhong Huang [a,*], Yanfeng Li [a], Ming J. Zuo [a,b], Min Xie [c]

[a] School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China
[b] Department of Mechanical Engineering, University of Alberta, Edmonton, AB, Canada T6G 2G8
[c] Department of System Engineering and Engineering Management, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China

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ABSTRACT

The rapidly increasing pace and continuously evolving reliability requirements of new products have made life cycle reliability assessment of new products an imperative yet difficult work. While much work has been done to separately estimate reliability of new products in specific stages, a gap exists in carrying out life cycle reliability assessment throughout all life cycle stages. We present a Bayesian model updating approach (BMUA) for life cycle reliability assessment of new products. Novel features of this approach are the development of Bayesian information toolkits by separately including "reliability improvement factor" and "information fusion factor", which allow the integration of subjective information in a specific life cycle stage and the transition of integrated information between adjacent life cycle stages. They lead to the unique characteristics of the BMUA in which information generated throughout life cycle stages are integrated coherently. To illustrate the approach, an application to the life cycle reliability assessment of a newly developed Gantry Machining Center is shown.

1. Introduction

Modern industrial societies have been characterized by the ever-increasing pace of new products appearing on the market. There is also an ever-increasing reliability requirement for these newly developed products. To deliver a new product with high reliability, it is necessary for the companies/manufacturers to track and manage its reliability throughout its life cycle. This requires coherent reliability assessment of the new product as life cycle stages move on. Accordingly, a life cycle reliability assessment approach is needed to cope with the rapidly increasing pace and continuously evolving reliability requirements of new products. Generally, the life cycle reliability assessment of a new product requires effective use of different types of data and information available throughout the life cycle. However, as the advancement of modern technology and the aggravation of market competition continue, available data for reliability assessment of a new product are extremely sparse and sometimes contain subjective information. As a result, it is impossible to obtain an accurate estimation of reliability using classical methods, which generally pertain to large sample sizes or abundant reliability data.

Alternatively, the Bayesian method is becoming more accepted in reliability engineering. Numerous articles have discussed the reliability assessment with different data types and reliability information, which form the foundation of life cycle reliability assessment. Walls and Quigley [30], Seth [29], Yadav et al. [37], and Augustine et al. [2] have proposed specific methodologies to deal with subjective information in reliability assessment. Hamada et al. [14], and Graves et al. [13] developed hierarchical Bayesian methods for assessing system reliability with multilevel binomial data. Huang et al. [16], Briand and Huzurbazar [5], Xu and Tang [36], Zhong et al. [39], and Reese et al. [27] have presented models and approaches for reliability assessment with lifetime data for different system structures subjected to various reliability information situations. Furthermore, Ching and Leu [6] developed a framework for estimating time-varying reliabilities with condition-state data sets. Wang et al. [32] presented a Bayesian updating mechanism to deal with reliability assessment with evolving, insufficient, and subjective data sets. Wilson et al. [33] and Anderson-Cook [1] described Bayesian approaches for reliability assessment of complex systems by combining multilevel heterogeneous binomial data, lifetime data, and degradation data.

These models and methodologies have formed a solid foundation for life cycle reliability assessment of new products. Further adopting the Bayesian approach for reliability assessment of new products in different life cycle stages, the following papers have appeared in the literature. Yadav et al. [38] proposed a framework for capturing subjective information from different sources for

* Corresponding author. Tel.: +86 28 6183 0248; fax: +86 28 6183 0227.
E-mail address: hzhuang@uestc.edu.cn (H.-Z. Huang).

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reliability assessment in the development stage of new products. Johnson et al. [19] applied the hierarchical Bayesian model to assess the reliability of complex products in the early in-service stage. Xing et al. [35] proposed a dynamic Bayesian evaluation method for reliability evaluation of a binomial system throughout the development stage. Quigley and Walls [17] proposed a coherent inference framework for reliability estimation during the product development stage by considering both Bayes and empirical Bayes inferences. Considering the Bayesian method in the study of product life cycle, Ho and Huang [15] presented a Bayesian decision model to assist the optimal life decision for new products during the life cycle.

These papers concentrated exclusively on specific life cycle stages for particular types of products. However, little attention has been given to the life cycle reliability assessment of new products. Traditionally, product reliability assessment in a specific life cycle stage is investigated, e.g., the development stage (Yadav et al., [38]) and the early launch stage (Johnson et al., [19]). Each of these methods is effective in a specific life cycle stage for particular types of products. However, when dealing with life cycle reliability assessment of new products, the applications of these methods face difficulties, and there is no systematic approach reported. Since the existing methods are based on different assumptions for different types of products, they can hardly be combined, and the information obtained in different life cycle stages can hardly be integrated. Moreover, this inconsistency between these models may lead to inaccurate estimations and result in poor design or unnecessary investment for new products.

In this paper, a comprehensive Bayesian model updating approach (BMUA) is proposed to deal with life cycle reliability assessment of new products. The BMUA consists of an information integration framework, two Bayesian information toolkits, and the corresponding Bayesian reliability models. Three critical aspects are highlighted in the proposed BMUA: (1) life cycle stages are investigated as a whole; (2) reliability models developed in different life cycle stages are interrelated; and (3) information generated in different life cycle stages is integrated and transited comprehensively under the information integration framework with the Bayesian models and information toolkits.

The paper is organized as follows. The general Bayesian model updating approach is presented in Section 2 with specific descriptions of the framework and critical steps. Two indispensable information toolkits for the BMUA are developed in Section 3. In Section 4, an application example of the proposed BMUA is illustrated for the life cycle reliability assessment of a newly developed Gantry Machining Center. We then summarize the proposed BMUA in Section 5.

2. A general Bayesian model updating approach

The life cycle of a new product consists of multiple stages. In particular, the electronic and manufacturing industries have demonstrated the multiple-stage nature of product life cycles. We note that there are many ways to define the multiple stages of a new product's life cycle (Yang, [40]; Murthy et al., [21]). Both marketing literature [11] and production literature have developed relative ways to deal with the multiple life cycle stages for particular problems, e.g., Cooper and Kleinschmidt [9] and Cohen et al. [10]. For the purpose of this paper, the life cycle is divided into three major stages. Our interest is in constructing a general methodology for life cycle reliability assessment of new products. We are especially concerned with building the BMUA framework, integration of information in each life cycle stage, and transition of integrated information between different life cycle stages.

2.1. Multi-stage product life cycle

In the field of reliability engineering, product life cycle is interconnected with the development process, status, and reliability requirements of a product. It can be divided into multiple stages based on particular perspectives of reliability engineering (Yang, [40]). For example, to specify the reliability for new product development, Murthy et al. [21,22] developed a new model of product life cycle in which the life cycle was divided into eight phases and grouped into three stages. Similarly, targeting on the life cycle reliability assessment in this paper, the product life cycle is divided into three stages. Each stage is defined according to the status of the product, work contents, techniques, and available data/information for reliability assessment. Fig. 1 provides a pictorial representation of the product life cycle. The definitions of particular stages are presented below.

Stage I (*Predevelopment*): As described in Fig. 1, this stage is concerned with a non-physical conceptualization of the product. No physical prototype is built for testing. Major considerations in this stage are analysis, evaluation, and comparison of potential design options and determining the final design of the product. Technologies for product design and assessment are implemented through computational modeling and simulation. Specifically, computer added engineering (CAE) technology is used for design and preanalysis of the product, encompassing simulation, validation, and optimization of the product. It provides information to support performance assessment and decision-making. As a result, reliability information in this stage mainly includes the knowledge and experience of experts gained through CAE technology, and the outputs of computational modeling and simulation, e.g., a stress analysis of components using finite element analysis (FEA). The historical data of existing similar product are also included as prior information in this stage. Reliability assessment of the product mainly depends on subjective information and available simulation data gathered from the design tools or techniques.

Stage II (*Development*): This stage deals with the physical embodiment of the product through research, development, and prototyping. Both the component and product prototypes are built. Major objectives at this stage are improving design and verifying that the desired performance is met through testing, problem detecting, and design improvement. Major technologies used include environmental tests, reliability tests, and preproduction demonstrations. Among these tests, the environmental test, environmental stress screening test, reliability growth test, accelerated life test (ALT), and accelerated degradation test (ADT) are closely related to the reliability assessment of the product. CAE

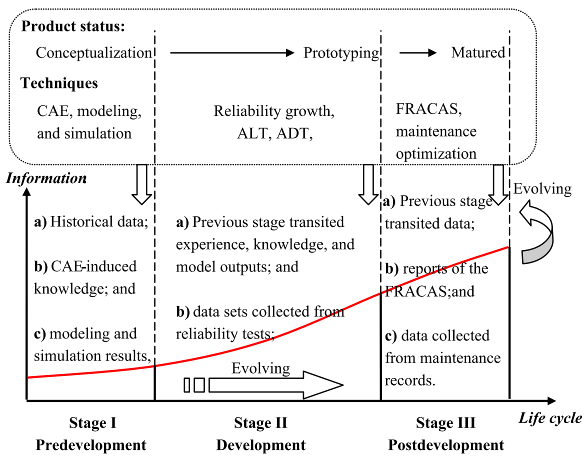

Fig. 1. Life cycle definition of a new product from the perspective of reliability assessment.

is also implemented to utilize the experience and knowledge of experts. Reliability assessment in this stage mainly depends on data collected from reliability tests. The knowledge and experience gathered by experts and the outputs of the model for reliability assessment in the predevelopment stage are also included as prior information.

Stage III (*Postdevelopment*): This stage is concerned with the remainder of product life cycle, which includes production and support of the product. Two major obligations at this stage are retaining the designed-in performance during production and maintaining the in-service performance throughout the warranty phase. Statistical process control, acceptance inspection and testing, built-in monitoring, and maintenance optimization techniques are implemented in this stage. Failure reporting, analysis, and corrective action system (FRACAS) is used to construct the reliability database for the product with the information gathered via these techniques. Accordingly, reliability assessment in this stage mainly depends on the data sets collected by the FRACAS and relative maintenance records. The information transited from the development stage is also integrated in this stage as prior information for reliability assessment, which includes the outputs of the model for reliability assessment in the development stage and knowledge and experience obtained by experts throughout the development of the product.

2.2. *The proposed Bayesian model updating approach*

The general Bayesian model updating approach (BMUA) consists of an information integration framework and three Bayesian models. Fig. 2 depicts a descriptive framework of the general BMUA. The information integration framework is demonstrated by the arrows of information flow, which is a coherent flow for information integration and transition throughout the life cycle. Three Bayesian models are highlighted in the rectangles with rounded corners, which are the information processor and reliability estimator in the life cycle stages. The information available in each life cycle stage is integrated and transited under the information integration framework, which is carried out by updating these Bayesian models throughout the life cycle stages. Reliability assessment of the product throughout the life cycle is based on the Bayesian model in each life cycle stage.

2.2.1. *The information integration framework for the BMUA*

As depicted in Fig. 2, the available information in each life cycle stage includes subjective information and objective information. The contents of information vary from stage to stage. The information integration framework integrates both types of information in different life cycle stages and accommodates the

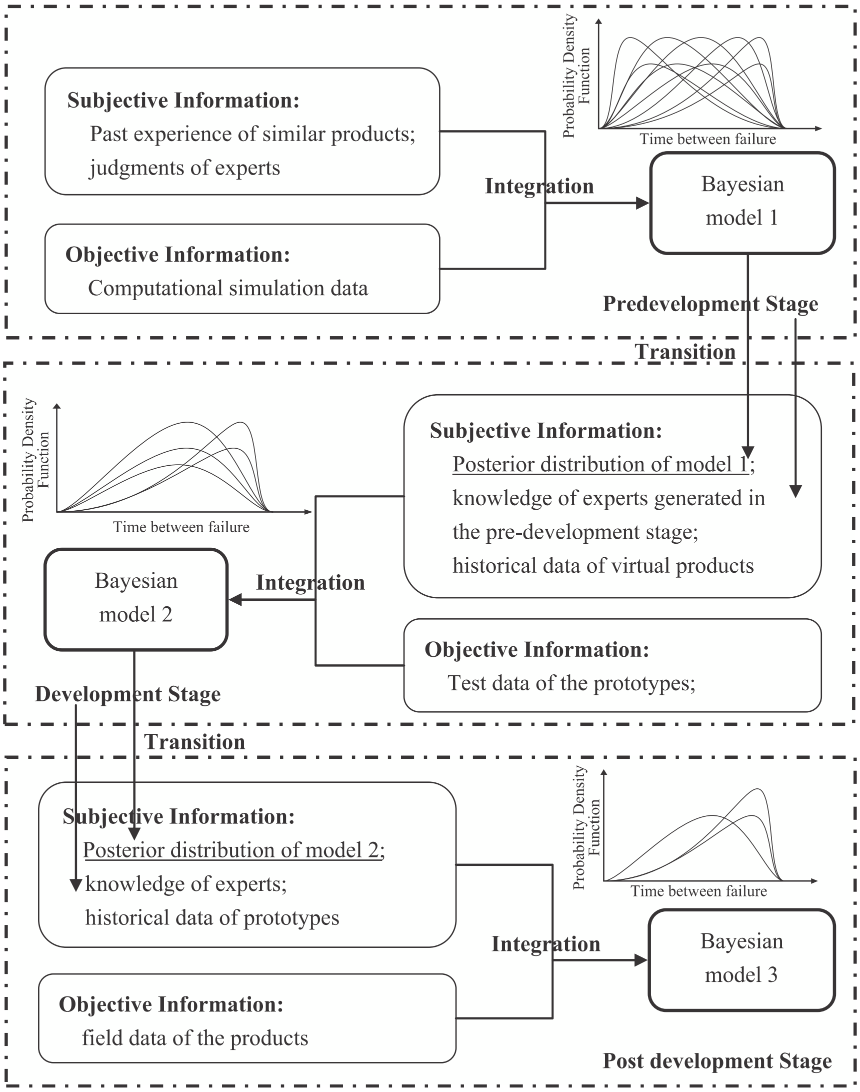

Fig. 2. The framework of Bayesian model updating approach (BMUA).

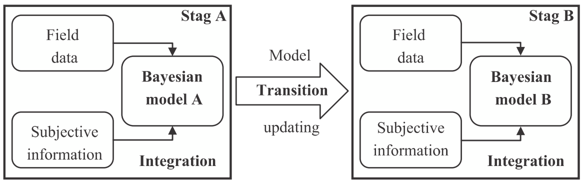

Fig. 3. An example of information integration framework.

flow of information between these stages. Fig. 3 depicts a simple information integration framework.

Suppose that a reliability model is chosen for the product. First, the subjective information and field data in stage A are integrated by constructing Bayesian model A. This is a process of information integration in the framework. During this process, the subjective information is quantified to the prior distribution of model parameters. The derived prior distribution is then updated by the field data available in stage A using the Bayesian theory. After the information is integrated in stage A, an information connect is constructed between these two adjacent stages. This is a process of information transition between adjacent stages in the framework. A model updating strategy is applied to fulfill this information transition. This model updating strategy gradually reduces the uncertainty in model A and replaces it with a more precise one (model B) as more detailed information is obtained. This is accomplished by incorporating the outputs of model A and the knowledge gained in stage A with the field data in stage B using the Bayesian method. The data and information from stage A are transited as the prior information in stage B. The updated Bayesian model (model B) is constructed based on this transited prior information and the available field data in stage B. Meanwhile, these Bayesian models in different life cycle stages are suggested for the reliability estimation when they are built.

In short, the information generated throughout the life cycle stages is gradually integrated, transited, and accumulated by constructing and updating the Bayesian models. Within the information integration framework, two critical aspects are handled specifically in this research: (1) quantifying the subjective information into prior distribution of model parameters in a specific stage; and (2) transiting the integrated information from one stage to the next stage for prior derivation. These lead to development of two toolkits for the BMUA in Section 3.

2.2.2. *The Bayesian models for the BMUA*

As mentioned earlier, the available information in each life cycle stage includes knowledge and experience of experts, historical data of similar existing products, and field data obtained in that stage. To use such information for reliability assessment of the product, Bayesian models are constructed within the framework of the BMUA. These models act as information integrators and reliability estimators in the BMUA. Meanwhile, the implementation of the information integration framework discussed above is also based on the derivation and formulation of these Bayesian models.

Generally, let a random variable T with probability density function $f(t;\theta)$, $\theta \in \Theta$ denote the lifetime of a product. Mathematically, the Bayesian theory can be expressed as

$$p(\theta|T_s) = \frac{l(T_s|\theta)\pi(\theta)}{\int_{\Theta} l(T_s|\theta)\pi(\theta)d\theta} \tag{1}$$

where $\pi(\theta)$ is the prior distribution, which describes the quantified subjective information in a specific stage before any field data of T_s is obtained. This information is contained in the subjective information boxes in Fig. 2. $l(T_s|\theta)$ is the likelihood function, which describes the objective information obtained in that stage. It is related to the specific reliability model in that stage. This information is contained in the objective information boxes in Fig. 2. $p(\theta|T_s)$ is the posterior distribution of model parameters, which describes the integrated result of the subjective information $\pi(\theta)$ and the objective information $l(T_s|\theta)$.

For a specific life cycle stage in Fig. 2, suppose $f(t;\theta)$, $\theta \in \Theta$ is chosen as the reliability model in this stage. The subjective information describes prior knowledge about the product, including knowledge and experience of experts, historical data of similar existing products, and even the posterior distributions of the previous model. This subjective information is quantified as the prior distributions of model parameters. When field data are obtained, the prior knowledge is updated with these field data using the Bayesian theory. A Bayesian model for this stage is constructed based on this prior distribution and field data as Eq. (1). The subjective information and objective information are integrated through this Bayesian model and depicted by the posterior distribution. Reliability assessment of the product in this stage is obtained based on this posterior distribution $p(\theta|T_s)$ and the related reliability model $f(t;\theta)$, $\theta \in \Theta$.

To adopt the Bayesian theory in the life cycle reliability assessment in this paper, two aspects are emphasized in different life cycle stages: (1) derivation of the prior distribution, and (2) formulation of the Bayesian model for reliability assessment. Since the formulation of the Bayesian model is a case-based process that relates to the choice of reliability model and the specific form of prior distribution, details for this aspect are demonstrated in the case study. The derivation of prior distribution corresponds to the two critical aspects highlighted in the subsection above. These aspects collectively lead to the development of the two toolkits in Section 3.

2.3. *Critical steps for the BMUA*

To implement the BMUA presented above, the following steps are taken.

Step 1: Identify the reliability index throughout the life cycle stages.
Because reliability characteristics of different products vary a lot, different reliability indexes will be investigated throughout their life cycles. Generally, the reliability index for a new product is chosen according to the evaluation criteria of product performance or based on contractual agreements between customers and manufacturers, such as the MTBF for the repairable products (e.g., [4]), MTTF for the non-repairable products, and the failure probability or failure rate for one-time use products (e.g., [19]).
Step 2: Select a reliability model for the product in the predevelopment stage, gather subjective information and objective information for reliability assessment.
The reliability model in this stage is chosen according to the reliability indexes one is interested in and the specific information available in the predevelopment stage. For instance, if the reliability index in question is MTBF and the available information is lifetime data, a lifetime model such as exponential distribution, Weibull distribution, or lognormal distribution can be chosen. Meanwhile, gathering subjective information should be compatible with the reliability model chosen in this stage. It is a process that considers what information is available and whether this information can be related to the reliability indexes. Some representative sources of information are described in the definition of life cycle stages in Section 2.1 and Fig. 1.
Step 3: Quantify the knowledge and experience gathered from experts, derive prior distribution, construct Bayesian model 1,

and then carry out the reliability assessment in the predevelopment stage using model 1.

Because field data are sparse in the predevelopment stage, the prior distribution derived from subjective information has a remarkable effect on estimation in this stage. To handle the subjective information quantification, an information integration toolkit is developed for the BMUA in Section 3. Construction of the Bayesian model 1 and assessment of the product are implemented based on the Bayesian procedure described in Eq. (1).

Step 4: Choose a reliability model and gather reliability information for the development stage.

This step is similar to Step 2. However, the subjective information in this stage should include the outputs of Bayesian model 1 and the knowledge and experience of experts generated in the predevelopment stage. An information transition is constructed between two adjacent stages as described by the information flow in Fig. 2.

Step 5: Quantify the subjective information gathered in the predevelopment stage, derive the prior distribution according to the quantified subjective information and outputs of model 1, construct Bayesian model 2, and then carry out the reliability assessment of the product in the development stage using model 2.

In this stage, two kinds of prior information should be combined coherently: subjective information from the predevelopment stage and outputs of Bayesian model 1. Both kinds of prior information are transited from the previous stage, but in different forms via different ways. An information transition toolkit is developed for the BMUA to integrate these kinds of information in Section 3. The construction of the Bayesian model 2 and the reliability assessment are based on the Bayesian procedure described in Eq. (1).

Step 6: Choose a reliability model for the product in the postdevelopment stage and gather subjective information and objective information.

This step is similar to Step 4. The subjective information in this stage should include the outputs of the Bayesian model 2 and the knowledge and experience of experts generated in the development stage as depicted in Fig. 2.

Step 7: Similar to Step 5, quantify subjective information, derive prior distribution, construct the Bayesian model 3, and carry out the reliability assessment in the postdevelopment stage using model 3.

Major differences between steps 7 and 5 are the details of the prior information and field data as illustrated in Fig. 2. The basic process and technique are similar to step 5.

Step 8: Keep updating the reliability assessment of the product using model 3 when new field data are available in the postdevelopment stage.

The available data in the postdevelopment stage is evolving with the progress of product monitoring and maintenance as the life cycle moves on. When new field data are obtained, the estimation results should be updated. This is implemented by re-executing step 7 with new prior distribution and field data. The new prior distribution is obtained using the information transition toolkit by substituting outputs of the Bayesian model 2 in step 6 with the outputs of the Bayesian model 3 in step 7. The field data are the new data sets available in this stage.

3. Bayesian information toolkits for the BMUA

Clearly, derivation of prior distributions is critical for the information integration framework and Bayesian models in the proposed BMUA. Both Fig. 2 and the steps of the BMUA highlight that the connection between adjacent life cycle stages is constructed through a coherent information transition. This is implemented by derivation of prior distributions for the Bayesian models through quantification, integration, and transition of subjective information throughout the life cycle. Accordingly, an information integration toolkit and an information transition toolkit are developed for the BMUA in this section.

3.1. Information integration toolkit

An information integration toolkit is constructed to deal with the derivation of prior distributions with the subjective information highlighted in Fig. 2. It is developed by defining an intermediate quantity and incorporating the probability encoding methods with transformations of random variables. Specifically, three critical aspects are involved in the procedure of this toolkit: (1) systematically eliciting subjective data from subjects (e.g., knowledge and experience of experts) in which an intermediate quantity is defined, (2) modeling the subjective data with statistical distributions in which the probability encoding method is used, and (3) combining this statistical distribution with the information of historical data in which the transformations of random variables are used.

As demonstrated in Fig. 2, the prior information in each life cycle stage is composed of subjective information of experts, historical information of existing products (e.g., existing similar products, virtual products, and prototypes), and the outputs of previous models. The information integration toolkit is developed for deriving prior distribution with the former two sources of prior information. These two sources are interconnected, and both are subjective representations of the product characteristics. Generally, subjective information of experts is obtained as opinions/judgments assigned for a meaningful and representative quantity of a product [34]. Meanwhile, the historical information of existing products is in the form of the reliability index of interest, such as the MTBF of existing products. Accordingly, the opinions/judgments of experts about newly developed products are delivered through their subjective comparisons of the reliability indexes between the newly developed products and existing similar products. To elicit the subject data of experts through this kind of subjective comparison, an intermediate quantity called the reliability improvement factor λ_{RI} is introduced.

$$\lambda_{RI} = \frac{RI_N - RI_E}{RI_E} \tag{2}$$

where RI_N is the reliability index of the new product chosen at step 1 of the BMUA as described in Section 2.3 and RI_E is the reliability index of existing products, which is a constant derived from historical data of similar products.

The reliability improvement factor is a measure of the reliability difference between new products and existing products. It is defined through the ratio of reliability difference between these two types of products against the reliability of the existing products. It is a quantitative description of the extent of the difference between these types of products. The reason for introducing this intermediate quantity lies in the fact that subjective information of experts is elicited through their judgments on this quantity, which is a direct and meaningful way of eliciting subjective data from subjects. Meanwhile, a connection is naturally constructed between the subjective information of experts and the historical information of existing products through Eq. (2).

To systematically elicit subjective data from experts and quantify the reliability improvement factor, the probability encoding method [31] is adopted. This method employs an interview process based on a series of questions, for which the answers can be represented as points on the cumulative distribution function of the quantity surveyed. In this paper, the experts are required to respond by specifying points on the probability scale for fixed values of the reliability improvement factor (e.g., a probability of 0.35 is assigned for the fixed value $\lambda_{RI}=0.1$ by experts, meaning the probability $P(\lambda_{RI} \le 0.1)=0.35$). When the interview process is finished, a group of cumulative probability values for fixed values of the reliability improvement factor is obtained. By fitting these subjective data to a probability distribution, the probability distribution of reliability improvement factor is obtained as $\pi(\lambda_{RI})$.

To obtain the prior distribution for the Bayesian model, the distribution $\pi(\lambda_{RI})$ should be transformed to the probability distribution of the model parameter. The method for *transformations of random variables* is used to map the distribution of the reliability improvement factor into the prior distribution of model parameter. Suppose the parameter of the reliability model is θ. The reliability index under this model is described by the functional relationship between RI_N and θ as $RI_N=r(\theta)$. By substituting this function into Eq. (2), the functional relationship between λ_{RI} and θ is obtained as $\lambda_{RI}=g(\theta)$. Given the probability distribution of λ_{RI}, the prior distribution of the parameter θ is generated through the *methods for transformations of random variables* ([28], p. 59), as following.

$$\pi(\theta)=\pi_{\lambda_{RI}}(g(\theta))\left|\frac{d\lambda_{RI}}{d\theta}\right| \tag{3}$$

where $\pi_{\lambda_{RI}}(\lambda_{RI})$ is the probability distribution of the reliability improvement index λ_{RI}, $g(\theta)$ is the functional relationship between the parameter θ and λ_{RI} under a specific reliability model, $\pi_{\lambda_{RI}}(g(\theta))$ is the distribution obtained by substituting λ_{RI} in the distribution $\pi_{\lambda_{RI}}(\lambda_{RI})$ with function $g(\theta)$, and $|d\lambda_{RI}/d\theta|$ is the associated Jacobian of the transformation.

A further demonstration of the information integration toolkit will be presented in Section 4.1. For more details about the probability encoding method, please refer to Wallsten and Budescu [31] and O'Hagan et al. [24]. As discussed, the information integration toolkit is developed specifically for the BMUA in this paper. It is important to mention that the proposed BMUA can be extended to broader applications with a practical case-based choice of methods for prior derivation using subjective information. For more information related to the prior distribution elicitation with subjective information, please refer to the works by Bedford et al. [3], Gutierrez-Pulido et al. [12], and Seth [29].

3.2. Information transition toolkit

As discussed, the subjective information in a life cycle stage is quantified separately. The presence of more than one prior distribution on one model parameter occurs quite naturally under the proposed BUMA framework (e.g., for the model parameters in the development stage, we have the first group of prior distribution generated through the information integration toolkit and the second group obtained from the outputs of previous Bayesian model). The information transition toolkit is developed to combine these distributions into a single prior distribution for the model parameters. It is developed by incorporating the Bayesian Chi-squared test with the linear opinion pool method. Specifically, the linear opinion pool method is used as the basic combing framework, which is a weighted linear combination of the prior distributions. An information fusion factor is defined to quantify the weights for the linear pooling. Meanwhile, the Bayesian Chi-squared test is used to determine the information fusion factor. The weakness of the linear opinion pool method on weight determination is eliminated by implementing the Bayesian Chi-squared test on each prior distribution against the field data in the present stage.

As demonstrated in Fig. 2, the information gathered in the previous stage is passed to the present stage for prior derivation. It includes the outputs (posterior distributions) of the previous Bayesian model as $\pi_1(\theta)$ and subjective information integrated using the information integration toolkit discussed above as $\pi_2(\theta)$. Under the proposed BMUA, the linear opinion pool method is used to combine these two groups of prior distributions. It is one of the most widely used formal approaches for combining probability distribution in the field of expert judgments [8,7]. It is given as

$$\pi(\theta)=\lambda_{IF}\pi_1(\theta)+(1-\lambda_{IF})\pi_2(\theta) \tag{4}$$

where $\pi_1(\theta)$ and $\pi_2(\theta)$ are the individual priors, λ_{IF} and $1-\lambda_{IF}$ are the weights assigned for the corresponding two priors, and $\pi(\theta)$ represents the combined probability distribution.

Given the two groups of prior distributions $\pi_1(\theta)$ and $\pi_2(\theta)$, the weights for these distributions are defined through the introduction of an information fusion factor as

$$\lambda_{IF}=\frac{B_P^1}{B_P^1+B_P^2} \tag{5}$$

where B_p^1 and B_p^2 are the Bayesian model fitting factors for the prior distribution $\pi_1(\theta)$ and $\pi_2(\theta)$, respectively. Both are defined based on the Bayesian χ^2 goodness-of-fit proposed by Johnson [18]:

Let $T_s=(t_1,t_2,t_3,...,t_n)$ denote the field data in this stage with cumulative distribution function $F(t;\theta)$. Let $0=a_0<a_1<...<a_K=1$ denote K equally spaced quantiles from a uniform distribution, and define $p_j=a_j-a_{j-1}$ and $K\approx n^{0.4}$, where n is the sample size of the observation T_s. Let $m_j(\tilde{\theta})$, $j=1,...,K$ denote the number of observations t_i, $i=1,...,n$ that fall into the interval $[a_{j-1},a_j]$, for which $a_{j-1}<F(t_i|\tilde{\theta})\le a_j$, and $\tilde{\theta}$ is a random sample from the prior distribution $\pi(\theta)$. The Bayesian Chi-squared test statistic for the random sample is defined as

$$R^B(\tilde{\theta})=\sum_{j=1}^{K}\frac{(m_j(\tilde{\theta})-np_j)^2}{np_j} \tag{6}$$

For large n, the distribution of $R^B(\tilde{\theta})$ follows the Chi-squared distribution with $K-1$ degrees of freedom. Based on the above Bayesian Chi-squared test statistic, we define B_p as

$$B_p=\Pr\left(\sum_{j=1}^{K}\frac{(m_j(\theta)-np_j)}{np_j}<\chi^2_{K-1,0.95}\right) \tag{7}$$

where $\chi^2_{K-1,0.95}$ is the 0.95 quantile of the reference Chi-squared distribution with $K-1$ degrees of freedom and $\Pr(A)$ is the probability of event A.

The idea of this toolkit is that B_p is a measure of the prior distribution fitting to the field data in each stage. The prior distribution that fits the field data more closely will result in a bigger B_p value. A higher weight will be assigned to this prior distribution in the linear weighted pooling. Accordingly, the combined prior distribution inherits the major belief of these two types of prior distributions and maintains the characteristics of the relevant field data.

A further demonstration of the information integration toolkit will be presented in Section 4.2. It is important to mention that this toolkit is developed specifically for the BMUA framework, which is intended to be easily understood and used for practical engineering. There may be a seemingly larger variance involved through the framework of linear opinion pool; however it is the direct effect of combining multiple sources of prior information

Table 1
The subjective information in the predevelopment stage.

Prior information derived from the warranty data of similar GMCs								
MTBF of existing similar GMCs	563 (h)							
95% percentile interval of the shape parameter β	[0.75,1.83]							
Judgments of experts for the reliability improvement index λ_{RI}								
Fixed values of λ_{RI}	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Cumulative probability values	0.35	0.72	0.83	0.90	0.95	0.97	0.98	0.99

because they are divergent in a certain way. A large sample of $\tilde{\theta}$ from the prior distribution $\pi(\theta)$ for calculating Eq. (7) could alleviate the effect of this enlarged variance. This information transition toolkit can also be extended to a wide application with a suitable choice of method for combining prior distributions. For more information on this topic, please refer to the work of Poole and Raftery [25], Clemen [7], and Ranjan and Gneiting [26].

4. A case study

A newly developed Gantry Machining Center (GMC) by company M is used to demonstrate the application of the BMUA. To launch a newly developed GMC with high reliability, company M needs to track and manage the reliability of the GMC throughout its life cycle. In this section, the BMUA procedure is carried out stage by stage for the life cycle reliability assessment of the newly developed GMC to illustrate its application in detail.

4.1. Reliability assessment of the GMC in the predevelopment stage (steps 1–3)

First, the MTBF is specified as the reliability index and the Weibull distribution is chosen to model the lifetime distribution of the GMC, as $T\sim$weibull (β,η). The prior information and the field data about the GMC are given in Tables 1 and 2, in which the field data are collected from the reliability simulation of the GMC during the design process.

Next, the information integration toolkit is adopted to derive the prior distribution with the subjective information given in Table 1. The subjective data for the reliability improvement index are fitted to a beta distribution as $\lambda_{RI}\sim$beta (1.86,10.2). A uniform distribution on the 95% percentile interval of the shape parameter is specified as the prior distribution of the shape parameter as $\beta\sim$uniform (0.75,1.87). It can incorporate information from existing similar products and maintain the diffusion of the prior distribution for the shape parameter.

The scale parameter η can be derived by combining the probability distribution of the λ_{RI} and the prior distribution of β through the relationship described in Eqs. (2) and (3), where RI_E is the MTBF of the existing GMC with Weibull distribution.

$$\mathrm{MTBF}_N=\eta\Gamma(1+(1/\beta))=(1+\lambda_{RI})\mathrm{MTBF}_E$$

$$\pi(\eta)=\pi(\eta(\lambda_{RI},\beta)|\lambda_{RI},\beta),\eta(\lambda_{RI},\beta)=\frac{(1+\lambda_{RI})}{\Gamma(1+1/\beta)}\times 563 \tag{8}$$

With the derived prior distributions and the field data given in Table 2, the Bayesian model in this stage is constructed as

$$p(\beta,\eta|T_1)=\frac{l(T_1|\beta,\eta)\pi(\beta,\eta)}{\iint_{\substack{\beta_2>\beta>\beta_1\\ \eta>0}} l(T_1|\beta,\eta)\pi(\beta,\eta)d\beta d\eta} \tag{9}$$

where $p(\beta,\eta|T_1)$ is the joint posterior distribution for the model parameters, $l(T_1|\beta,\eta)$ is the likelihood function of field data T_1, which is a product of the density function of Weibull distribution at each failure point given in Table 2, and $\pi(\beta,\eta)$ denotes the joint prior distribution of the model parameters derived above, which does not have an analytical form but can be sampled using the Monte Carlo method following Eq. (8). The interval $\beta_2>\beta>\beta_1$, $\eta>0$ indicates the derived intervals for the model parameters in their prior distributions.

Table 2
The objective information in the predevelopment stage.

Time between failures of the GMC T_1 (h)							
71	1882	1215	268	132	681	986	421
2038	1790	74	640	212	923	386	112

Since the posterior is complicated and no close form expression can be obtained, the MCMC methods are used to generate samples from the posterior. The assessment of the MTBF is implemented through sample-based posterior analysis by substituting the generated samples $(\tilde{\beta},\tilde{\eta})$ into$\mathrm{MTBF}_N=\eta\Gamma(1+(1/\beta))$. In this study, 20,000 representative samples are generated from the posterior distribution, and the samples for the MTBF are obtained as well. These samples are summarized in Table 3. The generated samples are fitted to approximate distributions as fitted posterior distributions in Table 3 to deliver an intuitional view.

4.2. Reliability assessment of the GMC in the development stage (steps 4–5)

The Weibull distribution is chosen to describe the lifetime of the GMC in this stage. The prior information from the outputs of the previous model is presented in Table 3 as $\pi_1(\beta)$, $\pi_1(\eta)$. The prior information gathered from experts and the existing product in the predevelopment stage is incorporated using the information integration toolkit as

$$\pi(\lambda_{RI})=\mathrm{beta}(3.28,12.33),\ \pi_2(\beta)=\mathrm{uniform}(0.7891,1.4949),$$
$$\pi_2(\eta)=\pi(\eta(\lambda_{RI},\beta)|\lambda_{RI},\beta),\ \eta(\lambda_{RI},\beta)=\frac{(1+\lambda_{RI})}{\Gamma(1+1/\beta)}\times 661.4 \tag{10}$$

Then, to incorporate these two kinds of prior distributions, the information transition toolkit is used. The Bayesian χ^2 goodness-of-fit B_p for these two kinds of prior distributions are obtained by simulating Eqs. (5) and (6) with the prior distributions given above and the field data given in Table 4, where $K=3\approx n^{0.4}$ and $n=12$. The information fusion factor in this stage is then obtained as

$$B_P^1=0.9392,\quad B_P^2=0.4997,\quad \lambda_{IF}=\frac{B_P^1}{B_P^1+B_P^2}=0.6527 \tag{11}$$

With the prior distribution and the information fusion factor given above, together with the field data in Table 4, the Bayesian model in this stage is constructed as

$$p(\beta,\eta|T_2)=\frac{\lambda_{IF}l(T_2|\beta,\eta)\pi_1(\beta,\eta)+(1-\lambda_{IF})l(T_2|\beta,\eta)\pi_2(\beta,\eta)}{\iint_{\substack{\beta>0\\ \eta>0}}[\lambda_{IF}l(T_2|\beta,\eta)\pi_1(\beta,\eta)+(1-\lambda_{IF})l(T_2|\beta,\eta)\pi_2(\beta,\eta)]d\beta d\eta} \tag{12}$$

Table 3
Summary statistics of the estimated results in the predevelopment stage.

Parameters	Posterior			Posterior percentiles		Fitted posterior distributions
	Mean	SD	Median	2.5%	97.5%	
MTBF	661.4	57.13	652.7	577.1	793.1	Lognormal (6.4907,0.0843)
β	1.097	0.1845	1.082	0.7891	1.494	Lognormal (0.0785,0.1667)
η	674.3	73.9	668.2	544.2	835.0	Lognormal (6.5078,0.1086)

Table 4
The objective information in the development stage.

Time between failures of the GMC′ prototype T_2 (h)					
97	39	1747	35	266	2030
14	11	1521	9	960	376

Table 5
Summary statistics of the estimated results in the development stage.

Parameters	Posterior			Posterior percentiles		Fitted posterior distributions
	Mean	SD	Median	2.5%	97.5%	
MTBF	733.4	63.21	729.4	621.6	868.8	Lognormal (6.5941,0.0855)
β	0.933	0.09604	0.9275	0.7614	1.137	Lognormal (− 0.0746,0.1025)
η	703.4	53.73	701.0	604.9	815.2	Lognormal (6.5530,0.0762)

Table 6
The objective information in the postdevelopment stage.

Time between failures of the GMC gathered from the users T_3 (h)						
450	179	361	274	306	518	484
1257	1197	1108	379	206	258	621

where $p(\beta,\eta|T_2)$ is the joint posterior distribution for the model parameters in the development stage, $l(T_2|\beta,\eta)$ is the likelihood function of the field data given in Table 4, and $\pi_1(\beta,\eta)$ and $\pi_2(\beta,\eta)$ separately denote the prior distributions derived from the outputs of the previous model and the transited information from previous stage.

Finally, the estimation is carried out by generating samples from the joint posterior distribution using the MCMC methods, similar to the predevelopment stage. The estimated results in this stage are summarized in Table 5.

4.3. Reliability assessment of the GMC in the postdevelopment stage (steps 6–7)

The Weibull distribution is chosen in this stage as well. The subjective information includes the outputs of the previous model given in Table 5 as $\pi_1(\beta)$, $\pi_1(\eta)$ and the opinions of experts gathered in the development stage. By adopting the information integration toolkit and the information transition toolkit, the prior distributions in this stage are obtained as

$$\pi(\lambda_{RI}) = \text{normal}(0.04,0.01),\ \ \pi_2(\beta) = \text{uniform}(0.7641,1.137),$$
$$\pi_2(\eta) = \pi(\eta(\lambda_{RI},\beta)|\lambda_{RI},\beta),\ \ \eta(\lambda_{RI},\beta) = \frac{(1+\lambda_{RI})}{\Gamma(1+1/\beta)} \times 733.4;$$
$$B_P^1 = 0.5857,\ \ B_P^2 = 0.6806,\ \ \lambda_{IF} = \frac{B_P^1}{B_P^1+B_P^2} = 0.4625 \tag{13}$$

With these prior distributions obtained above and the field data given in Table 6, the Bayesian model in this stage is constructed, having the same form of Eq. (12). By adopting the MCMC method, the estimated results of the GMC are obtained and presented in Table 7.

4.4. Model updating in the postdevelopment stage with new field data (step 8)

When new field data are obtained in the postdevelopment stage as in Table 8, the estimation of the GMC is updated by substituting the prior distribution derived from the outputs of the development stage with the outputs of the early postdevelopment stage in Table 7. The prior distribution derived from the opinions of the experts remains unchanged. By adopting the information transition toolkit, the information fusion factor in this stage is obtained as

$$B_P^1 = 0.9532,\ B_P^2 = 0.8418,\ \lambda_{IF} = \frac{B_P^1}{B_P^1+B_P^2} = 0.5301 \tag{14}$$

Finally, the Bayesian model in this stage is updated with these prior distributions and new field data, taking the same form of Eq. (12). By adopting the MCMC method, the updated estimates of the GMC are obtained and summarized in Table 9.

4.5. Analysis of the results

To demonstrate the effectiveness of the proposed BMUA, a model diagnostic of the BMUA, a vertical comparison within different life

Table 7
Summary statistics of the estimated results in the postdevelopment stage.

Parameters	Posterior			Posterior percentiles		Fitted posterior distributions
	Mean	SD	Median	2.5%	97.5%	
MTBF	701.4	36.57	698.2	638.9	783.7	Lognormal (6.5518,0.0515)
β	0.9849	0.06963	0.9888	0.844	1.113	Weibull (1.0170,15.4490)
η	694.2	37.55	692.3	624.9	775.4	Lognormal (6.5413,0.0537)

Table 8
The new field data in the postdevelopment stage.

The new field data of the GMC gathered from the users T_3^3 (h)									
430	41	330	102	41	232	333	761	773	126
576	734	943	704	695	1449	580	253	1481	598

Table 9
Summary statistics of the estimated results with new field data.

Parameters	Posterior			Posterior percentiles		Fitted posterior distributions
	Mean	SD	Median	2.5%	97.5%	
MTBF	691.9	29.44	688.8	643.2	760.1	Gamma (562.5758,1.2299)
β	0.9949	0.05966	0.999	0.8735	1.096	Weibull (1.0223,19.1355)
η	688.6	31.49	687.2	630.7	756.9	Lognormal (6.5336,0.0454)

cycle stages, and a horizontal comparison between different reliability assessment methods are carried out in this subsection.

First, a model diagnostic is carried out for the BMUA, implementing the Bayesian version of Person's Chi-squared goodness-of-fit test proposed by Johnson [18]. Twenty thousand samples are generated from the posterior distributions of the model parameters in each life cycle stage. Together with the field data presented above, the Bayesian Chi-squared goodness-of-fit test results for each stage are obtained. There are 22.97% of these simulated values that exceed the 0.95 quantile of the Chi-squared distribution in the postdevelopment stage. This suggests some lack of model fitting by the BMUA for the early stage of the postdevelopment stage. This is due to the domination of the subjective information in that stage. However, it is mitigated by the model updating with the BMUA in the postdevelopment stage with new field data, with the new results having only 2.35% values exceeding the same quantile under the same model diagnostic. Furthermore, the exceeded portions of the simulated values in the predevelopment stage and the development stage are merely 0% and 0.14%, respectively. Accordingly, the fitness of the BMUA for the life cycle reliability of the GMC can be verified through this kind of model diagnostic.

We then carry out a vertical comparison between different life cycle stages. The estimated MTBFs throughout the life cycle stage are presented in Fig. 4. The posterior distributions obtained by the proposed BMUA becomes more concentrated as the life cycle stages move on, which indicates that the uncertainty of estimation results are getting smaller and the corresponding precision is getting higher. This improvement of precision is attributed to the effectiveness of the information integration and transition of the proposed BMUA.

To carry out a horizontal comparison, we compare the estimated 95 percentile interval of the MTBF between the proposed BMUA and the classical methods, such as the maximum likelihood method (MLE) and the classical Bayesian method. By using "classical", we mean the Bayesian method without information transition between different life cycle stages (e.g., the methods treat the life cycle stages separately which follow the strategies introduced by Yadav et al., [38] and Johnson et al., [19]). The results are presented in Table 10. The estimated intervals obtained by the proposed BMUA are included in the ones by the classical methods. The estimated results from these methods are compatible, but the proposed BMUA possesses the smallest intervals, which indicates that the proposed BMUA has the highest precision. A more direct comparison can be obtained from Fig. 4, in which the posterior density of the MTBF by the BMUA has a narrower spread than the results by the classical Bayesian without coherent information fusion throughout life cycle stages. Accordingly, the effectiveness of the information integration and transition in the BMUA is verified through these vertical and horizontal comparisons.

5. Conclusions

A Bayesian model updating approach is developed to deal with life cycle reliability assessment of new products. The BMUA is constructed with an information integration framework, two information toolkits, and three Bayesian models. The information generated throughout the life cycle stages is integrated and transited under the information integration framework. The Bayesian models and the information toolkits are adopted to implement the information integration and transition throughout the life cycle stages. The proposed BMUA is demonstrated with the application to the life cycle reliability assessment of a newly developed GMC. The estimated results of the BMUA are analyzed and compared with classical methods. The effectiveness of the proposed BMUA is demonstrated by the evolving precision of estimated results stage by stage. It is also verified by the comparison of the accuracy of the estimated results between the BMUA and classical methods.

It is important to note that the case study is employed solely for illustrative purposes. The authors have made many assumptions and approximations to demonstrate the proposed techniques.

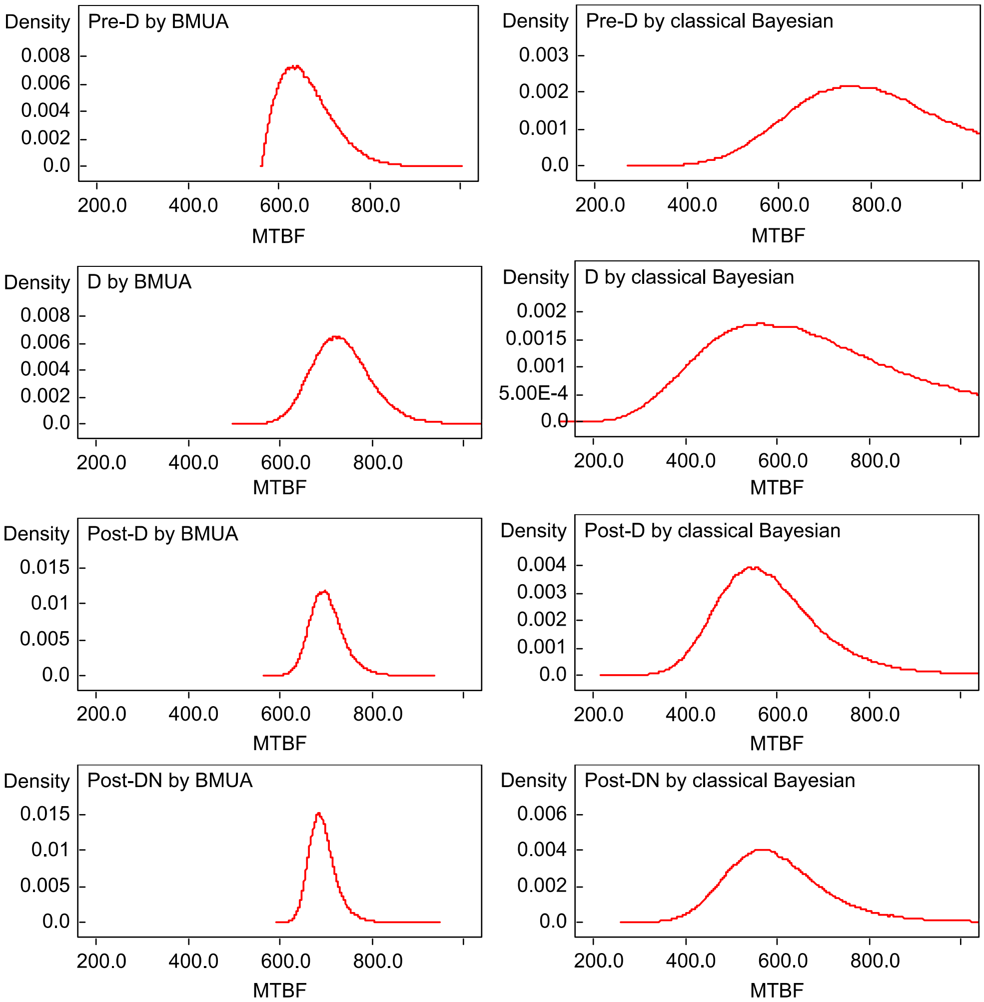

Fig. 4. Posterior distribution of the MTBF in different life cycle stages with the proposed BMUA and the classical Bayesian method.

Table 10
Comparison of the estimated MTBF between the proposed BMUA and classical methods.

Life cycle stages	The BMUA		The MLE		Bayesian without information fusion	
	2.5%	97.5%	2.5%	97.5%	2.5%	97.5%
Predevelopment	577.1	793.1	574.9	1104.3	518.9	1411.0
Development	621.6	868.8	587.3	1141.6	345.5	1539.0
Postdevelopment	638.9	783.7	420.2	761.4	404.0	867.3
Postdevelopment with new data	643.2	760.1	442.9	758.9	421.5	871.3

The calculation of the Bayesian models in the case study is implemented using the Markov chain Monte Carlo (MCMC) method and WinBUGS software [20,23] is applied.

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