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Probabilistic Physics of Failure-based framework for fatigue life prediction of aircraft gas turbine discs under uncertainty

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ABSTRACT

A probabilistic Physics of Failure-based framework for fatigue life prediction of aircraft gas turbine discs operating under uncertainty is developed. The framework incorporates the overall uncertainties appearing in a structural integrity assessment. A comprehensive uncertainty quantification (UQ) procedure is presented to quantify multiple types of uncertainty using multiplicative and additive UQ methods. In addition, the factors that contribute the most to the resulting output uncertainty are investigated and identified for uncertainty reduction in decision-making. A high prediction accuracy of the proposed framework is validated through a comparison of model predictions to the experimental results of GH4133 superalloy and full-scale tests of aero engine high-pressure turbine discs.

1. Introduction

High thrust-to-weight ratios and low costs are required in aero engine design to ensure flight safety and reliability. It means that higher stresses and temperatures will be created by engine hot section components such as turbine discs and blades, which are fracture critical engine components. As a result, considerable research has been conducted to improve materials and manufacturing processes to optimize component lifetime and reduce costs. Life-cycle assessments (LCA), particularly in life-cycle prediction endeavors, require damage prognostics for the health and capability of components [1]. In dynamic environments, stresses in engine hot section components constantly vary, making low cycle fatigue (LCF) failure a critical issue when designing these components [2–4]. LCF failure occurs under the influence of diverse uncertainties such as load variations in usage, material properties, geometry variations within tolerances, and other uncontrolled variations. Probabilistic fatigue models are required to account for these uncertainties. Accordingly, it is important to predict the remaining fatigue lives of components by using a probability distribution that is robust against unavoidable variations. Probabilistic LCF life prediction has been extensively studied, with many efforts implemented through either Physics of Failure (PoF)-based or data-driven techniques. This paper will focus on a probabilistic PoF-based method using both PoF-based modeling techniques and probabilistic techniques. Moreover, to enhance working life as well as the structural reliability, a practical algorithm for fatigue reliability analysis will be established.

Deterministic methods were introduced to improve resistance of materials against fatigue for hot section components [5–8]. However, the fatigue process is usually random in nature. Typically, component usage and material properties are probabilistic, meaning that the fatigue resistance of a component under a particular loading regime is difficult to guarantee. Evaluating the variability of the problem enables us to evaluate the risk of failure and minimize the chances of oversizing the component as described in [9]. Using a stochastic modeling technique, the probabilistic LCF life prediction of these components is primarily based on material property data, stress and strain analyzes, periodic inspection/monitoring (I/M) for defects, and modeling of damage accumulation [10].

Although the presence of uncertainty in hot section component life prediction is clearly a practical issue, few studies have explored the problem. Stochastic modeling characterizes material properties, model parameters and test parameters as random variables. By integrating the results of finite element analysis (FEA), LCF life assessments, material anomaly data, anomaly detection probabilities, and I/M strategies, probabilistic life predictions were performed using DARWIN to obtain the fracture probability for rotor discs [11–14]. Lu et al. [15] derived the safe life of an aircraft

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Nomenclature

D_t	accumulated damage
E	Young's modulus
F_p	multiplicative error of model prediction to the real value
F_t	multiplicative error of experimental result to the real value
F_{pt}	multiplicative error of experimental result to the model prediction
K_p	safety factor
$L(\cdot)$	likelihood function
$LN(\cdot)$	lognormal distribution function
N_f	number of cycles to failure
N_{fp}	model prediction
N_{ft}	experimental result
N_{real}	real fatigue life
R_{strain}	strain ratio
$\mathbf{X}$	random variables
Y	the logarithmic tested life
Z_j	random variables representing different sources of uncertainty
$\tilde{Y}$	the prediction of logarithmic fatigue life
$\mathbf{x}$	input variables
$f_{\mathbf{X}}(\mathbf{X}, \mathbf{\Theta}_f)$	parameterized probabilistic sub-model
$g_i(\mathbf{X}, \mathbf{\Theta}_g)$	parameterized physical sub-model
$\mathbf{\Theta}_f$	parameters of the probabilistic sub-model
$\mathbf{\Theta}_g$	parameters of the physical sub-models
ϕ_i	random variables representing different sources of scatter
ε	prediction error
$\sigma_{\lim}$	stress endurance limit
n'	cyclic strain hardening exponent
b_p	mean, error of model prediction to the real value
b_t	mean, error of experimental result to the real value
δ_{pred}	fatigue life prediction uncertainty in log-life
γ_p	coverage factor
s_p	standard deviation, error of model prediction to the real value
s_t	standard deviation, error of experimental result to the real value

turbine disc by incorporating the uncertainties in parameters that influenced creep and fatigue life. Larsen et al. [16] explored the fundamental variability and uncertainty in the microstructural-based limits of fatigue life in the design of aircraft engine rotors. Research has shown a potential opportunity for reducing uncertainty in the life-cycle prediction and management of hot section components. During crack growth modeling, Wei et al. [17] developed a linear superposition method for creep-fatigue-oxidation crack growth analysis under high temperatures. In addition, the uncertainties within parameters for the mechanisms of fatigue, creep, and oxidation crack growth were considered and evaluated using Monte Carlo simulations. Guided by the issues of uncertainty and model errors, Shankar et al. [18] quantified the uncertainties for probabilistic crack growth analysis by connecting FEA results, fatigue crack growth laws and a surrogate model using a Bayesian network. The majority of these published investigations on probabilistic fatigue failure modeling only considered the variability resulting from material/component properties, loading conditions and model parameters without considering the statistical uncertainty, model uncertainty and errors. Recently, a Bayesian framework for probabilistic LCF life prediction had been derived and applied to a number of LCF tests on aircraft turbine disc alloys, which then outputted the quantified uncertainty value in the form of uncertainty bounds [19].

Different types of uncertainties and errors in fatigue reliability analysis can be combined in nonlinear, nested or iterative manners. Systematically dealing with these uncertainties and errors significantly affects the robustness of uncertainty quantification (UQ) and model validation. These combined effects often lead to a significant scatter of hot section component life. For example, probabilistic simulations of fatigue cracking in a turbine disc performed by Hudak et al. [10] indicated that the variability of usage can lead to more than six times the variability in fatigue life and introduce 100 times more variability in the failure probability of a given life. Thus, this paper attempts to account for the variability, statistical uncertainty, model uncertainty and errors that influence damage modeling and develop a probabilistic framework to incorporate these uncertainties in LCF life prediction.

Through quantitative characterization and the reduction of uncertainties in applications, UQ measures the input uncertainty effects on response metrics of interest. It is crucial to achieve validated predictive results in engineering structural integrity assessment. In this study, an indispensable step for developing a practical probabilistic method is to appropriately incorporate and quantify the uncertainty. Mathematical structures such as interval analysis, evidence theory and probability theory are often used to characterize uncertainty. Moreover, a single variable or a vector of uncertain variables or a vector function of uncertainty variables can be used for each structure. Several quantitative approaches have been developed to account for various sources of uncertainty and to quantify consistency between model predictions and experimental observations [20–22]. Both classical statistics and Bayesian theory are used for this purpose [22]; the former are implemented by calculating statistics of model error and the latter are used to characterize the consistency level of the model supported by data, as well as compute the probability of the model being correct. However, there is no systematically practical and mathematically sound method for dealing with the problem of model validation and updating by leveraging new available information that can be used to reduce model uncertainties in a stable manner. The characteristics of Bayesian theory for data analysis give rise to the capability for high-precision LCA, of which the uncertainty is presented at various degrees, and data are combined through different stages.

To fulfill the aforementioned needs in an efficient way by combining classical and Bayesian statistics-based methods, this paper presents a formal assignment for the uncertainties associated with inputs, modeling and simulations along with an updating procedure for prediction improvement and reducing these uncertainties. This study presents a comprehensive UQ procedure to quantify the overall uncertainties in a structural integrity assessment when input probability distributions must be estimated from test data. It allows for quantifying and propagating overall uncertainties through a model for the life prediction and obtaining their joint effects on the predicted fatigue life distribution during decision makings.

The rest of this paper is organized as follows. Section 2 reviews the LCF failure criterion developed for the reliability evaluation and health management of hot section components. Section 3 discusses different types of uncertainties and errors and presents a probabilistic LCF life prediction framework under uncertainty. Section 4 describes the quantification of uncertainty and errors in a fatigue reliability analysis. Experimental data of a Ni-base superalloy GH4133 (turbine disc alloy) under different loading conditions and full-scale tests of aero engine high-pressure turbine discs are used to illustrate the proposed framework in Section 5. The advantages of the proposed framework are also discussed. Finally, concluding remarks are presented in Section 6.

2. LCF failure criteria for gas turbine hot section components

The primary failure mode of hot section components is LCF at high temperatures, which is an interaction of fatigue, creep and environmental corrosion and oxidation. Reliable LCF life predictions depend on the following factors: 1) accurate constitutive deformation laws, which should be validated under various loading conditions; 2) sufficient experimental data; and 3) physics-based identification of relationships between cumulative damage and deformation history. There have been numerous investigations on fatigue life prediction methods for these components [2,5,23], and several criteria for LCF and thermo-mechanical fatigue (TMF) life prediction of specimens/components at room/high temperatures have been developed [24]. These can be divided into five categories: classical fatigue criteria, fracture mechanics-based methods, damage mechanisms, energy criteria and ductility exhaustion-based criteria.

The classical criteria for LCF life prediction can be mathematically presented as a relationship between the number of cycles to failure N_f of a specimen/component and a function related to its material properties, geometry, loading waveform, and damage driving parameters:

$$\Phi(\sigma,\varepsilon,\cdots,P_{(\cdot)})=f(N_f,A,B,\cdots) \tag{1}$$

where Φ is a function related to the specimen/component deformation history, which considers the stress–strain relationship during fatigue; $P_{(\cdot)}$ denotes the parameters related to loading conditions; and A and B are material-dependent constants.

One of the most widely used classical methods is the Coffin-Manson (CM) criterion, which predicts the LCF life based on the plastic strain amplitude. Based on the CM criterion, Morrow [25], Smith-Watson-Topper [26], Ostergren [27], He et al. [28], Wang [29], Zhu et al. [30] applied mean stress and frequency effects to the classical methods. However, these methods treated the changing parameters as material constants, which is the primary disadvantage of these methods.

By characterizing the crack growth behavior under cyclic loading, fracture mechanics-based methods are commonly based on cyclic J-integrals that are used extensively for fatigue life prediction and component reliability evaluation [31]. A number of life prediction models [14,32–35] related to fracture mechanics have been proposed for hot section components; these include the deterministic fracture mechanics approach, the probabilistic fracture mechanics approach, and their combinations. FEA is the most commonly used approach for calculating the fracture mechanics parameters for gas turbine components [36,37]. The limitations of fracture mechanics-based methods on crack initiation predictions resulted in the introduction of micromechanics models. These models were originally developed by Kachanov and Rabotnov [38] through the introduction of the concept of effective stress and were named continuum damage mechanics (CDM)-based approaches. Fracture mechanics-based methods account for discrete crack initiation and propagation in components, whereas CDM-based approaches disregard these discrete flaws by treating the material as a continuum where damage evolution eventually causes a fracture. As a result, CDM-based life prediction methods have been developed for stress rupture and creep analyzes of materials and structures [39–41]. Through the implementation of micromechanics-based fracture models, where the damage state at the crack tip depends on the critical conditions required for fracture, CDM-based approaches provide a delicate description of nonlinear creep and/or fatigue damage accumulation. Failure occurs when the cumulative damage reaches the specimen's/component's critical damage. The most significant CDM-based model was introduced by Chaboche [42,43]. Because damage evolves with the actual damage state, the damage is accumulated nonlinearly. The major disadvantage of Chaboche's model is the difficulty in implementing it on complex structural systems because of the time-consuming modeling process and high computational efforts. CDM-based approaches have been recently extended to obtain a more realistic life prediction of high-temperature components and develop interaction laws in failure mechanism modeling of creep-fatigue interactions [44–47].

Microstructure plays an important role in the modeling of crack initiation, as well as subsequent short crack growth, particularly on small cracks that are comparable to typical microstructural features. Because metallurgical considerations and physical mechanisms are increasingly considered in the quest for an accurate life prediction, the creep and fatigue properties of high-temperature components have been analyzed in metallurgical and microstructural forms [48–51]. Moreover, metallurgical- and microstructural-based models have been presented for fatigue failure analysis [52–54]. Using crack initiation and micro-crack growth laws, Rémy et al. [52] developed a damage model to predict LCF lifetime for single-crystal superalloys at high temperatures. By analyzing the interactions between various microstructural parameters and LCF test data, Maderbacher et al. [53] put forward a new fatigue model that depended on temperature and microstructure to predict LCF strain-life curves. Using a dissipated energy model to predict LCF life, Gloanec et al. [54] investigated the impacts of both intrinsic and extrinsic parameters including microstructure, temperature and/or strain ratio on model prediction results.

Recent research indicates that the energy criteria for fatigue endurance calculations are robust and may reduce the scatter observed in other methods. Energy criteria have been developed for fatigue and creep-fatigue life prediction based on plastic strain energy, elastic strain energy or total strain energies. The energy criterion depends on the selection of strain energy density (SED) per cycle as the damage parameter. The SED method assumes that for a given stress concentration point, the SED in confined plasticity is the same as the SED at that point calculated in elasticity. The plastic SED-based criterion is predominantly used to characterize the LCF behavior of materials operating at high levels of stress/strain; its shortcoming lies in the fact that it cannot be used under high cycle fatigue conditions. To date, energy criteria have been applied in the life prediction of high-temperature components [5,8,24,55–58]. The energy criteria provided a physics-based method to predict hot section component life. However, there is generally a large variance in experimental fatigue results and the modeling of failure mechanisms [59,60]. These results indicate that a non-deterministic feature of fatigue failure mechanism must be considered for a life-cycle assessment.

Because they use both strain and stress, energy criteria appear robust and reduce the amount of scatter when compared with other methods. However, the failure data of hot section components is often insufficient and fatigue test sample sizes tend to be small. Moreover, multiple types of uncertainty and error appear at various stages of normal operating conditions. Thus, this paper attempts to incorporate different sources of uncertainty and error systematically to improve robustness in model calibration and model validation for probabilistic LCF life prediction. The aforementioned deterministic LCF life prediction methods present an initial development of probabilistic life prediction methods.

3. Probabilistic LCF life prediction framework under uncertainty

The fatigue cracking process is an inherently random process because of material variability and microstructural irregularities. In addition, various sources of uncertainty and errors arising from a simplified representation of the actual physical process (primarily through semi-empirical or empirical models) and/or sparse information on material properties, environmental conditions, and loading profiles contribute to stochastic behavior during physics-based failure mechanism modeling. This leads to the estimated crack size at a given time being treated as random. Thus, the appropriate treatment of uncertainty in an LCF regime has become a significant topic with widespread interest [18,61–64].

The various sources of uncertainty, based on their differences in origin, modeling and effects, can be broadly divided into two categories: the epistemic uncertainty and the aleatory uncertainty. The first derives from the inherent variation resulting from a physical process or an environmental condition [65]. For this type of uncertainty, additional knowledge is aimed not at reducing the uncertainty but at better quantifying the actual physical state of the process. The latter category derives where knowledge or information is lacking in modeling the processes. The acquirement of new information of the physical process, better use of the data or better modeling methods often leads to a reduction of epistemic uncertainty.

Structural integrity issues are often investigated under the confines of a model universe that includes physical and probabilistic model sets. These models can include various alternatives that are generally used to mathematically model reality to solve the problem. The model universe may include intrinsic uncertain quantities; moreover, the probabilistic models are sometimes invariably imperfect and lead to additional uncertainties. A critical aspect in the development of a model universe is accurately modeling these uncertainties. The features and characteristics of uncertainties must be studied within the model universe. Due to the nature of engineering issues, the process for setting up a model universe is largely subjective. Thus, an engineer's conceived models must be proven close to reality.

To provide a basis for discussion in this paper, a model universe for LCF life prediction issue should involve the following elements: 1) an input variable set $\mathbf{x}=(x_1,x_2,\cdots,x_n)$, which are the outcomes of basic random variables $\mathbf{X}=(X_1,X_2,\cdots,X_n)$; 2) a parameterized probabilistic sub-model $f_{\mathbf{X}}(\mathbf{X},\boldsymbol{\Theta}_f)$, which characterizes the probability distribution of random vector $\mathbf{X}$; and 3) a parameterized physical sub-model set $y_i=g_i(\mathbf{X},\boldsymbol{\Theta}_g), i=1,2,\cdots,k$, which describes the relations between $\mathbf{x}$ and k derived quantities $\mathbf{y}=(y_1,y_2,\cdots,y_k)$. According to the life prediction issue of mechanical components, the random variables $\mathbf{X}$ can be directly observed or determined from experimental data, including material properties (e.g., ductility, toughness, and endurance limit), load characteristics (e.g., plastic strain range, maximum stress, and strain ratio), environmental effects (e.g., temperature) and geometry. The derived variables $\mathbf{y}$ cannot be directly obtained, save those obtained from laboratory or field studies; they are derived quantities such as deformations, stresses, and measures of cumulative damage. The sub-models $f_{\mathbf{X}}(\mathbf{X},\boldsymbol{\Theta}_f)$ and $g_i(\mathbf{X},\boldsymbol{\Theta}_g)$, are generalized mathematical models of reality without considering variability and often contain model errors that are usually obtained through the fitting process of observed data. In this paper, these elements are used to investigate the UQ issue and evaluate their relevance to life prediction under different loading conditions.

Based on the elements mentioned above, various sources of uncertainty are categorized as follows:

1. Physical variability

- Variability or scatter in random variables $\mathbf{X}$, including the uncertainty results from the environment, material properties, test procedures, and number of cycles to failure of specimens/components with same material and/or experimental conditions.
- For this variability, multiple-repeat observations of a physical quantity often do not result in identical results.

2. Statistical uncertainty

- Uncertainty results from data-driven statistical estimation for model parameters according to available data, including statistical uncertainty within the estimation results of parameters $\boldsymbol{\Theta}_f$ in the probabilistic sub-models and the parameters $\boldsymbol{\Theta}_g$ in the physical sub-models. Because the observations of the variables are imperfect characterizations, there may be observation bias within the observed data.
- Measurement errors of observations, based on which of the parameters $\boldsymbol{\Theta}_f$ and $\boldsymbol{\Theta}_g$ are estimated.
- Insufficient data for the characterization of input random variables, such as the uncertainty caused by selecting the probability distribution type and parameters.

3. Model uncertainty and errors

- Uncertainty arises from the intentional simplification of relationships used in various models to represent practical relationships or actual phenomena of interest. This occurs because models cannot be sufficiently perfect without introducing uncertainty into either probabilistic or physical models. The uncertainty in the LCF life prediction model originates from the selection of probabilistic sub-model form $f_{\mathbf{X}}(\mathbf{X},\boldsymbol{\Theta}_f)$ to characterize the distribution of random variables $\mathbf{X}$ and the physical sub-model form $g_i(\mathbf{X},\boldsymbol{\Theta}_g)$ to describe the derived variables.
- Model errors result from the deviations between the model and the system that generated the measurements; these can be partitioned in two parts following their nature: stochastic model errors and systematic model errors. Stochastic model errors in experimental measurements are caused by unknown/unpredictable experimental changes. Systematic model errors in experimental observations typically come from the measuring instruments. If the experiment is repeated under the same conditions, the stochastic model errors vary from one realization to the other, while the systematic model errors remain the same. These two types of errors must be treated in different ways.
- The complexity of fatigue phenomena necessitates using simplified models for fatigue life prediction. A tradeoff between the simplicity and reliability of the model is indispensable. Furthermore, a comparison of uncertainty from model errors and other uncertainties is useful.

As mentioned above, the separation of the aleatory and epistemic uncertainties relies heavily on the modeler. Moreover, it is easy to find which uncertainty can be moderated and which uncertainty is less prone to be reduced. Better comprehension of the characteristics of uncertainties is critical to address life prediction issues properly.

During the probabilistic modeling of failure mechanism, models are developed to understand physical behaviors and predict or simulate the responses of physical processes. To approximate a practical physical situation, a model is often constructed as "a reduced and parsimonious representation of a physical, chemical, or biological system in a mathematical, abstract, numerical, or experimental form" [66]. By making certain assumptions, it is impossible for a model to completely depict the actual situation. Except for the simplifying assumption, models formulized for a particular physical system may vary due to a modeler's preference, model user requirements, or economic matters. Thus, these models are far from being perfect for the characterization of system behaviors, and model uncertainty is unavoidable. Thus, model uncertainty quantification is practically a requirement in problems where the predictions from engineering models are significantly different. Assuming that fatigue model parameters are random variables, the uncertainty can be characterized using corresponding probability distributions to quantify the uncertainty under various loading conditions. An important step in the probabilistic LCF life prediction lies in the accurate prediction of the probability distributions of model parameters.

The probabilistic LCF life prediction process for mechanical components can be depicted as shown in Fig. 1. First, the parameters in engineering (deterministic) models must be determined; these are the foundations of fatigue life predictions under a particular uncertainty. Second, when new information is available, the knowledge of fatigue failure can be evolved and reused for LCF life prediction based on a different uncertainty.

Because the incorporation of different sources of uncertainty and error is difficult, this paper attempts to develop a comprehensive procedure to address these uncertainties and errors in the fatigue reliability analysis of hot section components.

4. Uncertainty quantification for probabilistic LCF life prediction

Because the life prediction process contains uncertainty, implementing this process from a probabilistic perspective is critical. Predicting life based on uncertain information requires the characterization of the uncertainty to tune risk levels as needed in a particular application. Compared with traditional methods, where the limitations of conservative safety factor-based methods can be difficult to overcome, probabilistic modeling methods can quantitatively handle uncertainty in LCF life predictions. This section quantifies three types of uncertainty and errors in a fatigue reliability analysis and presents a simple method for predicting uncertainty using multiplicative and additive UQ methods. The appropriate incorporation and quantification of uncertainty has been widely recognized as a fundamental issue in the probabilistic modeling of fatigue failure.

LCF life prediction uncertainty is affected by a number of uncertainty and error sources. In this analysis, terminology scatter is used to address aleatory uncertainty. By assuming that the contributions from all variation types, statistical uncertainties and model uncertainties, are independent, similar to the prediction squared error proposed in [67], a general model for characterizing different types of uncertainty within LCF life prediction can be derived as

$$Y = \tilde{Y} + \phi_1 + \phi_2 + \cdots + \phi_p + Z_1 + Z_2 + \cdots + Z_q \tag{2}$$

where $\phi_i (i = 1, 2, \cdots, p)$ and $Z_j (j = 1, 2, \cdots, q)$ are random variables representing different sources of scatter and uncertainty, respectively. Y represents the logarithmic tested life, thus $Y = \ln N_{ft}$; $\tilde{Y}$ represents the logarithmic predicted fatigue life, thus $\tilde{Y} = \ln N_{fp}$.

The logarithmic predicted fatigue life can be represented as

$$\tilde{Y} = \ln N_{fp} = f(\mathbf{X}, \mathbf{\Theta}, \mathbf{\Phi}) \tag{3}$$

where $f(\mathbf{X}, \mathbf{\Theta}, \mathbf{\Phi})$ is a fatigue life prediction model involving the damage driving parameters $\mathbf{X}$ (e.g., stress, and strain), the vector of estimated model parameters $\mathbf{\Theta} = \{\theta_1, \theta_2, \cdots, \theta_r\}$, and the modeled scatter $\mathbf{\Phi}$.

The prediction error ε is determined by

$$\varepsilon = Y - \tilde{Y} = \ln N_{ft} - \ln N_{fp} = g(\mathbf{X}, \mathbf{\Phi}) - f(\mathbf{X}, \mathbf{\Theta}, \mathbf{\Phi}) \tag{4}$$

where $g(\mathbf{X}, \mathbf{\Phi})$ is the real physical relation for the log-life that depends on the damage driving parameters $\mathbf{X}$ and the involved scatter $\mathbf{\Phi}$.

Thus, the prediction error according to Eq. (2) and Eq. (4) can be approximated by the sum of

$$\varepsilon = \phi_1 + \phi_2 + \cdots + \phi_p + Z_1 + Z_2 + \cdots + Z_q \tag{5}$$

where the random quantities ϕ_i and Z_j are assumed with zero mean and variances ν_i^2 and τ_i^2, respectively.

Through the logarithmic transformation, the logarithm of a lognormal random variable (such as the fatigue life) is normally distributed; the variance of the transformed variable can then be interpreted as the squared coefficient of variation for the original variable according to the Gauss's approximation formula [68]. The measures of variation ν_i^2 and τ_i^2 can thus be treated as coefficients of variation in the fatigue life:

$$Var(\ln N_f) \approx Var(N_f)\left(\frac{\partial \ln N_f}{\partial N_f}_{N_f = E[N_f]}\right)^2 = \frac{Var(N_f)}{E[N_f]^2} \tag{6}$$

Eq. (6) is often used to estimate the uncertainty from the viewpoint of engineering judgments, as it easily relates to the percentage uncertainty. In this analysis, the variances and covariance of ϕ_i and Z_j are used in place of their exact distributions. The uncertainty within a quantity corresponding to log-life can be estimated using a sensitivity coefficient. The maximum likelihood method is commonly used to estimate the variances in statistical uncertainty. However, the original data are required for this statistical method. More details and explanations on the quantification of uncertainty are provided in the following section.

4.1. Estimation of physical variability

In general, the scatter or physical variability can be estimated through experimental examination and calculation of the sample standard deviation. However, it is often impossible or economically unaffordable to implement experiments for hot section components. In this case, previous design experience can be utilized.

There are several possible quantities that must be estimated, including (1) the parameters and errors of the LCF life prediction model, (2) the material properties, and (3) the variance of the output and measurement error. In this case study of an aircraft gas turbine disc material (GH4133 superalloy), the evaluation of the physical variability is based on 10 group fatigue tests at 400 °C and 500 °C with a strain ratio of $R_\varepsilon = -1$. The physical variability for LCF life of GH4133 originates from material properties and experimental uncertainty. Based on the LCF life data obtained from other experiments, the calculated coefficient of variation is used to represent an uncertainty regarding the experimental results. With the coefficient of variation determined from 10 group tests, a lognormal distribution was then fitted to this coefficient of variation by $\nu_{life} = 0.202577$, which gives the estimated scatter in log-life.

The strength of a component relies not only on its material properties but also its geometry and quality of assembly. In general, statistical loads and material properties can be developed from test data; however, in practice they are often based on limited test data or even expert opinion and judgment. This can result in a scatter or uncertainty concerning the actual underlying probability distributions for these random quantities. This scatter and uncertainty often has a direct impact on component life prediction. The random variables of the material properties for

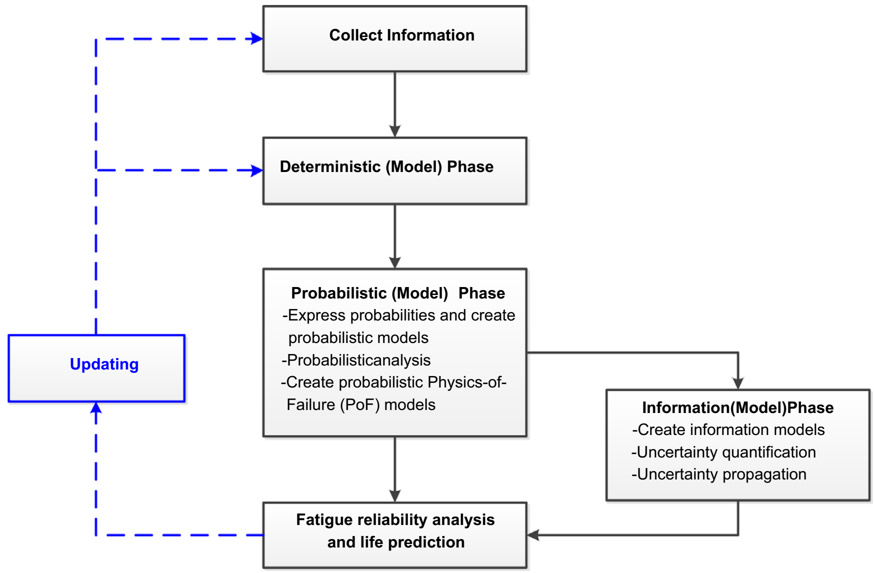

Fig. 1. Probabilistic LCF life prediction framework.

Table 1
Random material constants of GH4133.

Random variables	Distribution	Mean value (MPa)	Standard deviation
Young's modulus E	Normal	1.992×10^5	7.0×10^3
Stress endurance limit σ_{lim}	Normal	4.207×10^2	17.33
Cyclic strain hardening exponent n'	Normal	0.1005	0.006093

GH4133 were determined from an experimental data analysis in [69,70] and are shown in Table 1.

Because the loads that a component experiences during its service time are often difficult to obtain, the scatter and uncertainty in these loads must be quantified. Experience from service measurements often provides a rough estimate of potential load testing scenarios, so variations must be considered. Because the load/strength issue may override the remaining properties and lead to negligible variability in strength, the load scatter and uncertainty must be estimated. In the case study, existing load scatter and uncertainty for the test equipment was assigned an uncertainty percentage based on the testing machine manufacturer. Moreover, the load scatter and uncertainty due to individual use can be estimated based on previous engineering experience and/or differences in load spectrum. Based on the service life of a first-stage turbine disc of an aero engine installed in two types of aircrafts listed in [71], the load scatter and uncertainty for this disc can be obtained as $\nu_{load} = 0.206142$.

4.2. Statistical uncertainty

Standard statistical methods such as the maximum likelihood method and the bootstrap method are often used to obtain the variances of estimates. In the case of the physics-based LCF life prediction models shown in Eqs. (1) and (3), the model parameters can be fitted using a least squares method. As discussed in [72], the prediction uncertainty for a LCF life prediction model with more variables can be obtained by

$$\tau = s\sqrt{1 + \frac{r}{n}} \tag{7}$$

where r is the number of model parameters. Eq. (7) can be used to approximate the statistical uncertainty, particularly for situations where the original data are not available. Moreover, the statistical uncertainty can be roughly expressed as $\tau = s\sqrt{r/n}$. In our case study, a two-parameter generalized damage parameter (GDP) [56] was used based on 61 tests. As a result, the statistical uncertainty can be estimated by $\tau_{stat} = s\sqrt{2/61}$, where the scatter within log-life is obtained from 10 group fatigue tests as previously mentioned in Section 4.1, i.e., $s = \nu_{life} = 0.202577$.

4.3. Model uncertainty

During PoF-based reliability assessments, physical models are developed to predict component life under various failure mechanisms. These models are generally given as parametric forms with parameters derived through data-driven estimation methods. Generating a physical model is a mathematical simplification of a complicated phenomenon that utilizes various assumptions. Except for simplifying assumptions, model choices may vary depending on the subjective decision-making process of model construction; these choices are often not valid for all situations.

If dealt with appropriately, model uncertainty can be used to characterize the actual value of a quantity with a degree of certainty based on the model prediction. In general, this uncertainty can be measured by independently performing experiments and assessing the consistency of the model results to actual observations from these experiments. Note that the observations of experiments themselves may also contain uncertainties that may originate from measurement errors and/or the inadequate precision of measurement facilities. Thus, model uncertainty can originate from inherent epistemic uncertainty within model parameters and/or the propagated aleatory/epistemic uncertainty through model inputs.

The UQ associated with model prediction results not only serves as a criterion of confidence for the model prediction but also facilitates the estimation of other qualities, including the probability of crossing a predefined physical limit. This study incorporates model uncertainty into the life prediction of aircraft turbine discs. The experimental results can be used as representations of practical values because the errors of measurement facilities and variations of test conditions may be small enough to be disregarded. The model predictions can also be viewed as

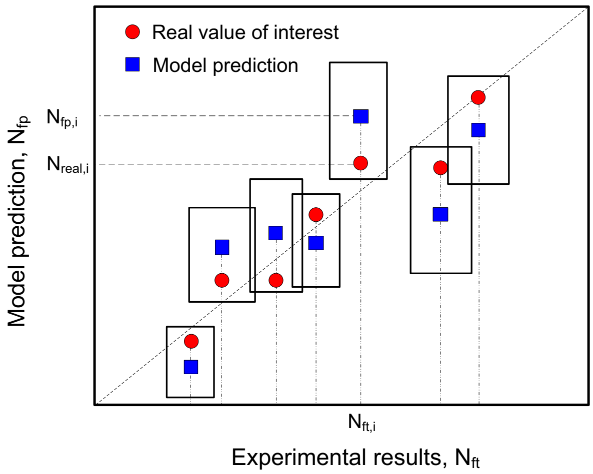

Fig. 2. Apparent scatter of uncertain data.

another representation of actual values based on the physical or empirical modeling processes presented in Fig. 2.

In Fig. 2, the uncertainty in the experimental data determines the position of the uncertain data points as presented on the N_{ft} axes. The uncertainty within the model outputs makes the location of data points uncertain on the N_{fp} axes. Moreover, the actual value of the relevant quantity is believed to reside somewhere in the rectangle, which represents the uncertainty in a particular location of every experimental result and model prediction pair. Thus, the two questions that must be answered are 1) how precise are the predictions, and 2) how does one implement a probabilistic quantification of that belief. Because both model predictions and experimental results provide information on physical variables (such as fatigue life during life prediction), the comparison is implemented through the relative error between the model predictions and the experimental results, thereby providing information for handling and mitigating uncertainty. Through the proper manipulation of this error, model predictions can then be considered independent samples from a population of model uncertainty that follows a particular statistical distribution.

As shown in Fig. 2, updating output with independent experimental data helps incorporate uncertainties that are not presented in the probability distributions developed for model inputs and parameters. According to the black-box viewpoint [19,73], the errors introduced by experimental measurements were assumed to be independent from the errors resulting from model predictions. When comparing the aforementioned representations (model prediction and experimental result) to actual values, a multiplicative error for each test $i=(1,2,\cdots,k)$ is used to describe the experimental error $F_{t,i}$ and the model prediction error $F_{p,i}$. During LCF life prediction, the ratio of real-life and model predictions or experimental results is assumed to be a random variable following a lognormal distribution, which is defined and represented, respectively, as

$$\frac{N_{real,i}}{N_{ft,i}}=F_{t,i} \quad F_t \sim LN(b_t,s_t) \tag{8}$$

and

$$\frac{N_{real,i}}{N_{fp,i}}=F_{p,i} \quad F_p \sim LN(b_p,s_p) \tag{9}$$

Combining Eqs. (8) and (9), the relationship between experimental uncertainty and model uncertainty can be derived as

$$\frac{N_{ft,i}}{N_{fp,i}}=\frac{F_{p,i}}{F_{t,i}}=F_{pt,i} \tag{10}$$

Assuming that F_p and F_t are independent leads to

$$F_{pt} \sim LN\left(b_p-b_t,\sqrt{s_p^2+s_t^2}\right) \tag{11}$$

The model uncertainty is characterized through a Bayesian inference where the experimental results are compared. Within this process, knowledge is consecutively evolved and the uncertainty is continually reduced by leveraging the Bayesian updating of the gradually gathered new information. Given a sample of a number of cycles to failure $\mathbf{N_{ft}}=\{N_{ft,1},N_{ft,2},\cdots,N_{ft,n}\}$ and model predictions $\mathbf{N_{fp}}=\{N_{fp,1},N_{fp,2},\cdots,N_{fp,n}\}$, the posterior joint distribution can be obtained by

$$\pi(b_p,s_p|N_{ft,i},N_{fp,i},b_t,s_t)=\frac{\pi_0(b_p,s_p)\times L(N_{ft,i},N_{fp,i},b_t,s_t|b_p,s_p)}{\int_{s_p}\int_{b_p}\pi_0(b_p,s_p)\times L(N_{ft,i},N_{fp,i},b_t,s_t|b_p,s_p)\,db_p ds_p} \tag{12}$$

with the likelihood function

$$L(N_{ft,i},N_{fp,i},b_t,s_t|b_p,s_p)=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\left(\frac{N_{ft,i}}{N_{fp,i}}\right)\sqrt{s_p^2+s_t^2}}\exp\left(-\frac{1}{2}\times\frac{\left[\ln\left(\frac{N_{ft,i}}{N_{fp,i}}\right)-(b_p-b_t)\right]^2}{s_p^2+s_t^2}\right) \tag{13}$$

where $\pi_0(b_p,s_p)$ is the joint prior distribution of parameters b_p and s_p and $\pi(b_p,s_p|N_{ft,i},N_{fp,i},b_t,s_t)$ is the joint posterior distribution of these parameters.

When new data becomes available, Eq. (12) is later used to update the posterior distributions of parameters b_p and s_p generated based on previous data. Using Eq. (9), if considering only the model uncertainty, the life distribution N_{real} can be easily summarized from samples of $\pi_0(b_p,s_p)$ using a Markov Chain Monte Carlo simulation as

$$N_{real} \sim LN(\ln(N_{fp})+b_p,s_p) \tag{14}$$

Because the validation tests are performed under conditions that are expected to give rise to the same model errors as those generated in-service. The updated model uncertainty can be presented as the random scatter together with the model uncertainty. However, it is often impossible to implement validation tests under practical engineering conditions. This can result in additional uncertainties to the life prediction results such as the variance of remaining model uncertainty, which comes from the randomness and unpredictability of future working conditions that cannot be reflected in the validation tests.

Experimental uncertainty can be depicted as the variation-edge in the horizontal direction of the surrounding rectangle for every data point presented in Fig. 2. The model uncertainty will be overestimated when the experimental uncertainty is not subtracted from the obvious scatter of data. Thus, combined with the experimental uncertainty obtained in Section 4.1, a black-box approach based on experimental results and model predictions with a Bayesian inference framework is constructed to obtain a robust quantification of model uncertainty.

4.4. LCF life prediction uncertainty

According to Eq. (5), the LCF life prediction uncertainty is obtained by summing all the contributions as

$$\delta_{pred}=\sqrt{\nu_1^2+\nu_2^2+\cdots+\nu_p^2+\tau_1^2+\tau_2^2+\cdots+\tau_q^2} \tag{15}$$

During probabilistic LCF life prediction, all types of scatter and uncertainty characterized by an associated standard deviation are assumed to be statistically independent. Consequently, there is no covariance term in Eq. (15); it is simply the sum of these covariance terms using Eq. (15) to correlate the scatters and uncertainties. This permits the combination of different scatter and uncertainty sources into a single uncertainty measure.

The implementation of life prediction and reliability analysis results will be described in the following section. A critical goal is to gain information for particular points from which uncertainties can be significantly reduced. In this paper, model uncertainty appearing in the model parameters is by far the most significant source of uncertainty. Accordingly, it is critical to further investigate the model parameters and reduce model uncertainty during the decision-making process.

4.5. Efficient probabilistic analysis

In this section, the uncertainty within LCF life prediction is utilized and given as a prediction interval or presented in the form of a safety factor for the log-life. Through the formulization of a prediction interval under normal approximation, we have

$$\ln N_f = \ln N_{fp} \pm \gamma_p \delta_{pred} \tag{16}$$

where δ_{pred} is the uncertainty within LCF life prediction; the factor γ_p is a coverage factor that expands the coverage of the uncertainty estimate depending on the percent confidence desired in the estimate and the quantity of observations available for computing the random and bias variances, such as $\gamma_{0.025}=2$ for a 95% interval and $\gamma_{0.001}=3$ for a 99.8% interval. The coverage factor is greater for a higher desired confidence level and/or a smaller number of observations.

The life safety factor can be defined as the ratio of the median life over a low quantile life based on the prediction interval

$$K_p = \frac{N_{fp,0.5}}{N_{fp,p}} \tag{17}$$

where p is the specimen/component failure probability.

Combined with Eq. (16), the safety factor can be obtained by

$$K_p = \exp\left(\gamma_p \cdot \delta_{pred}\right) \tag{18}$$

Moreover, experiments can be used to update the predicted fatigue life by correcting the systematic error and reducing the model uncertainty.

According to LCF life predictions based on the prediction uncertainty, Eq. (16) requires a single standard deviation measure (1σ) for each random uncertainty source. Our approach determines the largest or worst-case error in a parameter from experiments that isolate a single source of uncertainty and identify it as the single standard deviation measure associated with that particular source. This method is a conservative approach because the largest or worst-case error measure is typically considered to be three standard deviations (3σ). One can choose how much conservatism is built into the UQ.

Another objective for a probabilistic analysis is quantifying the fatigue reliability based on the random variable inputs. Fatigue reliability R_f is defined by the probability of total fatigue damage at a given lifetime being less than one

$$R_f = \iint_{D<1,-\infty<\sigma_{\lim}<+\infty} f(D_t, \sigma_{\lim}) dD_t d\sigma_{\lim} \tag{19}$$

where $f(D_t, \sigma_{\lim})$ is the joint probability density function (PDF) of the cumulative damage and the endurance limit.

Based on the conditional distribution of total fatigue damage, Eq. (19) can be rearranged as

$$R_f = \iint f\left(D_t | \sigma_{\lim}\right) f(\sigma_{\lim}) dD_t d\sigma_{\lim} = \int_{-\infty<\sigma_{\lim}<+\infty} P(D_t<1|\sigma_{\lim}) f(\sigma_{\lim}) d\sigma_{\lim} \tag{20}$$

where the conditional reliability $P(D_t<1|\sigma_{\lim})$ can be obtained while the conditional total fatigue damage can be determined as a normal random variable using the central limit theorem for any given $\sigma_{\lim}$.

By characterizing the input random variables, a probabilistic interpretation can be developed to capture the aforementioned scatter and uncertainty. Typically, fatigue life and reliability can be obtained using Eq. (16) and Eq. (20), respectively, to consider the contributions of each source of scatter and uncertainty. Moreover, fatigue reliability can be obtained by simulating input load variability via FE analysis using Eq. (20), where the distribution of total fatigue damage is calculated based on the quantified uncertainty in this analysis; this will be further investigated.

5. Probabilistic LCF life prediction and experimental data comparison

To predict LCF life probabilistically, the combination of physics-based and data-driven approaches considers the proper fracture mechanism while correcting for material variations and model uncertainty using data-driven model updating. In general, the degree of consistency between model outputs and test observations can be measured by model validation. This section aims to validate and assure confidence in the life prediction of aircraft turbine engine discs. One important aspect in this process is the rational, clear treatment of all types of uncertainties and errors. By accounting for these uncertainties in the systematic manner discussed in Section 3, the probability distribution of fatigue life can be calculated. Moreover, the model performance can be quantitatively judged using the methods developed in Sections 2 and 4.

5.1. Turbine disc alloy GH4133 tests at 500 °C and 400 °C

Using the method developed to estimate model uncertainty in Section 4.3, the model uncertainty bounds (a 2.5–97.5% prediction interval) for the life prediction of GH4133 by the GDP method are approximately $[-7.96\%, +3.10\%]$ and the model uncertainty can be approximated by $\tau_{model}=0.028214$ when the sample size is sufficiently large. Multiple types of scatter and uncertainty, as well as their contributions given as a standard deviation of the log-life over the uncertainty of life prediction, are summarized and shown in Table 2. The sum of the uncertainty measure for an aircraft turbine disc can be computed using Eq. (15).

Using the GDP method, the 95% prediction intervals of LCF life prediction for aircraft turbine disc alloy specimens according to Eq. (16) are compared with the observed results in Figs. 3–5 under different loading conditions. Using probability distributions of inputs and quantified uncertainty along with the GDP life prediction model leads to a predicted life distribution that can be expressed with the lower bound (2.5%) and upper bound (97.5%) of the predicted life.

As observed in the 2.5–97.5% bounded figures, most of the predicted LCF life intervals using the GDP method for GH4133 are bounded within a factor of ± 2 compared to the tests at 500 °C, $R_{strain}=-1$ and 400 °C, $R_{strain}=0$. However, the prediction intervals for GH4133 at 400 °C and $R_{strain}=-1$ tend to be conservative for tests above 10^4 cycles. The probabilistic LCF life predictions using the GDP method demonstrated consistency with the experimental results by means and bounds (2.5% and 97.5%). Based on both the upper and lower bounds of the predicted fatigue life, the predictions incorporate the contributions from different scatter and uncertainty sources and characterize the statistical nature of fatigue life through UQ.

Using the previously developed Bayesian framework for UQ in [19], the GDP method predicts the LCF life with the uncertainty bound $[-10.01\%, +8\%]$, and the LCF prediction uncertainty δ_{pred} as approximately 0.04595 for the GH4133 superalloy. Compared with the LCF prediction uncertainty of $\delta_{pred}=0.292701$ as shown in Table 2, the difference between the two results arises from the consideration of load scatter and uncertainty due to individual usage based on previous engineering experience in this analysis. Additionally, the proposed framework using the multiplicative and

Table 2
Table summarizing the sources of uncertainty of GH4133.

Type of scatter and uncertainty	Scatter and uncertainty
Physical variability (experimental uncertainty)	0.202577
Load scatter	0.206142
Statistical uncertainty (two-parameter model)	0.036681
Model uncertainty	0.028214
Sum	0.292701

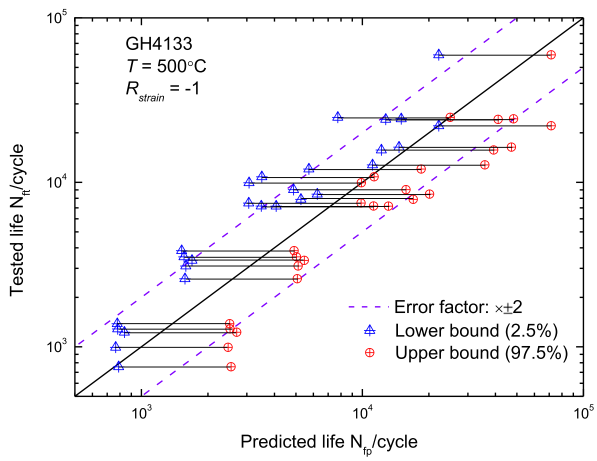

Fig. 3. Prediction intervals versus experimental life for GH4133 at 500 °C and $R_{strain} = -1$.

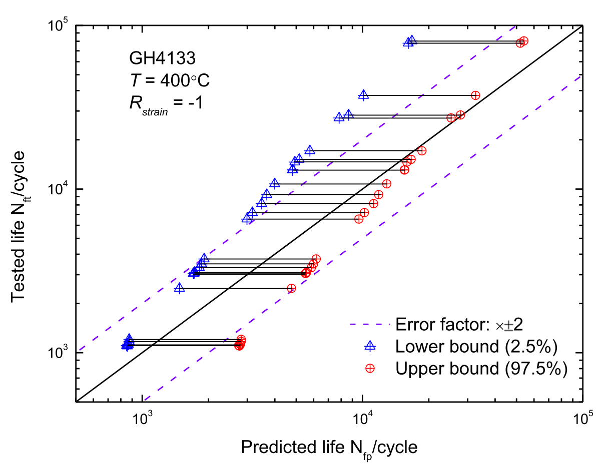

Fig. 4. Prediction intervals versus experimental life for GH4133 at 400 °C and $R_{strain} = -1$.

additive UQ method can directly identify the factors that contribute most to the resulting output uncertainty and offer a useful reference for uncertainty reduction in decision-making, which has the merit of simplicity for engineering applications.

5.2. Full-scale turbine disc tests

In this section, the proposed framework is verified using full-scale tests of high-pressure turbine discs from an aero engine (see Fig. 6a) used in a particular aircraft. This disc works under high temperatures and experiences high rotational velocity. This disc is used to accurately settle the ring of the rotating blades. It is also used to deliver the energy absorbed by the rotating blades through the shaft by connecting the turbine to the compressor. High rotational velocity often gives rise to significant centrifugal forces in discs. Moreover, their high-temperature working conditions often reduce disc material strength.

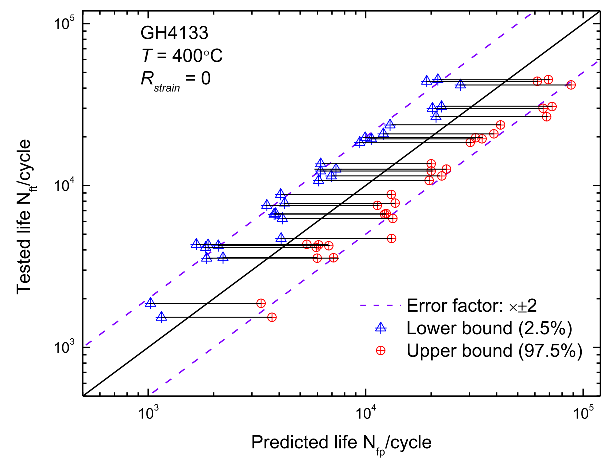

Fig. 5. Prediction intervals versus experimental life for GH4133 at 400 °C and $R_{strain} = 0$.

High-temperature LCF tests for full-scale turbine discs were conducted at the vertical disc test facility of the China Gas Turbine Establishment. The spinning facility includes a vacuum case, data acquisition and control systems, and a main power system, as shown in Fig. 6. The performance indicators of this test facility are listed below in Table 3. Tests on three GH4133 superalloy turbine discs with the same geometry were carried out under the following loading conditions: a rotation speed range of 2000–12710–2000 rpm (rotations per minute) and a temperature distribution of 250 °C ± 10 °C at the hub zone and 270 °C ± 10 °C at the rim section. These tests were performed with a triangular rotation speed spectrum, as shown in Fig. 7. The speed control precision is approximately ±50 rpm for the rotation speed range of 2000–4000 rpm and ±20 rpm for the rotation speed range of 4000–12710 rpm. In these tests, service life is determined for the disc under stress and temperature profiles similar to those endured by the engine, where the disc is deemed a ‘failure’ when an ‘engineering crack’ reaches approximately 0.78 mm in length at the surface of disc-critical regions such as the assembly holes, the dovetail-rim area, and the hub zone.

Based on the physical variability mentioned in Section 4.1, the coefficient of variation of this high-pressure turbine disc can be estimated as $\nu_{life} = 0.166227$ based on the three disc tests. These variations can be attributed to material scatter and production scatter because of different assembly, processing and/or supplier effects, geometry and assembling quality. Similarly, using the GDP method for the LCF life prediction of high-pressure turbine discs, the sources of uncertainty are summarized as shown in Table 4; the LCF life prediction uncertainty can be estimated as $\delta_{pred} = 0.298903$.

Using the GDP method, the 95% prediction intervals of LCF life prediction of high-pressure turbine discs according to Eq. (16) are subsequently compared with those obtained from full-scale fatigue tests in Fig. 8. Good consistency is clearly demonstrated between the observed test results and the calculated prediction intervals using the GDP method by the mean, upper and lower bounds.

Under the assumption that the contributions from scatter and uncertainty sources are independent, although contributions from particular sources may vary for different applications, it is worth noting that LCF life prediction uncertainty is dominated by the model uncertainty, load scatter and uncertainty and physical variability for the experimental data sets of GH4133 and high-pressure turbine discs outlined above. For the same test data,

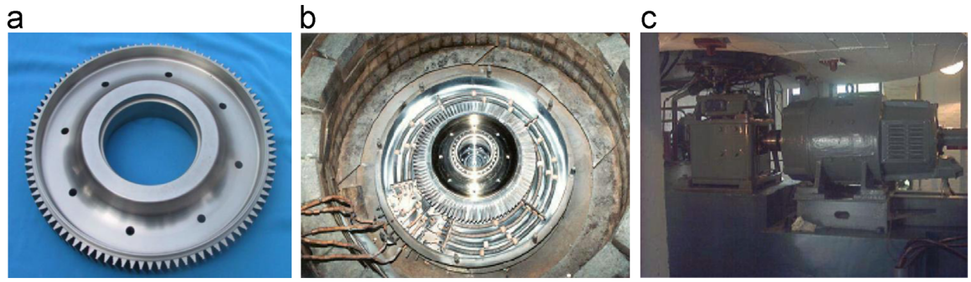

Fig. 6. The vertical disc test facility, (a) high-pressure turbine disc for tests, (b) the vacuum case, and (c) the main power system.

Table 3
Performance indicators of the vertical disc test facility.

Category	Performance indicator
Test type	Strain measurement, overspeed, over-temperature, fracture and LCF test
Maximum diameter of the rotor	1200 mm
Rotation speed range	0–25000 rpm
Maximum temperature	800 °C ± 10 °C
Rotation speed spectrum	Trapezoidal, triangular and random waveforms

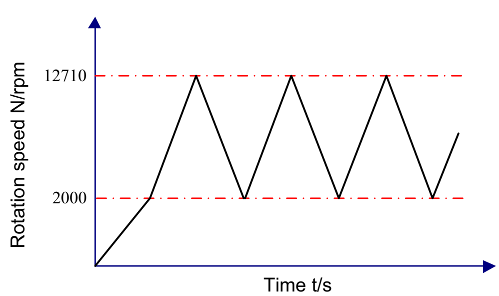

Fig. 7. Rotation speed spectrum for the full-scale turbine disc test.

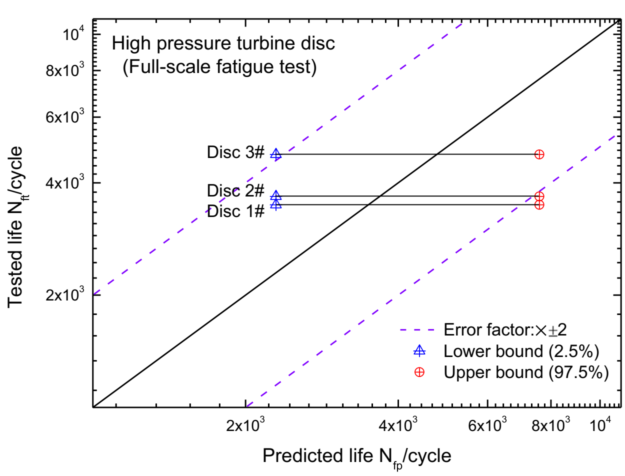

Fig. 8. Prediction intervals versus experimental life for high-pressure turbine discs.

using different fatigue models often results in different life prediction uncertainties according to Eq. (16); tighter uncertainty bounds generally give rise to better model selection with the same information and knowledge. Thus, this work introduces a theoretical foundation for model selection in the life prediction of engineering components.

In this study, the proposed framework is capable of representing the propagation of various uncertainties within the life

Table 4
Table summarizing the sources of uncertainty of high-pressure turbine discs.

Type of scatter and uncertainty	Scatter and uncertainty
Physical variability (experimental uncertainty)	0.166227
Load scatter	0.206142
Statistical uncertainty (two-parameter model)	0.135724
Model uncertainty	0.028214
Sum	0.298903

prediction model, as well as determining the joint effect of these uncertainties on the predicted fatigue life distribution. The framework can be further used to facilitate fatigue life prediction for most mechanical components by quantifying the uncertainties within total inputs and model uncertainty. Through full-scale testing of the discs and specimens, the predicted lifetime and reliability of the GH4133 superalloy can be obtained with high-precision and less uncertainty using numerical simulations via a stochastic FEA.

In terms of the stochastic nature of fatigue, the PDFs of the stress and strain associated with turbine discs can be obtained through FE analysis. Based on the FEA-based stress/strain analysis of turbine discs subjected to multiple levels of practical cyclic loadings, the fatigue reliability of the turbine disc can be calculated and further assessed.

6. Conclusions

To assess the LCF life of aircraft gas turbine discs, this paper conducted UQ using classical statistics and Bayesian theory to describe the failure properties of GH4133 superalloy and full-scale tests of aero engine high-pressure turbine discs. The findings of this paper are summarized as follows.

(1) A probabilistic PoF-based framework for LCF life prediction of aircraft gas turbine discs under uncertainty is developed. This framework can facilitate the uncertainty propagation through the life prediction model in a coherent way to assess their joint effects on the predicted fatigue life distributions.
(2) A comprehensive UQ procedure is presented by quantifying multiple types of uncertainty using multiplicative and additive UQ methods. With respect to the aircraft gas turbine discs, three categories of uncertainty exist in this UQ procedure: variability associated with loading conditions and material properties, statistical uncertainty originating from measurement errors and/or sparse data and/or I/M results, and model uncertainty and errors introduced during fatigue failure analysis and numerical simulations. Furthermore, the factors that contribute most to the resulting output uncertainty have been investigated and identified, thus providing a theoretical basis to select the most effective tests to reduce uncertainty.

(3) The prediction accuracy of the proposed framework is validated through a comparison of model prediction intervals to the experimental results of GH4133 and full-scale turbine disc tests. The results demonstrate the good consistency between the experimental observations and the model prediction intervals using the GDP method. Based on both the lower and upper bounds of the predicted LCF life, the calculated predictions incorporate contributions from different scatter and uncertainty sources and characterize the statistical nature of fatigue life. The proposed framework can be extended to any PoF-based LCF life prediction models and fatigue reliability analysis methods; this paper uses the GDP method solely for illustration purposes.

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