International Journal of Information Technology & Decision Making
Vol. 14, No. 5 (2015) 1017–1034

DOI: 10.1142/S0219622014500242

Quantification Classification Algorithm of Multiple Sources of Evidence

Jian-Ping Yang[*,†,‡], Hong-Zhong Huang[*,§],
Yu Liu[*,¶] and Yan-Feng Li[*,‖]

[*]*School of Mechanical, Electronic, and Industrial Engineering*
University of Electronic Science and Technology of China
No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu
Sichuan 611731, P. R. China

[†]*Unit 95389 of People's Liberation Army*
Guilin 541007, P. R. China
[‡]*yjp_28@163.com*
[§]*hzhuang@uestc.edu.cn*
[¶]*yuliu@uestc.edu.cn*
[‖]*lyfmoon@gmail.com*

Published 27 January 2014

Although Dempster–Shafer (D–S) evidence theory and its reasoning mechanism can deal with imprecise and uncertain information by combining cumulative evidences for changing prior opinions of new evidences, there is a deficiency in applying classical D–S evidence theory combination rule when conflict evidence appear — conflict evidence causes counter-intuitive results. To address this issue, alternative combination rules have been proposed for resolving the appeared conflicts of evidence. An underlying assumption is that conflict evidences exist, which, however, is not always true. Moreover, it has been verified that conflict factors may not be accurate to characterize the degree of conflict. Instead, the Jousselme distance has been regarded as a quantification criterion for the degree of conflict because of its promising properties. To avoid the counter-intuitive results, multiple sources of evidence should be classified first. This paper proposes a novel algorithm to quantify the classification of multiple sources of evidence based on a core vector method, and the algorithm is further verified by two examples. This study also explores the relationship between complementary information and conflicting evidence and discusses the stochastic interpretation of basic probability assignment functions.

Keywords: Dempster–Shafer evidence theory; evidence distance; evidence conflict; quantification classification.

1. Introduction

Probability theory has been widely used to characterize the information uncertainty in engineering systems. However, with the increased system complexity and engineering requirements, some limitations of probability theory have been recognized: (1) epistemic uncertainty of system, which is subjective, and reducible uncertainty

[§]Corresponding author.

resulting from the lack of knowledge or data about the system cannot be described[1]; (2) different types of information obtained from multiple sources cannot be aggregated using traditionally probability theory[1]; (3) disparity between imprecision and uncertainty must be declared in quantification of system uncertainty.[2] Thus, to overcome these limitations, some new theoretical frameworks have been proposed as alternatives to probability theory, such as possibility theory,[3,4] Dempster–Shafer (D–S) evidence theory,[5] fuzzy sets theory, and interval probability.[6–11]

D–S evidence theory was introduced by Shafer[5] based on Dempster's investigation.[12] It employs a confidence interval which is represented by the upper and lower limits called plausibility function (Pl) and belief function (Bel). Both limits are constructed by basic probability assignment (BPA) to characterize the uncertainty of system. However, it is different that the probability mass can be assigned to event sets in D–S evidence theory. In probability theory, mass probability can only be assigned to a single event. Because of the flexibility of the basic axioms in evidence theory, no further assumptions are needed to quantify the uncertain information of system. Moreover, the D–S evidence theory combination rule can aggregate information from multiple sources to a new belief assignment. Owing to these properties, it has been used in fault diagnosis,[13–16] reliability engineering,[17–22] classification,[23] object extraction,[24] knowledge discovery,[25] risk analysis,[26,27] and so on.

When D–S evidence theory gained popularity in many applications, Zadeh proposed criticisms that combination consequence may be counter-intuitive using classical D–S evidence theory combination rule on conflicting beliefs and provided a compelling counter example.[28] Since then, many alternative combination rules have been introduced to resolve this issue. Yager proposed a new combination algorithm that uses a ground probability mass assignment function instead of BPA and attributed conflict belief to universal set. It should be noted that there is no normalization factor in the ground probability mass assignment function.[29] Inagaki introduced a new combination rule with the goal of harmonizing D–S combination rule and Yager's rule.[30] Dubois and Prade investigated the empirical and axiomatic foundations of belief functions and possibility measures and proposed the disjunctive combination rule to cope with conflicting beliefs and belief combinations from specific information sources which may be nonreliable, non-exhaustive, inconsistent, and dependent.[31] Smets proposed the transferable belief model based on a subjective interpretation and then introduced an unnormalized combination rule.[32] Kallel and Hégarat-Mascle proposed a new parameterized combination rule that takes into account a partial non-distinctness of sources between two bodies of evidence. Because this method defined separable basic belief assignments and belief densities, it can be applied to a discrete or a continuous body of evidence.[33] According to classical Bayes analysis through a bijective mapping between Dirichlet distributions and BPA, Josang *et al.* proposed the cumulative rule and averaging rule of BPA based on the generalizations of the corresponding fusion operators for opinions.[34] Yang *et al.* proposed the paradox combination algorithm based on an absolute difference factor of two pieces of evidence and a relative difference factor of two pieces of evidence for a specific hypothesis with the consideration of local attributions to local conflict.[35]

These studies show that the D–S evidence theory has been widely used and deeply studied. However, there are two main questions that remain to be answered:

(1) How to quantify the degree of conflict between beliefs? The conflict factor, which is commonly regarded as the quantitative measure of the qualitative definition of conflict, may be attained as the wrong result of two bodies of evidence in classical D–S evidence theory.[36] In other words, the new quantification conflict criteria should be selected or constructed to measure the degree of difference between two bodies of evidence, instead of the conflict factor of the classical D–S evidence theory. Moreover, construction of these criteria serves as objective foundation about whether the combination result is counter-intuitive or not when using classical D–S combination rule. There are many papers that deal with the counter-intuitive results, a few of which, however, investigated the precondition of using the new combination rule — a common underlying assumption in these papers states that conflicts between two bodies of evidence are identified.
(2) How to classify multiple sources of evidence quantificationally? With the development of sensor techniques and information science and the demand of increased system complexity, many bodies of evidence representing the system character can be attained. To meet this trend, an efficient classification algorithm is a must in order to avoid the counter-intuitive results when multiple bodies of evidence are collected and aggregated.

For the first question, Liu proposed the distance between betting commitments of the two bodies of evidence according to the pignistic transformation of themselves to quantify the degree of conflict.[36] However, the properties of matrix distance such as non-negativity, nondegeneracy, symmetry, and triangle inequality are not represented. Fixsen and Mahler proposed a pseudo-distance,[37,38] which is a distance to a body of evidence where the event is a singleton and probability of event is equal to 1. This confuses total certainty about ignorance with total uncertainty about all the subsets.[39] Jousselme *et al.* proposed a new distance considering taking the maximum advantage of the information contained in the BPA.[39] The aim of the proposed distance is to represent a measure of performance (MOP) for identification of algorithms based on D–S evidence theory. The essence of MOP is to measure the degree of difference between two bodies of evidence. Moreover, the property of the metric distance is demonstrated and the maximum distance is calculated to be equal to 1. For the second question, many papers only represent the classification between two bodies of evidence.[25–33,36,37,39] These two bodies of evidence can be classified into two categories: conflicting and nonconflicting evidence.

In this paper, the Jousselme distance is adopted as the criterion to quantify the degree of conflict between two bodies of evidence. The relationship between conflicting evidence and information complementarity is analyzed. A novel algorithm to quantify the classification of multiple sources of evidence is proposed. The rest of the paper is organized as follows. Section 2 reviews the basis of D–S evidence theory. Section 3 describes the interpretation between conflicting evidence and information complementarity. Section 4 presents the stochastic interpretation for BPAs. Section 5 introduces the Jousselme distance of evidence and proposes our quantification classification algorithm. Section 6 verifies the novel algorithm through two examples. Finally, Sec. 7 concludes the main contribution of the paper.

2. D–S Evidence Theory

D–S evidence theory is represented by a finite nonempty exhaustive set of mutually exclusive possibilities called a frame of discernment, Θ; 2^Θ is the power set of Θ, which includes all the possible subsets of Θ. There are 2^n elements in 2^Θ, if Θ has n elements. Let q_i be the ith possibility and $i = 1, 2, \cdots, n$, then we have

$$\begin{aligned} 2^\Theta = \{&\emptyset, \{q_1\}, \ldots, \{q_n\}, \{q_1, q_2\}, \{q_1, q_3\}, \ldots, \{q_1, q_n\}, \ldots, \\ &\{q_{n-1}, q_n\}, \{q_1, q_2, q_3\}, \ldots, \{q_1, q_2, \ldots q_n\}\} \end{aligned} \tag{2.1}$$

In D–S evidence theory, three basic functions are adopted to build the elementary reasoning framework: the BPA function, the Bel function, and the Pl function.

The BPA function is a primitive of evidence theory, which is denoted by $m(X)$. The function $m(X)$ is a mapping: $m(X) : 2^\Theta \to [0, 1]$, and satisfies the following conditions:

$$m(\emptyset) = 0, \tag{2.2}$$

$$\sum_{X \in 2^\Theta} m(X) = 1. \tag{2.3}$$

Here, $m(X)$ expresses the precise probability in which the evidence corresponding to m supports proposition X. That is, $m(X)$ is a measure of the belief attributed exactly to hypothesis X and to none of the subsets of X. X is not only a single possible event but also a set of multiple possible events; $m(\emptyset) = 0$ means that the existing evidence supports no element of the domain; $m(X) = 1$ states that an existing evidence only supports x in the domain. $\sum_{x \in 2^\Theta} m(X) = 1$ guarantees the normalization of evidences. The BPA for an event set is the belief assigned for the whole set. In other words, it cannot be reassigned to any subsets of the event.

A Bel is often defined by the BPA function which is represented by $\mathrm{Bel}(X)$ in the following.

$$\mathrm{Bel}(X) = \sum_{Y \subseteq X} m(Y) \tag{2.4}$$

$\mathrm{Bel}(X)$ represents the total amount of probability that must be distributed among elements of X. It reflects inevitability and indicates the total degree of belief of X and constitutes a lower limit function on the probability of X.[40] Bel(X) is the lower limit of the posteriori confidence interval. On the other hand, the BPA function can be obtained from the Bel by means of the Möbius inversion formula

$$m(X) = \sum_{A \subseteq X} (-1)^{|X-A|} \mathrm{Bel}(A), \tag{2.5}$$

where $|\cdot|$ denotes the cardinality function. Obviously, when there are n events in Θ, the following formula is satisfied.

$$0 \le |A| \le n \quad A \in 2^{\Theta}. \tag{2.6}$$

A Pl is defined as follows:

$$\mathrm{Pl}(X) = 1 - \mathrm{Bel}(\overline{X}) = \sum_{Y \cap X \neq \Phi} m(Y), \tag{2.7}$$

where $\overline{X}$ is the negation of a hypothesis X. Pl(X) measures the maximal amount of probability that can be distributed among the elements in X. It describes the total degree of belief related to X and constitutes an upper limit function on the probability of X.[40] The same applies to Bel(X), and the Pl can be transformed into the BPA function using the following formula

$$m(X) = \sum_{A \subseteq X} (-1)^{|X|-|A|+1} \mathrm{Pl}(\overline{A}). \tag{2.8}$$

[Bel(X), Pl(X)] is the posteriori confidence interval which expresses the uncertainty of X. When the ignorance to proposition X is decreased, the length of interval is diminished. $[\mathrm{Bel}(X), \mathrm{Pl}(X)] = [0, 1]$ describes complete ignorance of proposition X. This is illustrated in Fig. 1.

The fusion of multiple sources of evidence can be performed by the Dempster's combination rule that is defined in Eq. (2.9) below. Given the two BPA functions, $m_i(X)$ and $m_j(Y)$, the Dempster's combination rule can be defined as

$$\begin{aligned} m(C) &= m_i(X) \oplus m_j(Y) \\ &= \begin{cases} 0, & X \cap Y = \varnothing \\ \dfrac{\sum_{X \cap Y = C, \forall X, Y \subseteq \Theta} m_i(X) \times m_j(Y)}{1 - \sum_{X \cap Y = \varnothing, \forall X, Y \subseteq \Theta} m_i(X) \times m_j(Y)}, & X \cap Y \neq \varnothing \end{cases}, \end{aligned} \tag{2.9}$$

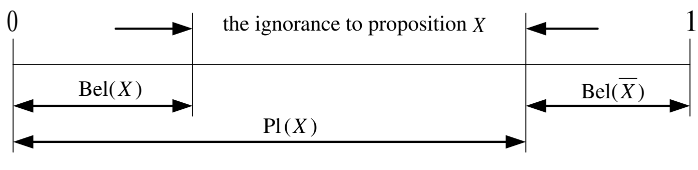

Fig. 1. Belief function and plausibility function.

where $m_{i(j)}(c)$ denotes the BPA of that is supported by ith (jth) evidence. Let

$$K_{ij} = \sum_{X \cap Y = \emptyset} m_i(X) \times m_j(Y). \tag{2.10}$$

K_{ij} satisfies $0 \leq K_{ij} \leq 1$ and is called the conflict factor, which expresses the degree of conflict between evidences ith and jth. $K_{ij} = 0$ means that evidence ith and jth have no any conflict, while $K_{ij} = 1$ or $0 < K_{ij} < 1$ indicates that two sources of evidence are in complete conflict or in partial conflict, respectively, to support an opinion.

3. Information Interpretation of Conflicting Evidence

Information from multiple sources of evidence may be redundant or complementary. Redundant information enhances the reliability of system analysis and aggregation in information fusion. Complementary information enlarges and improves the representation of system status. The system is more comprehensively and abundantly expressed. The status of system is reflected from the different perspective. For classical D–S evidence theory, only knowing that two bodies of evidence are conflicting is not enough to recognize which one is more important and sufficient, given that no more information of system is attained. In this situation, these two conflicting bodies of evidence may be complementary to each other in expressing system information. In other words, the evidence which is complementary on information cannot be aggregated using classical D–S evidence theory combination rule. When two bodies of evidence are nonconflicting with each other, they may include redundant information. When the combination rule of D–S evidence theory is used to aggregate the evidence involving redundant information, combination results will be rational and efficient. For instance, in Zadeh's famous compelling example, there are two bodies of evidence from two doctors. One doctor thinks that the patient has either meningitis with a probability of 0.99 or a brain tumor with a probability of 0.01. The second one believes that the probability of the patient suffering from a concussion and a brain tumor is 0.99 and 0.01, respectively.[28] In this situation, there is no more information about the two doctors. The two bodies of evidence can be considered as complementary.

4. Stochastic Interpretation for BPAs

With the brief review of D–S evidence theory, we can see that a body of evidence represents the basic belief assignment information through BPA in a situation at a given time. A BPA can be taken as a discrete random function whose variable is a probability distribution, $m(\cdot)$, of 2^Θ. Consequently, a BPA can be easily represented using vector notation whose elements are discrete random $m(\cdot)$ of 2^Θ and can be dealt with by elementary vector algebra.

Definition 1. Probability vector ($\mathbf{p}$): $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ satisfies the following conditions:

$$0 \leq p_i \leq 1, \quad i = 1, 2, \ldots, n, \tag{4.1}$$

$$\sum_{i=1}^{n} p_i = 1. \tag{4.2}$$

According to the definition of a BPA, it can be perceived that a BPA is a special case of probability vector which have 2^n elements and can be noted as $\mathbf{m} = (m(\emptyset), m(X_1), m(X_2), \ldots, m(X_{2^n-1}))$. Consequently, the elements of $\mathbf{m}$ satisfy:

$$\sum_{i=1}^{2^n-1} m(X_i) = 1 \quad \text{and} \quad 0 \leq m(X_i) \leq 1, \quad i = 1, 2, \ldots, 2^n - 1. \tag{4.3}$$

where $X_i \in 2^\Theta$ and $m(\emptyset) = 0$.

Definition 2. Stochastic matrix ($\mathbf{P}$):

$$\mathbf{P} = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix} \tag{4.4}$$

satisfies that every row vector is a probability vector and is called stochastic matrix. D$-$S evidence theory can fuse multiple sources of evidence and every source of evidence is expressed by the BPA function. Consequently, every source of evidence can be used as a row of the stochastic matrix. The BPAs of k independent sources of evidence can form a $k \times 2^n$ stochastic matrix. This matrix is also called as the mass stochastic matrix denoted as $\mathbf{M}$.

$$\mathbf{M} = \begin{pmatrix} m_1(\emptyset) & m_1(X_1) & \cdots & m_1(X_{2^n-1}) \\ \vdots & \vdots & \ddots & \vdots \\ m_k(\emptyset) & m_k(X_1) & \cdots & m_k(X_{2^n-1}) \end{pmatrix}, \tag{4.5}$$

where $X_i \in 2^\Theta$.

The matrix $\mathbf{M}$ characterizes all available information of the sources of evidence which has to be combined to solve the fusion problem. D–S combination rule can be applied to combine k rows of $\mathbf{M}$ to get fusion result. Simultaneously, it is easy to analyze the relation between different sources of evidence. When every element of the columns $\mathbf{M}$ in is equal to 0, the column can be deleted and then $\mathbf{M}$ can be simplified.

Definition 3. Core vector (evidence) $\mathbf{m}_c$:

There are multiple row vectors to express the different bodies of evidence in the matrix $\mathbf{M}$. The core vector, $\mathrm{m}_c, 1 \leq c \leq k$, is the row whose BPA function is the largest in the column. This column is the one whose summation of the BPA functions is the largest in the columns of matrix $\mathbf{M}$. In application of D–S evidence theory, the true result may be attained through combining multiple row vectors. The core vector is the closest to the real result of the vectors in the matrix $\mathbf{M}$.

The core vector can be attained through the following method.

(1) Multiple bodies of evidence can be expressed by the stochastic matrix $\mathbf{M}$. The column sum of matrix $\mathbf{M}$ can be computed and then the column whose column sum is maximum can be attained. Generally speaking, if there is more than one column whose sum is maximum, the column with the minimum number is attained. This column can be expressed by the following formula:

$$\alpha_{j_0} = \max_j \left(\sum_{i=1}^{k} m_{ij}(\cdot) \right). \tag{4.6}$$

(2) The maximum element of the maximum column sum can be computed. If there are more than one element, the element whose row number is minimum is attained. The row of this maximum element is core vector. The maximum can be noted as

$$m_{i_0 j_0} = \max_i \left(\frac{m_{ij_0}}{\max\limits_j (\sum_{i=1}^{k} m_{ij}(\cdot))} \right). \tag{4.7}$$

5. Quantification Classification Algorithm

5.1. *Jousselme distance of evidence*

D–S evidence theory has been adopted to quantify uncertainty in a number of domains: reliability engineering, design optimization, and so on. Chen *et al.* have classified various uncertainties into three types: fuzziness, discord, and nonspecificity.[41] Because D–S evidence theory is performed in the power set of frame of discernment, it can deal with two types of uncertainty: discord and nonspecificity. Jousselme *et al.*[39] proposed the novel principle distance in order to represent the nonspecificity discrepancy between two bodies of evidence. In other words, the novel distance can quantify the difference between sets.

Let $\mathbf{m}_1$ and $\mathbf{m}_2$ be two BPAs on the same frame of discernment Θ, containing N mutually exclusive and exhaustive hypotheses. The distance between $\mathbf{m}_1$ and $\mathbf{m}_2$ is:

$$d_{\mathrm{BPA}}(\mathbf{m}_1, \mathbf{m}_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T \underline{\underline{D}}(\mathbf{m}_1 - \mathbf{m}_2)}, \tag{5.1}$$

where $\underline{\underline{D}}$ is an $2^N \times 2^N$ positively defined matrix and adopted to describe the "similarity" between the subsets of Θ, whose elements are

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A,\ B \in 2^{\Theta}. \tag{5.2}$$

In Eq. (5.1), $\frac{1}{2}$ is needed to normalize d_{BPA}. It is very important that the distance satisfy a range limitation represented by the following formula.

$$0 \le d_{\mathrm{BPA}}(\mathbf{m}_1, \mathbf{m}_2) \le 1 \tag{5.3}$$

Moreover, Eq. (5.1) can be rewritten as

$$d_{\mathrm{BPA}}(\mathbf{m}_1, \mathbf{m}_2) = \sqrt{\frac{1}{2}(\|\mathbf{m}_1\|^2 + \|\mathbf{m}_2\|^2 - 2\langle \mathbf{m}_1, \mathbf{m}_2 \rangle)}, \tag{5.4}$$

where $\langle \mathbf{m}_1, \mathbf{m}_2 \rangle$ is the scalar product defined as

$$\langle \mathbf{m}_1, \mathbf{m}_2 \rangle = \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|}, \tag{5.5}$$

where $A_i, A_j \in 2^\Theta$ for $i, j = 1, 2, \ldots, 2^N$. Then $\|\mathbf{m}\|^2$ is the square norm of $\mathbf{m}$:$\|\mathbf{m}\|^2 = \langle \mathbf{m}, \mathbf{m} \rangle$.

5.2. *Classification of multiple bodies of evidence*

5.2.1. *Classification criterion*

The Jousselme distance of evidence describes and quantifies the difference of two bodies of evidence. This distance is defined with the power set, 2^Θ, and more flexible than the pseudo-distance defined by Fixsen and Mahler[37] which can only compute the distance of the singleton hypothesis of Θ between the two bodies of evidence. In other words, the distance proposed by Fixsen and Mahler does not describe the nonspecificity situation. It is deficient and localized to quantify the whole difference of basic belief assignment functions in the power set that the cardinality function is not equal to 1 between the two bodies of evidence. Meanwhile, the Jousselme distance satisfies all the metric requirements. The difference of two bodies of evidence can be efficiently described and quantified using this distance which has the range stated above. This range makes it possible that the differences among all the bodies of evidence in the Θ can be attained using the bounded values. Consequently, the Jousselme distance is selected in this paper. A threshold, δ, should be given using this distance to quantify the difference of multiple bodies of evidence. This can be represented as follows:

$$d_{\mathrm{BPA}}(\mathbf{m}_i, \mathbf{m}_j) \geq \delta, \tag{5.6}$$

where δ is a threshold of the positive real number, $0 < \delta < 1$. When Eq. (5.6) is satisfied, information of the two bodies of evidence $\mathbf{m}_i$, $\mathbf{m}_j$, is regarded as complementary on information and the two bodies of evidence are conflicting evidence. These two bodies of evidence cannot be combined by classical D–S evidence theory combination rule according to Sec. 3. In other words, the combined result may be counter-intuitive using classical D–S evidence theory combination rule. When Eq. (5.6) is not satisfied, information of the two bodies of evidence is redundant and classical D–S evidence theory combination rule can be used to combine these bodies of evidence. Consequently, d_{BPA} can be considered as the classification criterion for quantification measurement. Multiple bodies of evidence can be divided into two types: complementary information and redundant information using this measurement criterion. Moreover, there are some subjective factors in choosing the threshold, δ. The authors have observed that a value of the order of $\delta = 0.5$ gives completely satisfactory number results.

5.2.2. *Classification processing*

Multiple bodies of evidence can be expressed in the matrix $\mathbf{M}$. When evidence can be combined using the classical D–S evidence theory combination rule, multiple bodies of evidence can be classified in order to avoid the combined counter-intuitive results. The classification of evidence is, thus, transformed to the classification of vectors in the matrix $\mathbf{M}$. The following procedure is proposed for classification:

(1) The core vector, $\mathbf{m}_c^{(0)}$, is computed according to Sec. 4.
(2) The distance between $\mathbf{m}_c^{(0)}$ and other row vector is computed, respectively, and represented in core distance vector

$$\mathbf{d}_c^{(0)} = (d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_1), \ldots, d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_c), \ldots, d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_k)), \tag{5.7}$$

where $\mathbf{m}_c^{(0)}$ may be $\mathbf{m}_1$ or $\mathbf{m}_k$, $d_{\mathrm{BPA}}(\mathbf{m}_c^{(0)}, \mathbf{m}_c) = 0$.
(3) All of the row vectors are classified into two types: complementary and redundant information according to Sec. 5.2.1 in core distance vector. The two types are represented by set $C^{(0)}$ and $R^{(0)}$,

$$C^{(0)} = (\underbrace{\ldots, \mathbf{m}_l, \ldots}_{d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_i) \geq \delta}), \tag{5.8}$$

$$R^{(0)} = (\underbrace{\ldots, \mathbf{m}_j, \ldots}_{d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_j) < \delta}), \tag{5.9}$$

where $0 \leq l, j \leq k$. Here, n_c and n_r are adopted to express the number of the element of set $C^{(0)}$ and $R^{(0)}$, respectively, and $n_c + n_r = k$. According to the number of the element of set $C^{(0)}$, the following different conditions are analyzed:

(a) If $n_c = 0$, the information of all vectors in the matrix $\mathbf{M}$ is redundant, the classical D–S evidence theory combination rule is applied. The results of combination are rational and robust.
(b) If $n_c \neq 0$, the vectors in $C^{(0)}$ are regarded as the new core vector, respectively. The new classification results are obtained and represented as $\mathbf{m}_c^{(i)}$, $\mathbf{d}_c^{(i)}$, $C^{(i)}$ and $R^{(i)}$, where $0 \leq i \leq k$. When i is equal to n_c, the computational process is accomplished. The number of classification group, n_g, is equal to $n_c + 1$.

The flowchart of the proposed algorithm is shown in Fig. 2.

6. Numerical Examples

In this section, two numerical examples are presented to illustrate the new approach. The first one is adopted from Ref. 41 to address the procedure evidence classification and the second one applies the complicated case to express the efficiency and practical significance of the proposed method.

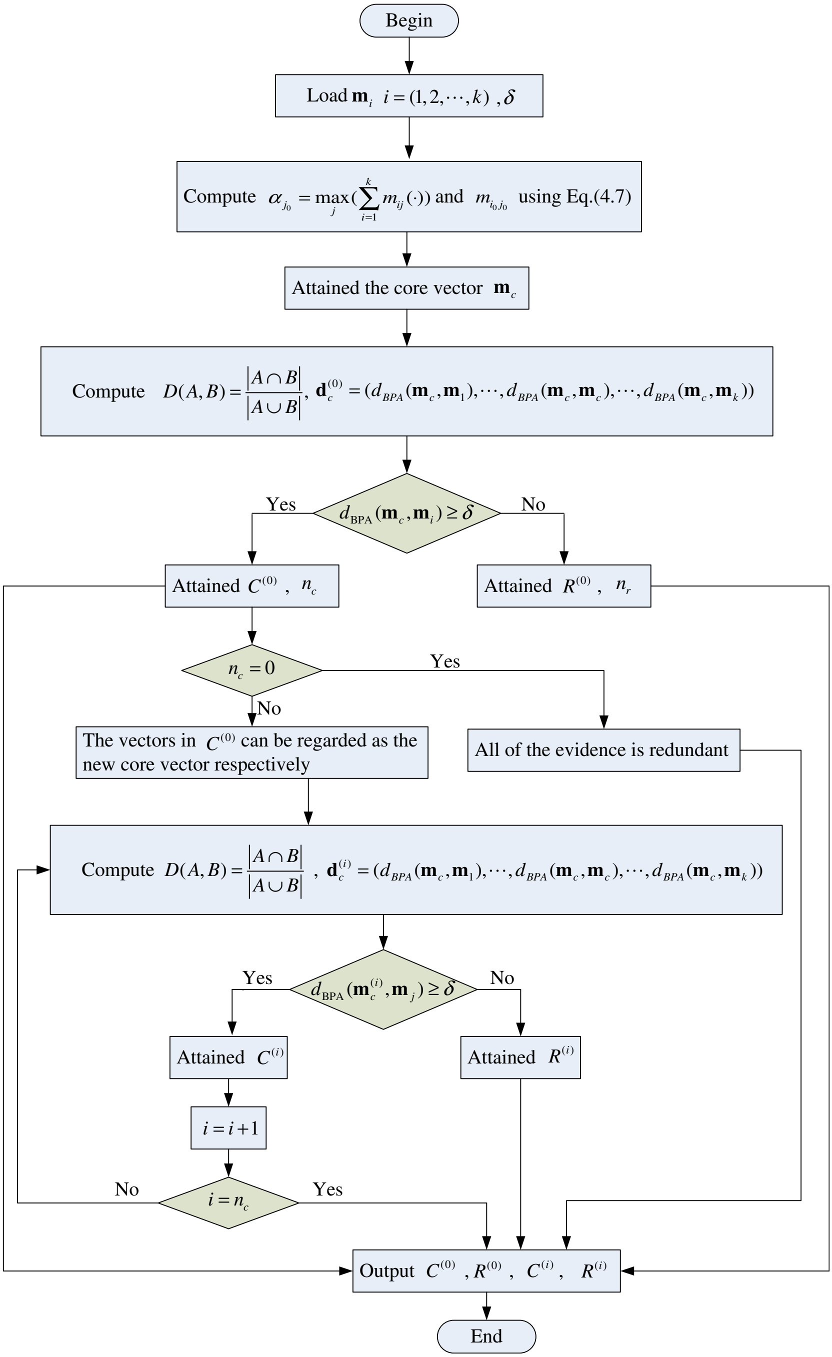

Fig. 2. Flowchart of the quantification classification procedure.

6.1. *Example study-I*

Example 1. There are five bodies of evidence $m_i, i = 1, \ldots 5$, which have been collected from a multisensor automatic target recognition system. The BPA functions of these bodies of evidence which have three targets have been expressed in the matrix $\mathbf{M}$.

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \\ \mathbf{m}_4 \\ \mathbf{m}_5 \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0.50 & 0.20 & 0.30 \\ 0.00 & 0.90 & 0.10 \\ 0.55 & 0.10 & 0.35 \\ 0.55 & 0.10 & 0.35 \\ 0.55 & 0.10 & 0.35 \end{pmatrix} \end{array}$$

First, the core vector should be attained. According to Sec. 4, $\alpha_{j_0} = \alpha_1 = 2.15$, $m_{i_0 j_0} = m_{31} = 0.55$, the core vector is $\mathbf{m}_3$. Second, the core distance vector $\mathbf{d}_c$ is computed as follows:

$$\begin{aligned} \mathbf{d}_c^{(0)} &= (d_{\mathrm{BPA}}(\mathbf{m}_3, \mathbf{m}_1), d_{\mathrm{BPA}}(\mathbf{m}_3, \mathbf{m}_2), d_{\mathrm{BPA}}(\mathbf{m}_3, \mathbf{m}_3), \\ &\quad\; d_{\mathrm{BPA}}(\mathbf{m}_3, \mathbf{m}_4), d_{\mathrm{BPA}}(\mathbf{m}_3, \mathbf{m}_5)) \\ &= (0.079,\ 0.550,\ 0.000,\ 0.000,\ 0.000) \end{aligned}$$

Finally, all the vectors are classified into complementary information and redundant information according to Sec. 5.2.1. Herein, $\delta = 0.5$. Set $C^{(0)}$ and $R^{(0)}$ can be represented as follows:

$$\begin{aligned} C^{(0)} &= (\underbrace{\ldots, \mathbf{m}_i, \ldots}_{d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_i) \geq \delta}) = (m_2), \\ R^{(0)} &= (\underbrace{\ldots, \mathbf{m}_c, \ldots}_{d_{\mathrm{BPA}}(\mathbf{m}_c, \mathbf{m}_j) < \delta}) = (\mathbf{m}_1, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5). \end{aligned}$$

Because $n_c = 1 \neq 0$, $\mathbf{m}_2$ is regarded as new core vector, according to Sec. 5.2.2(b). The core distance vector $\mathbf{d}_c^{(1)}$ can now be computed as follows:

$$\begin{aligned} \mathbf{d}_c^{(1)} &= (d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_1), d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_2), d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_4), d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_5)) \\ &= (0.557,\ 0.000,\ 0.550,\ 0.550) \end{aligned}$$

Set $C^{(1)}$ is attained as follows:

$$C^{(1)} = (\mathbf{m}_1, \mathbf{m}_4, \mathbf{m}_5).$$

From the above, $\mathbf{m}_2$ is complementary to the information of the other bodies of evidence. Consequently, $\mathbf{m}_2$ cannot be combined with other vectors in the matrix $\mathbf{M}$ using the classical D–S evidence theory combinational rule. Other vectors besides $\mathbf{m}_2$ can be combined with each other using the classical D–S evidence theory combinational rule. The result is identical with Ref. 41.

6.2. *Example study-II*

Ten bodies of evidence on Θ with $|\cdot| = 3$ are represented in the following matrix $\mathbf{M}$, using 1, 2, etc. to denote element 1, element 2 etc. in Θ as follows.

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \\ \mathbf{m}_4 \\ \mathbf{m}_5 \\ \mathbf{m}_6 \\ \mathbf{m}_7 \\ \mathbf{m}_8 \\ \mathbf{m}_9 \\ \mathbf{m}_{10} \end{pmatrix} = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 1,2 & 1,3 & 2,3 & \Theta \end{array} \\ \begin{pmatrix} 0.2129 & 0.1833 & 0.1842 & 0.1011 & 0.1568 & 0.0717 & 0.0900 \\ 0.0000 & 0.6807 & 0.0203 & 0.0000 & 0.0000 & 0.2990 & 0.0000 \\ 0.0696 & 0.2136 & 0.2556 & 0.1153 & 0.0723 & 0.0871 & 0.1865 \\ 0.7278 & 0.0000 & 0.1366 & 0.0000 & 0.1356 & 0.0000 & 0.0000 \\ 0.0129 & 0.2542 & 0.1488 & 0.1655 & 0.1440 & 0.1607 & 0.1139 \\ 0.2301 & 0.3685 & 0.1175 & 0.0451 & 0.0233 & 0.0644 & 0.1512 \\ 0.1693 & 0.1452 & 0.2069 & 0.2171 & 0.0212 & 0.2002 & 0.0400 \\ 0.0000 & 0.0581 & 0.6892 & 0.1428 & 0.0000 & 0.0191 & 0.0908 \\ 0.1522 & 0.1769 & 0.1376 & 0.1603 & 0.0826 & 0.1206 & 0.1700 \\ 0.1406 & 0.2671 & 0.0169 & 0.2653 & 0.1541 & 0.0091 & 0.1469 \end{pmatrix} \end{array}$$

(1) The core vector: according to Sec. 4, $\alpha_{j_0} = \alpha_2 = 2.3475$, $m_{i_0 j_0} = m_{22} = 0.6807$, the core vector is $\mathbf{m}_2$.

(2) The core distance vector $\mathbf{d}_c^{(0)}$ is computed as follows, based on Eq. (5.7). Herein, $\underline{\underline{D}}$ of the Jousselme distance should be first computed as:

$$\underline{\underline{D}} = \begin{pmatrix} 1.0000 & 0.0000 & 0.0000 & 0.5000 & 0.5000 & 0.0000 & 0.3333 \\ 0.0000 & 1.0000 & 0.0000 & 0.5000 & 0.0000 & 0.5000 & 0.3333 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.5000 & 0.5000 & 0.3333 \\ 0.5000 & 0.5000 & 0.0000 & 1.0000 & 0.3333 & 0.3333 & 0.6667 \\ 0.5000 & 0.0000 & 0.5000 & 0.3333 & 1.0000 & 0.3333 & 0.6667 \\ 0.0000 & 0.5000 & 0.5000 & 0.3333 & 0.3333 & 1.0000 & 0.6667 \\ 0.3333 & 0.3333 & 0.3333 & 0.6667 & 0.6667 & 0.6667 & 1.0000 \end{pmatrix}$$

$$\begin{aligned} \mathbf{d}_c^{(0)} &= (d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_1), d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_2), \ldots, d_{\mathrm{BPA}}(\mathbf{m}_2, \mathbf{m}_{10})) \\ &= (0.4941,\ 0.0000,\ 0.4212,\ 0.8278,\ 0.3631,\ 0.3555, \\ &\quad\ 0.4253,\ 0.6324,\ 0.4398,\ 0.4498) \end{aligned}$$

(3) In accordance with Sec. 5.2.1, the 10 bodies of evidence are classified into two types. Set $C^{(0)}$ and $R^{(0)}$ are represented as follows:

$$C^{(0)} = (\mathbf{m}_4, \mathbf{m}_8) \quad R^{(0)} = (\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_9, \mathbf{m}_{10}).$$

(4) Because $n_r \neq 0$, $\mathbf{m}_4$ and $\mathbf{m}_8$ should be regarded as the core vector according to Sec. 5.2.2(b). When $\mathbf{m}_4$ is the core vector, $d_c^{(1)}$ can be computed as follows:

$$\begin{aligned} \mathbf{d}_c^{(1)} &= (d_{\mathrm{BPA}}(\mathbf{m}_4, \mathbf{m}_1), d_{\mathrm{BPA}}(\mathbf{m}_4, \mathbf{m}_3), \ldots, d_{\mathrm{BPA}}(\mathbf{m}_4, \mathbf{m}_{10})) \\ &= (0.3947, 0.5276, 0.0000, 0.5653, 0.4891, 0.4761, 0.6360, 0.4648, 0.4720). \end{aligned}$$

Set $C^{(1)}$ and $R^{(1)}$ can be obtained as follows:

$$C^{(1)} = (\mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_5, \mathbf{m}_8) \quad R^{(1)} = (\mathbf{m}_1, \mathbf{m}_4, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_9, \mathbf{m}_{10}).$$

(5) When $\mathbf{m}_8$ is regarded as the core vector, the core vector, $d_c^{(2)}$, set $C^{(2)}$, and $R^{(2)}$ can be computed, respectively, as follows:

$$\begin{aligned} \mathbf{d}_c^{(2)} &= (d_{\text{BPA}}(\mathbf{m}_8, \mathbf{m}_1), d_{\text{BPA}}(\mathbf{m}_8, \mathbf{m}_3), d_{\text{BPA}}(\mathbf{m}_8, \mathbf{m}_5), \ldots, d_{\text{BPA}}(\mathbf{m}_8, \mathbf{m}_{10})) \\ &= (0.3639,\ 0.3059,\ 0.3707,\ 0.4539,\ 0.3554,\ 0.0000,\ 0.3920,\ 0.5204), \\ C^{(2)} &= (\mathbf{m}_2, \mathbf{m}_4, \mathbf{m}_{10}) \quad R^{(2)} = (\mathbf{m}_1, \mathbf{m}_3, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_8, \mathbf{m}_9). \end{aligned}$$

Finally, the three groups of evidence which are combined with the classical D–S evidence theory combination rule can be attained, as represented in Table 1. The groups of evidence which are not combined using combination rule are also included in Table 1. Because $i = n_c = 2$, the complete classification processing is accomplished and the three groups of evidence are attained.

(6) Classification consequence verification:
Multiple bodies of evidence have been classified and expressed in Fig. 1. To verify the accuracy of the classification, these classified bodies of evidence can be combined using the classical D–S evidence theory combination rule. These bodies of evidence which are redundant in information can be combined and the combination result is rational and robust. Simultaneously, the counter-intuitive results can also be obtained when these complementary bodies of evidence are combined. The combination results are analyzed as follows.

For Group 1, multiple bodies of evidence in set $R^{(0)}$ can be combined one by one and the combination consequence is represented in Table 2; $\mathbf{m}_j^i$ is denoted as the

Table 1. The classification consequence of 10 bodies of evidence.

Group	D–S combinational rule	Not D–S combinational rule
1	$\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_9, \mathbf{m}_{10}$	$\mathbf{m}_2, \mathbf{m}_4, \mathbf{m}_8$
2	$\mathbf{m}_1, \mathbf{m}_4, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_9, \mathbf{m}_{10}$	$\mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_8$
3	$\mathbf{m}_1, \mathbf{m}_3, \mathbf{m}_5, \mathbf{m}_6, \mathbf{m}_7, \mathbf{m}_8, \mathbf{m}_9$	$\mathbf{m}_2, \mathbf{m}_4, \mathbf{m}_8, \mathbf{m}_{10}$

Table 2. The combination results of multiple bodies of evidence in set $R^{(0)}$ of Group 1.

		$m(1)$	$m(2)$	$m(3)$	$m(12)$	$m(13)$	$m(23)$	$m(\Theta)$	K_{ij}
$\mathbf{m}_2^1$	$\mathbf{m}_1 \oplus \mathbf{m}_2$	0.0000	0.7077	0.2042	0.0000	0.0000	0.0880	0.0000	0.4508
$\mathbf{m}_3^1$	$\mathbf{m}_2^1 \oplus \mathbf{m}_3$	0.0000	0.7215	0.2404	0.0000	0.0000	0.0382	0.0000	0.3688
$\mathbf{m}_5^1$	$\mathbf{m}_3^1 \oplus \mathbf{m}_5$	0.0000	0.7659	0.2186	0.0000	0.0000	0.0155	0.0000	0.3250
$\mathbf{m}_6^1$	$\mathbf{m}_5^1 \oplus \mathbf{m}_6$	0.0000	0.8542	0.1401	0.0000	0.0000	0.0059	0.0000	0.4284
$\mathbf{m}_7^1$	$\mathbf{m}_6^1 \oplus \mathbf{m}_7$	0.0000	0.8833	0.1145	0.0000	0.0000	0.0024	0.0000	0.4150
$\mathbf{m}_9^1$	$\mathbf{m}_7^1 \oplus \mathbf{m}_9$	0.0000	0.9035	0.0960	0.0000	0.0000	0.0011	0.0000	0.3853
$\mathbf{m}_{10}^1$	$\mathbf{m}_9^1 \oplus \mathbf{m}_{10}$	0.0000	0.9524	0.0483	0.0000	0.0000	0.0003	0.0000	0.3463

Table 3. The combination results of multiple bodies of evidence in set $C^{(0)}$ of Group 1.

		$m(1)$	$m(2)$	$m(3)$	$m(12)$	$m(13)$	$m(23)$	$m(\Theta)$	K_{ij}
$\mathbf{m}_4^2$	$\mathbf{m}_2 \oplus \mathbf{m}_4$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.9131
$\mathbf{m}_8^2$	$\mathbf{m}_4^2 \oplus \mathbf{m}_8$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.2009

combined vector of $\mathbf{m}_i$ and $\mathbf{m}_j$. From this table, the combined results do not generate the counter-intuitive results of applying D–S evidence theory combination rule between the multiple bodies of evidence in set $C^{(0)}$ each other. Consequently, this can demonstrate that the classification algorithm is rational and valid.

Second, to set $C^{(0)}$ in Group 1, the combination can be expressed as provided in Table 3; $\mathbf{m}_2$ strongly supports hypothesis 2 in the matrix $\mathbf{M}$, while the vector $\mathbf{m}_4$ almost completely supports hypothesis 1 in the matrix $\mathbf{M}$. Moreover, $d_{\text{BPA}}(m_2, m_4) = 0.8278$ and $k_{24} = 0.9131$. Consequently, these two bodies of evidence strongly contradict each other. Generally speaking, when this happens, it is believed that the two bodies of evidence are in conflict. In practical application, two bodies of evidence are regarded as complementary if enough information is not available to determine which one is more important and sufficient. Moreover, from the combination consequence of $\mathbf{m}_2$ and $\mathbf{m}_4$ in Table 3, $\mathbf{m}_4^2$ is absolutely sure that hypothesis 3 is the right hypothesis. This consequence is extremely counter-intuitive according to the two bodies of evidence, $\mathbf{m}_2$ and $\mathbf{m}_4$. Meanwhile, $\mathbf{m}_8$ largely supports hypothesis 3. This evidence is complementary to the two bodies of evidence, $\mathbf{m}_2$ and $\mathbf{m}_4$. These three bodies of evidence are strongly in conflict with each other. However, the combination result is absolutely sure about hypothesis 3 from Table 3. Consequently, this result is extremely irrational. The above analysis shows that $\mathbf{m}_2$, $\mathbf{m}_4$, and $\mathbf{m}_8$ are complementary to each other and cannot be combined using the classical D–S evidence theory combination rule. To avoid the counter-intuitive results, these bodies of evidence should be classified before using the traditional combination rule.

Based on the above analysis of the results of quantification classification, the proposed algorithm can completely produce valid classification results. The results of

Table 4. The combination results of multiple bodies of evidence in Group 2.

		$m(1)$	$m(2)$	$m(3)$	$m(12)$	$m(13)$	$m(23)$	$m(\Theta)$	K_{ij}
$\mathbf{m}_4^1$	$\mathbf{m}_1 \oplus \mathbf{m}_4$	0.7671	0.0000	0.1759	0.0000	0.0570	0.0000	0.0000	0.4124
$\mathbf{m}_6^1$	$\mathbf{m}_4^1 \oplus \mathbf{m}_6$	0.8131	0.0000	0.1647	0.0000	0.0224	0.0000	0.0000	0.5564
$\mathbf{m}_7^1$	$\mathbf{m}_6^1 \oplus \mathbf{m}_7$	0.8098	0.0000	0.1875	0.0000	0.0030	0.0000	0.0000	0.5399
$\mathbf{m}_9^1$	$\mathbf{m}_7^1 \oplus \mathbf{m}_9$	0.8257	0.0000	0.1738	0.0000	0.0014	0.0000	0.0000	0.4446
$\mathbf{m}_{10}^1$	$\mathbf{m}_9^1 \oplus \mathbf{m}_{10}$	0.9119	0.0000	0.0888	0.0000	0.0006	0.0000	0.0000	0.3594
$\mathbf{m}_3^2$	$\mathbf{m}_2 \oplus \mathbf{m}_3$	0.0000	0.7258	0.1574	0.0000	0.0000	0.1168	0.0000	0.2995
$\mathbf{m}_4^2$	$\mathbf{m}_3^2 \oplus \mathbf{m}_4$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.9254
$\mathbf{m}_5^2$	$\mathbf{m}_4^2 \oplus \mathbf{m}_5$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.4159
$\mathbf{m}_8^2$	$\mathbf{m}_5^2 \oplus \mathbf{m}_8$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.2009

Table 5. The combination results of multiple bodies of evidence in Group 3.

		$m(1)$	$m(2)$	$m(3)$	$m(12)$	$m(13)$	$m(23)$	$m(\Theta)$	K_{ij}
$\mathbf{m}_3^1$	$\mathbf{m}_1 \oplus \mathbf{m}_3$	0.2147	0.2737	0.3145	0.0609	0.0702	0.0409	0.0250	0.3290
$\mathbf{m}_5^1$	$\mathbf{m}_3^1 \oplus \mathbf{m}_5$	0.1837	0.3780	0.3417	0.0335	0.0344	0.0241	0.0045	0.3682
$\mathbf{m}_6^1$	$\mathbf{m}_5^1 \oplus \mathbf{m}_6$	0.1967	0.5110	0.2555	0.0131	0.0118	0.0106	0.0013	0.4834
$\mathbf{m}_7^1$	$\mathbf{m}_6^1 \oplus \mathbf{m}_7$	0.1745	0.5793	0.2328	0.0067	0.0014	0.0051	0.0001	0.4538
$\mathbf{m}_8^1$	$\mathbf{m}_7^1 \oplus \mathbf{m}_8$	0.0986	0.4370	0.4587	0.0038	0.0003	0.0014	0.0000	0.5845
$\mathbf{m}_9^1$	$\mathbf{m}_8^1 \oplus \mathbf{m}_9$	0.0996	0.4849	0.4124	0.0022	0.0001	0.0007	0.0000	0.4309
$\mathbf{m}_4^2$	$\mathbf{m}_2 \oplus \mathbf{m}_4$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.9131
$\mathbf{m}_8^2$	$\mathbf{m}_4^2 \oplus \mathbf{m}_8$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.2009
$\mathbf{m}_{10}^2$	$\mathbf{m}_8^2 \oplus \mathbf{m}_{10}$	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.6730

multiple bodies of evidence in set $R^{(0)}$ are rational and reasonable and those bodies of evidence in set $C^{(0)}$ are counter-intuitive. This demonstrates that the proposed algorithm is efficient and robust in the quantification classification of multiple bodies of evidence.

For Groups 2 and 3, it is the same as Group 1 in that the quantification classification is efficient and its results are robust. For the completeness of the example, the classification combination results of Groups 2 and 3 are presented in Tables 4 and 5, respectively.

7. Conclusion

In this paper, the Jousselme distance has been employed to quantify the degree of conflict between two bodies of evidence, so as to resolve the deficiency of the conflict factor in classical D–S evidence theory. A stochastic interpretation of BPA function is described and analyzed. Based on this, multiple sources of evidence are represented as a stochastic matrix. This is convenient for computing and dealing with multiple sources of evidence as well. Classification is performed before multiple sources of evidence are combined in order to avoid the generation of the counter-intuitive results. Consequently, the core vector of the stochastic matrix is defined. A novel quantification classification algorithm is proposed based on the Jousselme distance and the core vector. Multiple sources of evidence are classified in two types. Finally, two numerical examples are used to illustrate the detailed classification procedure and to demonstrate the accuracy and efficiency of the algorithm.

Acknowledgments

This research was partially supported by the National Natural Science Foundation of China under the contract number 51075061, and the Specialized Research Fund for the Doctoral Program of Higher Education of China under the contract number 20120185110032.

References

1. K. Sentz and S. Ferson, *Combination of Evidence in Dempster-Shafer Theory* (Sandia National Laboratories, New Mexico, 2002).
2. C. R. Parikh, M. J. Pont and N. B. Jones, Application of Dempster-Shafer theory in condition monitoring applications: A case study, *Pattern Recognition Letters* **22**(6) (2001) 777–785.
3. D. Dubois, H. Prade and P. Smets, A definition of subjective possibility, *International Journal of Approximate Reasoning* **48**(2) (2008) 352–364.
4. D. Dubois, H. M. Prade, H. Farreny *et al.*, *Possibility Theory: An Approach to Computerized Processing of Uncertainty* (Plenum Press, New York, 1988).
5. G. Shafer, *A Mathematical Theory of Evidence* (Princeton University Press, New Jersey, 1976).
6. B. N. Nelson, P. D. Gader and J. M. Keller, Fuzzy set information fusion in land mine detection, in *SPIE Proc.*, Vol. 3710 (1999), pp. 1168–1178.
7. M. Oussalah, H. Maaref and C. Barret, New fusion methodology approach and application to mobile robotics: Investigation in the framework of possibility theory, *Information Fusion* **2**(1) (2001) 31–48.
8. C. Sossai, P. Bison, G. Chemello *et al.*, Sensor fusion for localization using possibility theory, *Control Engineering Practice* **7**(6) (1999) 773–782.
9. J. F. Crouzet and O. Strauss, Interval-valued probability density estimation based on quasi-continuous histograms: Proof of the conjecture, *Fuzzy Sets and Systems* **183**(1) (2011) 92–100.
10. P. Guo and H. Tanaka, Decision making with interval probabilities, *European Journal of Operational Research* **203**(2) (2010) 444–454.
11. P. Guo and Y. Wang, Eliciting dual interval probabilities from interval comparison matrices, *Information Sciences* **190**(1) (2012) 17–26.
12. A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, *Annals of Mathematical Statistics* **38**(2) (1967) 325–339.
13. X. Fan and M. J. Zuo, Fault diagnosis of machines based on D-S evidence theory, Part 1: D-S evidence theory and its improvement, *Pattern Recognition Letters* **27**(5) (2006) 366–376.
14. M. J. Zuo and X. F. Fan, Fault diagnosis of machines based on D-S evidence theory, Part 2: Application of the improved D-S evidence theory in gearbox fault diagnosis, *Pattern Recognition Letters* **27**(5) (2006) 377–385.
15. L. Oukhellou, A. Debiolles, T. Denoeux *et al.*, Fault diagnosis in railway track circuits using Dempster–Shafer classifier fusion, *Engineering Applications of Artificial Intelligence* **23** (2010) 117–128.
16. H. Luo, S. Yang, X. Hu *et al.*, Agent oriented intelligent fault diagnosis system using evidence theory, *Expert Systems with Applications* **39**(3) (2012) 2524–2531.
17. M. L. Koc, Risk assessment of a vertical breakwater using possibility and evidence theories, *Ocean Engineering* **36**(14) (2009) 1060–1066.
18. H. R. Bae, R. V. Grandhi and R. A. Canfield, An approximation approach for uncertainty quantification using evidence theory, *Reliability Engineering and System Safety* **86**(3) (2004) 215–225.
19. H. R. Bae, R. V. Grandhi and R. A. Canfield, Epistemic uncertainty quantification techniques including evidence theory for large-scale structures, *Computers and Structures* **82**(13) (2004) 1101–1112.
20. J. P. Yang, H. Z. Huang, L. P. He *et al.*, Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using Dempster-Shafer evidence theory under uncertainty, *Engineering Failure Analysis* **18**(8) (2011) 2084–2092.

21. J. P. Yang, H. Z. Huang, R. Sun *et al.*, Reliability analysis of aircraft servo-actuation systems using evidential networks, *International Journal of Turbo and Jet Engines* **19**(2) (2012) 59–68.
22. J. P. Yang, H. Z. Huang, Y. Liu *et al.*, Evidential networks for fault tree analysis with imprecise knowledge, *International Journal of Turbo and Jet Engines* **19**(2), (2012) 111–122.
23. I. Bloch, Defining belief functions using mathematical morphology application to image fusion under imprecision, *International Journal of Approximate Reasoning* **48**(2) (2008) 437–465.
24. F. Tupin, I. Bloch and H. Maitre, A first step towards automatic interpretation of SAR images using evidential fusion of several structure detectors, *IEEE Geoscience and Remote Sensing* **37**(3) (1999) 1327–1343.
25. D. Cai, M. F. Mctear and S. I. Mcclean, Knowledge discovery in distributed databases using evidence theory, *International Journal of Intelligent Systems* **15**(8) (2000) 745–761.
26. L. V. Utkin, Risk analysis under partial prior information and nonmonotone utility functions, *International Journal of Information Technology and Decision Making* **6**(4) (2007) 625–647.
27. M. Khodabandeh and A. Mohammad-Shahri, Two generalizations of aggregated uncertainty measure for evaluation of Dezert-Smarandache theory, *International Journal of Information Technology and Decision Making* **11**(1) (2012) 119–142.
28. L. A. Zadeh, A simple view of the Dempster-Shafer framework and new combination rules. *AI Magazine* **7** (1986) 85–90.
29. R. R. Yager, On the Dempster-Shafer framework and new combination rules, *Information Sciences* **41** (1987) 93–138.
30. T. Inagaki. Interdependence between safety-control policy and multiple-sensor schemes via Dempster-Shafer theory, *IEEE Transactions on Reliability* **40**(2) (1991) 182–188.
31. D. Dubois and H. Prade, Representation and combination of uncertainty with belief functions and possibility measures, *Computational Intelligence* **4**(3) (1988) 244–264.
32. P. Smets, Data fusion in the transferable belief model, in *Proc. 3rd Int. Conf. Information Fusion*, Vol. I, Paris, France (2000), pp. PS21–PS33.
33. A. Kallel and S. L. Hégarat-Mascle, Combination of partially non-distinct beliefs: The cautious-adaptive rule, *International Journal of Approximate Reasoning* **50**(7) (2009) 1000–1021.
34. A. Josang, J. Diaz and M. Rifqi, Cumulative and averaging fusion of beliefs, *Information Fusion* **11**(2) (2010) 192–200.
35. J. P. Yang, H. Z. Huang, Q. Miao *et al.*, A novel information fusion method based on Dempster-Shafer evidence theory for conflict resolution, *Intelligent Data Analysis* **15**(3) (2011) 399–411.
36. W. Liu, Analyzing the degree of conflict among belief functions, *Artificial Intelligence* **170**(11) (2006) 909–924.
37. D. Fixsen and R. P. S. Mahler, A Dempster-Shafer approach to Bayesian classification, in *Proc. 5th National Symposium on Sensor Fusion*, Vol. I, Orlando, USA (1992), pp. 213–231.
38. D. Fixsen and R. P. S. Mahler, The modified Dempster-Shafer approach to classification, *IEEE Transactions on Systems, Man, and Cybernetics, Part A* **27**(1) (1997) 96–104.
39. A. L. Jousselme, D. Grenier and E. Bosse, A new distance between two bodies of evidence, *Information Fusion* **2**(2) (2001) 91–101.
40. A. Ayoun and P. Smets, Data association in multi-target detection using the transferable belief model, *International Journal of Intelligent Systems* **16**(10) (2001) 1167–1182.
41. L. Z. Chen, W. K. Shi and Y. Deng *et al.*, A new fusion approach based on distance of evidence, *Journal of Zhejiang University Science A* **6**(5) (2005) 476–482.