Structural Safety 50 (2014) 57–65

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

A novel reliability method for structural systems with truncated random variables

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ARTICLE INFO

Article history:
Received 18 November 2013
Received in revised form 20 February 2014
Accepted 27 March 2014

Keywords:
Truncated random variables
Design variable
BP networks
Reliability analysis
Structural systems

ABSTRACT

Uncertainty is usually modeled using random variable with certain probability distribution. However, the probability distributions of many random variables are often truncated in engineering applications. In the procedure of reliability based design optimization for structural systems with truncated random variables, repeated function evaluations are required for different design points where the computational costs are extremely huge. In this paper, an efficient as well as novel reliability method is proposed for structural systems with truncated random variables which does not require repeated function evaluations for the different design points. Uniformly distributed samples are generated for truncated random variables in the supported intervals and design variables in the specified intervals to approximate cover the entire uncertain space fully. In order to avoid repeated function evaluations and improve computational efficiency, a surrogate model is established using back-propagation (BP) neural networks which can approximate the relationships between the inputs and system responses properly in almost entire uncertain space using the proposed given available data. The main advantages of the proposed method are high accuracy and effectiveness in estimating the probability of failure under different design points which requires neither large samples nor the repeated function evaluations when compared to the existing reliability methods. Four numerical examples are investigated to demonstrate the effectiveness and accuracy of the proposed method.

1. Introduction

Uncertainty widely exists in engineering practices which can be divided into aleatory and epistemic uncertainties [1–4]. Various uncertainties are usually modeled using random variables. It is well known that the supported intervals of many continuous random variables are $[-\infty, \infty]$ while is impossible in engineering practices. In order to handle the problem, the probability distributions of some random variables are usually truncated. Therefore, truncated variables are involved in many engineering applications. For example, the volatility of material properties and physical dimensions is modeled using truncated random variables in reliability engineering. The truncated exponential distribution is usually employed to model the earthquake magnitude [5,6].

In the reliability-based design optimization (RBDO), repeated reliability estimation is required for each configuration of the design variable. Therefore, reliability analysis is the key step in RBDO. Reported existing reliability analysis methods, such as the first/s order reliability method (FORM/SORM) and Monte Carlo simulation (MCS), can be employed to calculate the probability of failure for structural systems with truncated random variables. However, some studies pointed out that the convergence problem may arise when the standard algorithm of the FORM is employed for structural systems with truncated random variables [7]. To overcome the problem, a modification of the standard FORM algorithm have been proposed by Melchers et al. [7], while the most probable point (MPP) search is also required. Du and Hu [5] linearized limit-state function at the MPP and a reliability method for system with truncated random variable based on the first order saddle-point approximation is presented. They have proved that the accuracy of their proposed method is higher than the FORM while keeps the same efficiency. Despite these efforts, it is well known that the MPP search is an iterative optimization process, which is not only time-consuming for structural systems with implicit performance function, but also needs the repeated function evaluations. Sometimes it may fail when the MPP search process does not converge [8]. MCS can be used for structural systems with truncated random variables; however, the computational costs of the MCS are extremely huge because it requires large sample sizes

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http://dx.doi.org/10.1016/j.strusafe.2014.03.006
0167-4730/

and many repeated function evaluations [8,9]. The computational burden using MCS is extremely huge when the performance function cannot be defined explicitly.

Despite some efforts have been made, RBDO for structural systems with truncated random variables is a challenging problem. Many classical reliability methods (such as the FORM/SORM) can be employed to calculate the probability of failure for structural systems with one failure mode. However, engineering system often has multiple failure modes, and these failure modes are usually correlated each other because they depend on the same uncertain variables. Up to now, the research of system reliability analysis almost had been stagnant when compared to the significant advances of component reliability due to the complicated features and intersections for the multiple failure modes, as well as the many existing methods cannot estimate system probability of failure with high efficiency and accuracy [10,11]. Due to the difficulties, the bounds of system probability of failure are provided by many reported reliability methods, instead of its unique value [12–14]. In RBDO for structural systems with truncated random variables, sometimes it is difficult to use the classical reliability analysis methods (such as the FORM/SORM) due to the MPP search algorithm may completely breakdown [5,7], and the results usually are bounded rather than unique value. Furthermore, repeated function evaluations under different design points are required for many existing methods. Therefore, the computational burden by using these methods is very huge, especially when the finite element analysis (FEA) method is used. In order to avoid the MPP search and repeated function evaluations as well as the difficulties of modeling the multiple failure modes, an efficient reliability method is proposed for structural systems with truncated random variables in this paper. The proposed method is robustness which is suitable for structural systems with explicit or implicit functions and even multiple failure modes. The main advantages of the proposed method are high accuracy and effectiveness in estimating the probability of failure for structural systems under different design points because it does not require the MPP search and large samples, as well as repeated function evaluations.

This paper is organized as follows. Section 2 provides an efficient reliability method for structural systems with truncated distributions in details. Three engineering examples and one mathematical problem are investigated in Section 3 to demonstrate the accuracy and efficiency of the proposed method. A brief discussion and conclusions are provided in Section 4 of the paper.

2. The efficient proposed method for calculating the probability of failure under different design points

The problem of MPP search may breakdown, and the difficulties of modeling multiple failure modes, as well as repeated function evaluations, are main disadvantages of the existing reliability methods for structural systems with truncated random variables. The computational costs for repeated function evaluations are extremely huge, especially for systems with implicit performance functions. In this case, every calculation of system response is a complete finite element calculation which is time costly. To overcome the drawbacks of the existing reliability methods and the difficulties of modeling multiple failure modes as well as improve the accuracy, the combinations of surrogate model and MCS are considered for structural systems, and an efficient reliability method is proposed to calculate the probability of failure for structural systems with truncated random variables.

The following steps are suggested to calculate the probability of failure under different design points for structural systems with truncated random variables in the proposed method.

(1) Generating uniformly distributed samples for truncated random variables in the supported intervals and for design variables in the specified bounded intervals;
(2) Calculating system responses and using the available data to construct a back-propagation (BP) neural network;
(3) Given input samples and calculate the probability of failure under different design points using the trained BP network.

The detailed information for each step is given in the following subsections.

2.1. Generating uniformly distributed samples for truncated random variables in the supported intervals and design variables in the specified bounded intervals

Let $\widetilde{X}_i$ is a continuous random variable with the probability density function (PDF) and cumulative density function (CDF) $f(\widetilde{X}_i)$ and $F_{\widetilde{X}_i}(\widetilde{X}_i)$, respectively, and both of them have infinite supported intervals. Suppose X_i denotes the truncated random variable for $\widetilde{X}_i$ with the supported interval $[a_i, b_i]$, a_i and b_i are two constants, $-\infty < a_i \leqslant x \leqslant b_i < \infty$. The corresponding truncated PDF $f(X_i)$ and CDF $F_{X_i}(X_i)$ of X_i can be respectively given by

$$f(X_i) = \frac{f(\widetilde{X}_i)}{F_{\widetilde{X}_i}(b_i) - F_{\widetilde{X}_i}(a_i)} \tag{1}$$

$$F_{X_i}(x_i) = \frac{1}{F_{\widetilde{X}_i}(b_i) - F_{\widetilde{X}_i}(a_i)}\left[F_{\widetilde{X}_i}(x_i) - F_{\widetilde{X}_i}(a_i)\right] \tag{2}$$

For example, suppose $\widetilde{X}$ is the standard normal distribution, the CDFs for truncated random variable X with supported interval $[-2, 2]$ and $\widetilde{X}$ are shown in Fig. 1.

The generation of pseudo-random numbers is very important and common task in computer programming, which is very useful in developing MCS. The mostly used pseudo-random number generators is the linear congruential generator, which can be expressed as [15]

$$X_{n+1} = (cX_n + d) \bmod m \tag{3}$$

where c is called the multiplier, d is called the increment, and m is called the modulus of the generator.

Let u_i denote a uniformly distributed sample in interval $[0, 1]$, the ith uniformly distributed sample in interval $[a, b]$ can be generated by

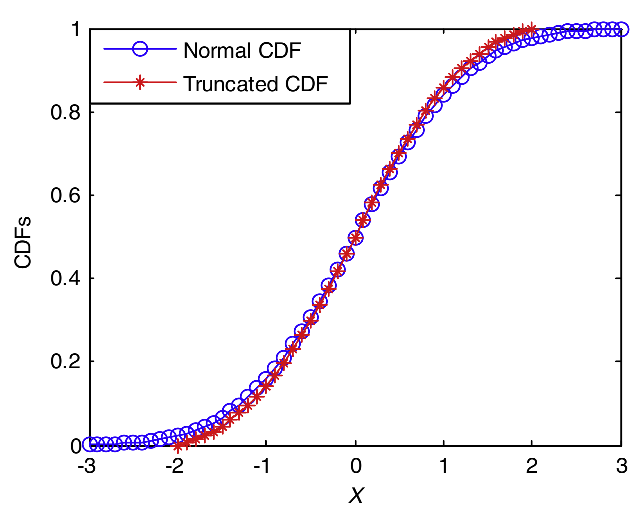

Fig. 1. The CDFs for truncated and standard normal distributions.

$$x_i = a + (b - a)u_i \tag{4}$$

where x_i is the *ith* sample for random variable X_i.

According to Eq. (4), the uniformly distributed samples in an arbitrary supported interval can be generated quickly.

Consider an arbitrary truncated random variable X_j, N uniformly distributed samples in the supported interval $[a_j, b_j]$ can be expressed as

$$(\bar{x}_j^1, \bar{x}_j^2, \cdots, \bar{x}_j^N) \tag{5}$$

where $a_j \leq \bar{x}_j^i \leq b_j, i = 1, 2, \cdots, N$.

Let Y_j be a design variable which belongs to the specified interval $[Y_j^L, Y_j^U]$. By the same way, N uniformly distributed samples for design variable Y_j in its specified interval $[Y_j^L, Y_j^U]$ can be expressed as

$$(\bar{y}_j^1, \bar{y}_j^2, \cdots, \bar{y}_j^N) \tag{6}$$

where $Y_j^L \leq \bar{y}_j^i \leq Y_j^U, i = 1, 2, \cdots, N$.

Uniformly distributed samples in the supported and specified intervals for both truncated random variables and design variables can almost cover the entire uncertain space fully. For example, X_1, X_2 are two truncated random variables which follow the Gumbel and the standard normal distributions, respectively, i.e. $X_1 \sim G(3, 1)$, $X_2 \sim N(0, 1)$, and the supported intervals for X_1, X_2 are respectively $[-5,5]$ and $[-3,3]$. 1000 samples in the uncertain space generated according to the corresponding CDFs by using MCS directly, and the proposed method are shown in Figs. 2 and 3, respectively. According to Figs. 2 and 3, we know that a small sizes of uniformly distributed samples can approximate cover the entire uncertain space properly and fully for both design and truncated random variables, while the samples generated by MCS according to the corresponding CDFs are concentrated on the mean values of variables which cannot cover the entire uncertain space fully and successfully.

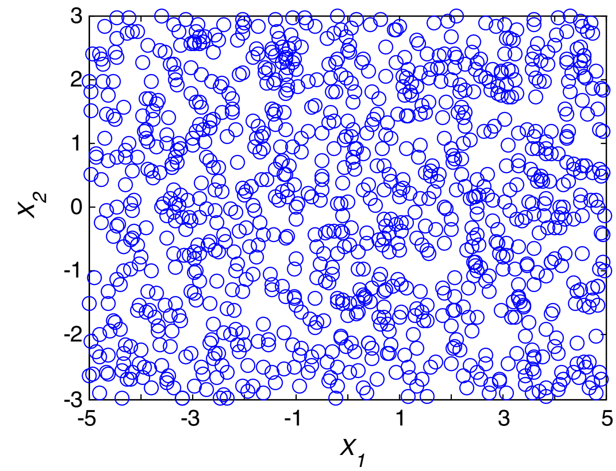

Fig. 3. Uniformly distributed samples in the supported intervals.

2.2. Calculating system responses and using the available data to construct a BP neural network

Neural network has been successfully applied in many fields such as earthquake magnitude prediction [16], reliability analysis [17], forecasting and fault diagnosis [18,19] etc. There are many types of neural networks while the BP neural network with three layers (input layer, hidden layer and output layer) is adopted for its capability of approximating any continuous functions properly. The diagram of a BP neural network with multi-inputs and one output or multi-outputs are shown in Figs. 4 and 5, respectively. The first and second inputs in Figs. 4 and 5 are designed for input random variables and design variables, respectively.

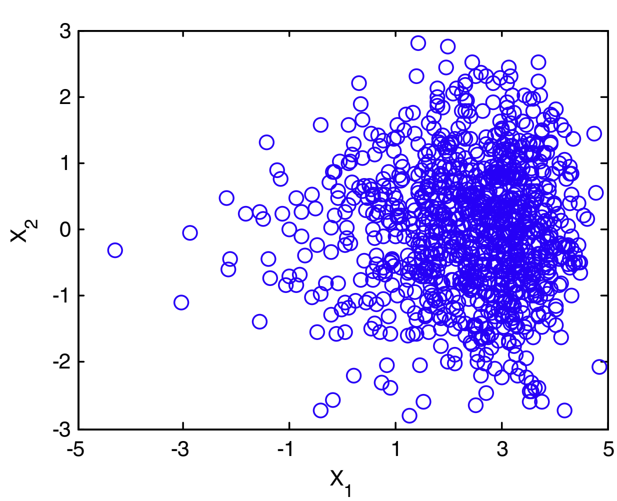

Fig. 2. Samples generated using MCS according to the corresponding CDFs.

In order to construct a BP neural network correctly, the available data for training the BP neural network should be given firstly. There are two cases should be respectively considered, i.e., one or multiple failure modes in structural systems.

(1) One failure mode in structural systems

Suppose the performance function of system is $Z = g(\mathbf{X}, \mathbf{Y})$, the *jth* system response under given input samples $\bar{\mathbf{x}}^j = (\bar{x}_1^j, \bar{x}_2^j, \cdots, \bar{x}_n^j)$ and $\bar{\mathbf{y}}^j = (\bar{y}_1^j, \bar{y}_2^j, \cdots, \bar{y}_p^j)$ can be calculated as

$$\bar{z}^j = g(\bar{\mathbf{x}}^j, \bar{\mathbf{y}}^j) \tag{7}$$

N samples of system responses can be expressed as

$$(\bar{z}^1, \bar{z}^2, \cdots, \bar{z}^N) \tag{8}$$

(2) Multiple failure modes in structural systems

Suppose there are m failure modes in system which can be expressed as

$$\begin{aligned} Z_1 &= g_1(\mathbf{X}, \mathbf{Y}) \\ Z_2 &= g_2(\mathbf{X}, \mathbf{Y}) \\ &\vdots \\ Z_m &= g_m(\mathbf{X}, \mathbf{Y}) \end{aligned} \tag{9}$$

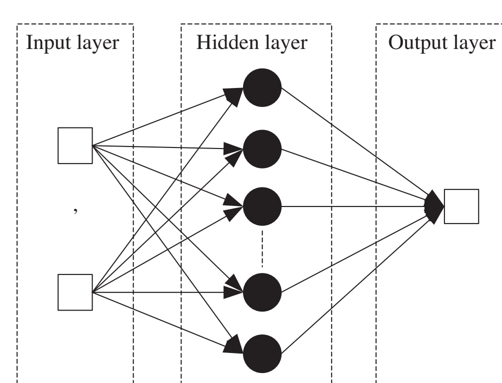

Fig. 4. A BP neural network with multi-inputs and one output.

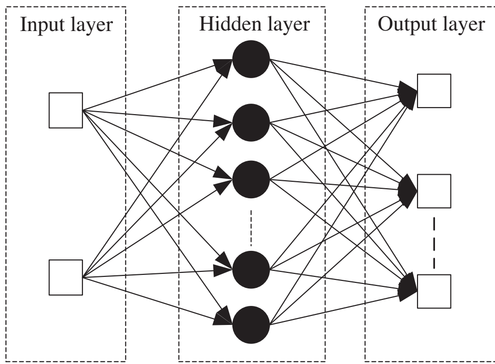

Fig. 5. A BP neural network with multi-inputs and multi-outputs.

Let $\mathbf{Z} = (Z_1, Z_2, \cdots, Z_m)$, N samples of system responses can be expressed as

$$(\bar{\mathbf{z}}^1, \bar{\mathbf{z}}^2, \cdots, \bar{\mathbf{z}}^N) \tag{10}$$

According to Eqs. 5, 6, 8, and 10, the available data of training BP neural network for structural systems with one or multiple failure modes can be respectively given by

$$(\bar{\mathbf{x}}^j, \bar{\mathbf{y}}^j \text{ and } \bar{z}^j, j = 1, 2, \cdots, N) \tag{11}$$

$$(\bar{\mathbf{x}}^j, \bar{\mathbf{y}}^j \text{ and } \bar{\mathbf{z}}^j, j = 1, 2, \cdots, N) \tag{12}$$

It should be noted that in engineering practices, the system performance function usually cannot be determined using an explicit function. Therefore, FEA method should be employed to calculate system responses. For more detailed information about FEA, please see Ref. [20].

The basic principles of constructing a BP neural network and learning algorithms as well as choosing the activation functions are omitted in this paper, but these can be found in Ref. [21].

Since the uniformly distributed samples of the supported and specified intervals have almost covered the entire uncertain space fully, so the BP neural network is trained using the available data which can approximate the relationships between the inputs and system responses properly in the entire uncertain space. The computational efficiency by the trained BP neural network is higher than other classical reliability methods because it not only does not need the MPP search and repeated performance function evaluations, but also the approximate system responses can be calculated quickly, especially when the performance function cannot be defined explicitly. In this case, FEA should be used to calculate the system responses. However, every calculation of system response is a complete finite element calculation which is time costly.

2.3. Calculate probability of failure under different design points using the trained BP neural network

Given a truncated random variable X_i with the supported interval $[a_i, b_i]$ and the truncated CDF F_{X_i}, the samples generated using MCS and the inverse transformation method can be expressed as [22]

$$x_i^j = F_{X_i}^{-1}(u_i^j) \tag{13}$$

where $F_{X_i}^{-1}$ is the inverse function of F_{X_i}, and u_i^j is uniformly distributed in interval $[0, 1]$. From Eq. (13), the samples for arbitrary truncated random variable can be generated using MCS and the inverse transformation method easily.

The sampling process is shown in Fig. 6.

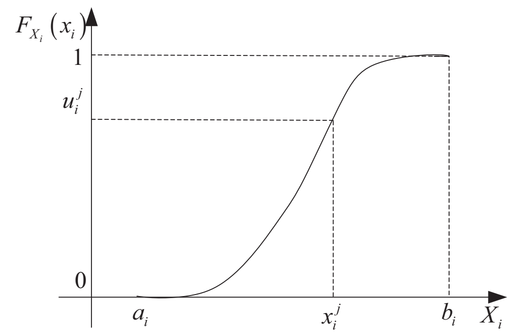

Fig. 6. Sampling method for truncated random variable.

Let $\mathbf{x}^j$ be the jth sample for truncated random vector $\mathbf{X} = (X_1, X_2, \cdots, X_n)$, and $\mathbf{x}^j = (x_1^j, x_2^j, \cdots, x_n^j)$. $\mathbf{y}^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_k=y_p^j)}$ denotes the jth sample for design vector $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_p)$ under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$. For example, Y_i is a design variable with the lower and upper bounds of Y_i^L and Y_i^U, respectively. $Y_i|_{Y_i=y_i^j}$ denotes the jth design point for Y_i is y_i^j, that is, $Y_i = y_i^j$, and y_i^j is an arbitrary value in the specified interval $[Y_i^L, Y_i^U]$. From the trained BP neural network with one or multi-outputs, the relationships between the inputs and system responses can be shown in Figs. 7 and 8, respectively.

For a system with one failure mode, the probability of failure under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$ can be calculated as

$$P_f|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} = \frac{1}{N}\sum_{j=1}^{N} I\left[z^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} < 0\right] \tag{14}$$

where $I[\bullet]$ is the indicator function, which equals to 1 if $[\bullet]$ is true and 0 if $[\bullet]$ is false.

For a series system with m failure modes, the occurrence of any failure mode will lead to failure of the system. The system

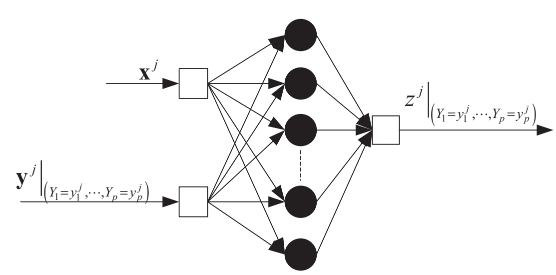

Fig. 7. The trained BP neural network with one output.

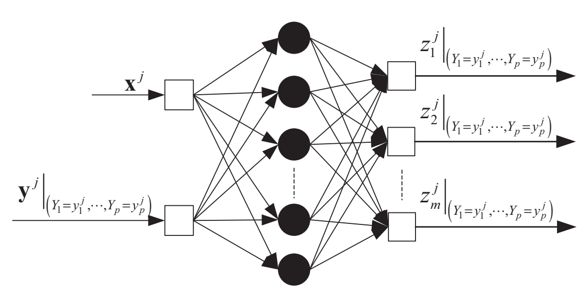

Fig. 8. The trained BP neural network with multi-outputs.

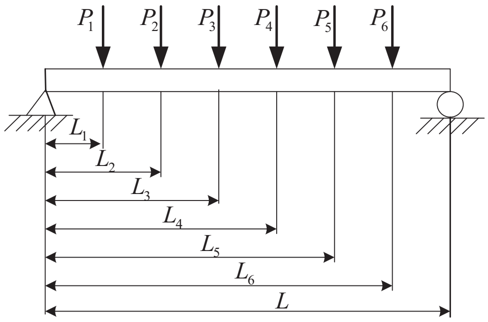

Fig. 9. A composite beam.

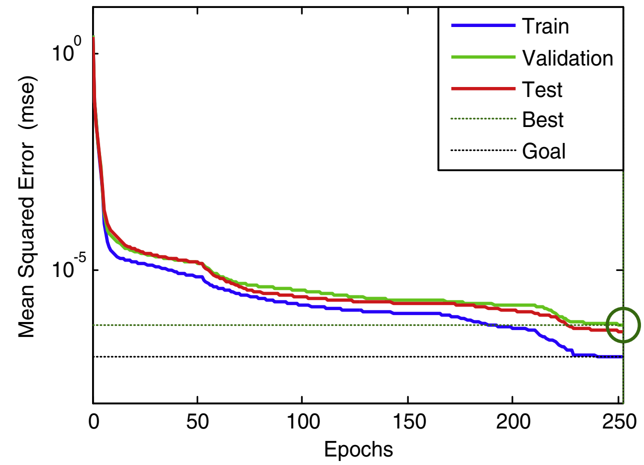

Fig. 10. Training process for the BP neural network.

Table 1
Detailed information of truncated random variables.

Variable	Mean value	Standard deviation	Distribution type	Supported interval
B	200	0.2	Normal	[199.4, 200.6]
C	80	0.2	Normal	[79.4, 80.6]
D	20	0.2	Normal	[19.4, 20.6]
L_1	200	1	Normal	[197, 203]
L_2	400	1	Normal	[397, 403]
L_3	600	1	Normal	[597, 603]
L_4	800	1	Normal	[797, 803]
L_5	1000	1	Normal	[997, 1003]
L_6	1200	1	Normal	[1197, 1203]
L	1400	1	Normal	[1394, 1403]
P_1	15	1.5	Gumbel	[5, 19]
P_2	15	1.5	Gumbel	[5, 19]
P_3	15	1.5	Gumbel	[5, 19]
P_4	15	1.5	Gumbel	[5, 19]
P_5	15	1.5	Gumbel	[5, 19]
P_6	15	1.5	Gumbel	[5, 19]
E_a	70	7	Normal	[49, 91]
E_w	8.75	0.875	Normal	[6.125,11.375]
S	25	2.5	Gumbel	[16, 35]

probability of failure under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$ can be calculated as

$$P_f|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} = \frac{1}{N}\sum_{j=1}^{N} I\left[\bigcup_{i=1}^{m} z_i^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} < 0\right] \tag{15}$$

Let

$$S_{\min}^j = \min\left[z_1^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)}, \cdots, z_m^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)}\right], \tag{16}$$

the probability of failure for a series system under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$ can be rewritten as

$$P_f^{\text{sys}}|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} = \frac{1}{N}\sum_{j=1}^{N} I\left[S_{\min}^j < 0\right] \tag{17}$$

For a parallel system with m failure modes, the occurrence of all failure modes will lead to failure of the system. The system probability of failure under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$ can be calculated as

$$P_f|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} = \frac{1}{N}\sum_{j=1}^{N} I\left[\bigcap_{i=1}^{m} z_i^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} < 0\right] \tag{18}$$

Let

$$S_{\max}^j = \max\left[z_1^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)}, \cdots, z_m^j|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)}\right], \tag{19}$$

the probability of failure for a parallel system under the design points $Y_1 = y_1^j, Y_2 = y_2^j, \cdots, Y_p = y_p^j$ can be rewritten as

$$P_f^{\text{sys}}|_{(Y_1=y_1^j, Y_2=y_2^j, \cdots, Y_p=y_p^j)} = \frac{1}{N}\sum_{j=1}^{N} I[S_{\max}^j < 0] \tag{20}$$

3. Numerical examples

In this Section, three engineering examples and one mathematical example are investigated to demonstrate the efficiency as well as the accuracy of the proposed method. The first example is a composite beam with 19 independent truncated random variables and one design variable; the second example is a cantilever beam-bar with three failure modes and one design variable, the third example is a parallel system with two failure modes and one design variable, and the fourth example is a truss system with two failure modes and two design variables. The numbers of hidden nodes in the constructed BP neural network are 10 and 20 for examples 2, 3 and 1, 4, respectively. The training function of all examples is "trainlm", and the transfer functions for hidden and output layers are "tansig" and "purelin", respectively. Furthermore, the results using MCS method with the large samples are used as a reference for both the accuracy and efficiency comparisons.

3.1. Example one-A composite beam

A composite beam with 19 independent variables, as shown in Fig. 9, is employed to demonstrate the effectiveness and accuracy of the proposed method for the structural system with only one failure mode. The maximum stress of the beam is calculated by [8]

$$\sigma_{\max} = \frac{\left[\frac{\sum_{i=1}^{6} P_i(L-L_i)}{L} L_3 - P_1(L_2 - L_1) - P_2(L_3 - L_2)\right]\left[\frac{0.5AB^2 + \frac{E_a}{E_w}DC(B+D)}{AB + \frac{E_a}{E_w}}\right]}{\frac{1}{12}AB^3 + AB\left\{\left[\frac{0.5AB^2 + \frac{E_a}{E_w}DC(B+D)}{AB + \frac{E_a}{E_w}DC}\right] - 0.5B\right\}^2 + \frac{1}{12}\frac{E_a}{E_w}CD^3 + \frac{E_a}{E_w}CD\left\{0.5D + B - \left[\frac{0.5AB^2 + \frac{E_a}{E_w}DC(B+D)}{AB\frac{E_a}{E_w}DC}\right]\right\}}$$

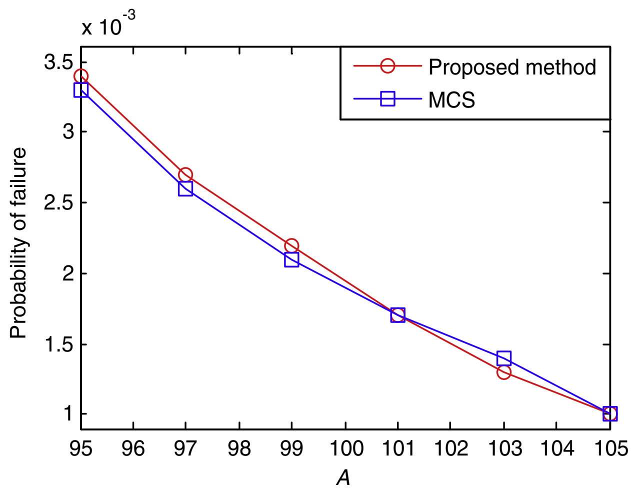

Fig. 11. Probabilities of failure under different design points.

Table 2
Probabilities of failure under different design points.

A(design point)	Proposed methodP_f^{sys}	MCS P_f^{sys}
95	0.0034	0.0033
97	0.0027	0.0026
99	0.0022	0.0021
101	0.0017	0.0017
103	0.0013	0.0014
105	0.0010	0.0010
Total function evaluations	1000	1,000,000 × 6

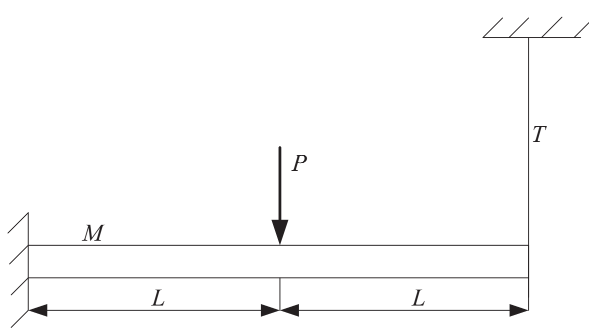

Fig. 12. A cantilever beam-bar.

Table 3
Detailed information of truncated random variables.

Variable	Mean value	Standard deviation	Distribution type	Supported interval
T	250	100	Normal	[0, 550]
M	200	50	Normal	[50, 350]
P	60	10	Normal	[30, 90]

where $L_1, L_2, \cdots, L_6$and L are corresponding length from the left node measured in mm; E_a, E_w are Young's modulus in GPa; $P_1, P_2, \cdots, P_6$ are applied loads at six different locations along the beam in kN, A, C, B and D are wide and high of the beam in mm, respectively. In order to keep the composite beam in safe domain, the allowable strength should bigger than the maximum stress. Therefore, the performance function can be determined by

$$Z = S - \sigma_{\max}$$

where S is the allowable strength.

A is taken as a design variable with the lower and upper bounds of 95 and 105, respectively. The detailed information of the truncated random variables is given in Table 1.

This example with the high nonlinear performance function is used to demonstrate the accuracy of the proposed method; 1000 uniformly distributed samples are respectively generated for the design and truncated random variables in the specified intervals and supported intervals, the corresponding system responses and input uniformly distributed samples are used as available data to train the BP neural network. The training process and the probability of failure under different design points are shown in Figs. 10 and 11 and Table 2, respectively.

From Table 2 and Fig. 11, we can conclude that a small sample sizes can cover the entire uncertain space properly and fully, and the constructed BP neural network can approximate the relationships between the inputs and outputs properly by using the proposed available data. The results calculated by the proposed method are almost identical to the results calculated using MCS, while the total computational costs are significantly reduced. For example, 1,000,000 numbers of samples are used to calculate the probability of failure under each design point, then the total numbers of function evaluations are 1,000,000 × 6 for MCS while the total function evaluations by the proposed method are only 1000.

3.2. Example two – A cantilever beam-bar

A cantilever beam-bar system [11], shown in Fig. 12, is employed to demonstrate the accuracy and effectiveness of the proposed method for structural systems with multiple failure modes. In this example, three failure modes are considered, and the corresponding performance function for three failure modes are determined by

$$Z_1 = T - 5P/16$$
$$Z_2 = M - LP/3$$
$$Z_3 = M + 2LT - LP$$

This cantilever beam-bar system is considered as a series system because any failure mode occurs; the system is failing. L is taken as a design variable with the lower and upper bounds of 4.5 and 5.5, respectively. The detailed information of the truncated random variables is given in Table 3.

In this example, 500 uniformly distributed samples are respectively generated for the design and truncated random variables in the specified intervals and supported intervals, the corresponding system responses and input uniformly distributed samples are used as available data to train the BP neural network. The training process and the probability of failure under different design points are shown in Fig. 13 and 14 and Table 4, respectively.

From Table 4 and Fig. 14 we know that the results obtained by the proposed method under different design points are almost identical to the results calculated using MCS, while the total function evaluations of the proposed method are only 500 × 3 when compared to the MCS with 1,000,000 × 3 × 6.

3.3. Example 3 – a parallel system

Consider a parallel system with two failure modes, and the corresponding performance functions are given by

$$Z_1 = X_1X_2/X_4 + X_3 - 10$$
$$Z_2 = X_1^2/X_3 + 2X_2 - X_4 - 6$$

X_4 is taken as a design variable with the lower and upper bounds 3 and 7, respectively. X_1, X_2, X_3 are truncated random variables, and

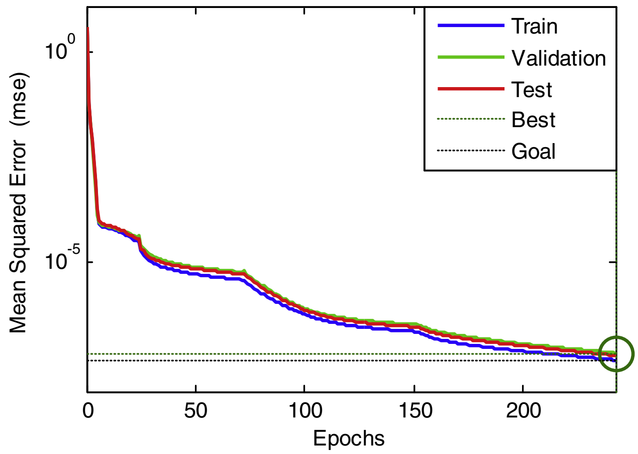

Fig. 13. Training process for the BP neural network.

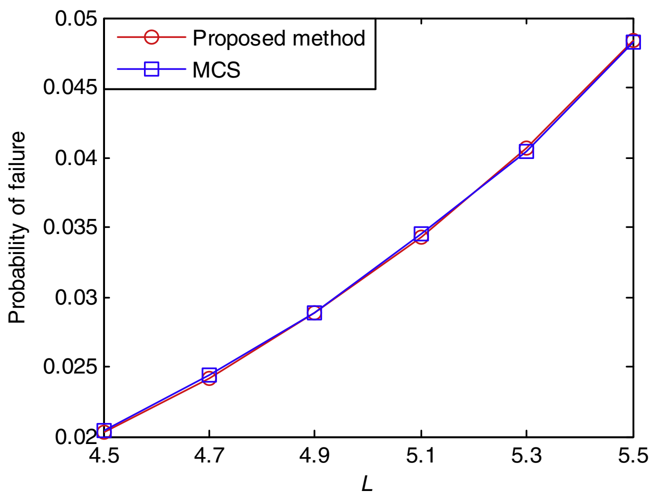

Fig. 14. Probabilities of failure under different design points.

Table 4
Probabilities of failure under different design points.

L(design point)	Proposed method P_f^{sys}	MCS P_f^{sys}
4.5	0.0203	0.0204
4.7	0.0242	0.0244
4.9	0.0288	0.0289
5.1	0.0343	0.0345
5.3	0.0407	0.0404
5.5	0.0484	0.0483
Total function evaluations	500×3	$1{,}000{,}000 \times 3 \times 6$

the detailed information of these truncated random variables are shown in Table 5.

A parallel system with two failure modes is considered in this example, 250 uniformly distributed samples are respectively generated for the design and truncated random variables in the specified intervals and supported intervals, the corresponding system responses and input uniformly distributed samples are used as available data to train the BP neural network. The training process and the system probability of failure under different design points are shown in Figs. 15 and 16 and Table 6, respectively.

From Table 6 and Fig. 16, we know that the results obtained by the proposed method under different design points are almost identical to the results calculated using MCS, while the total function evaluations of the proposed method are only $250 \times 2$when compared to the MCS with $1{,}000{,}000 \times 2 \times 5$.

Table 5
Detailed information of truncated random variables.

Variable	Mean value	Standard deviation	Distribution type	Supported interval
X_1	2	1	Normal	[0, 5]
X_2	5	1	Normal	[2, 8]
X_3	10	1	Normal	[7, 13]

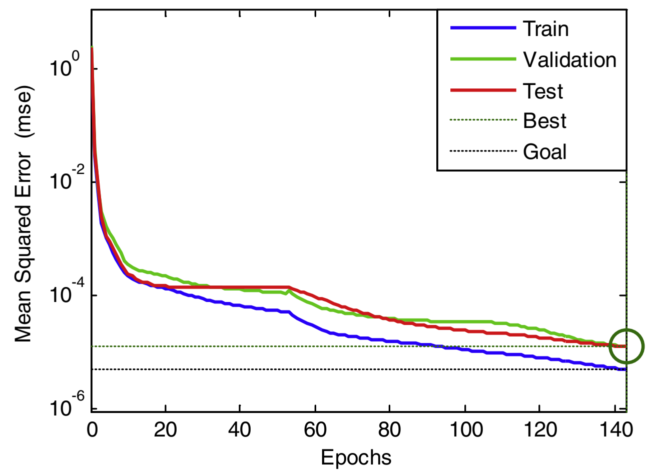

Fig. 15. Training process for the BP neural network.

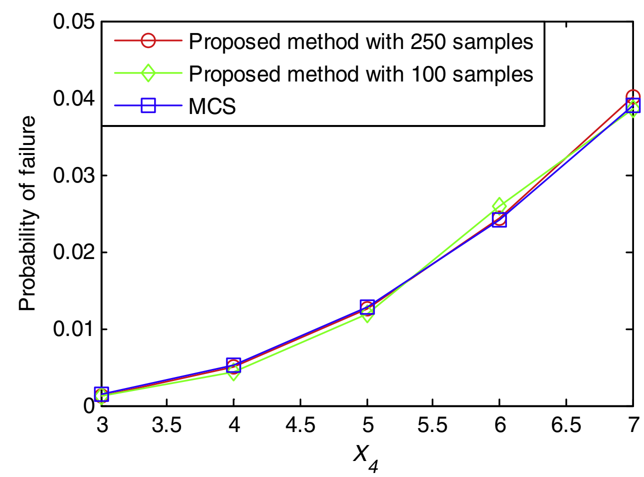

Fig. 16. Probabilities of failure under different design points.

Table 6
Probabilities of failure under different design points.

X_4(design point)	Proposed method P_f^{SYS}	MCS P_f^{SYS}
3	0.0141	0.0135
4	0.0310	0.0308
5	0.0539	0.0557
6	0.0821	0.0836
7	0.1158	0.1121
Total function evaluations	250×2	$1{,}000{,}000 \times 2 \times 5$

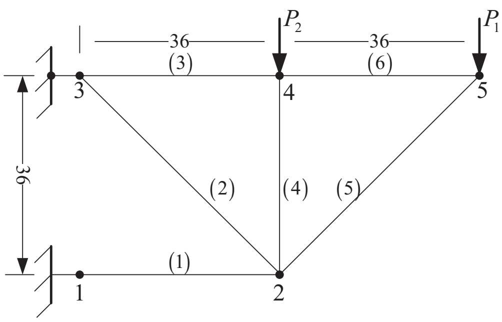

Fig. 17. A truss with six members.

Table 7
Detailed information of truncated random variables.

Variable	Mean value	Standard deviation	Distribution type	Supported interval
P_1	495	10	Normal	[465, 525]
P_2	500	10	Normal	[470, 530]
$A_1 \sim A_4$	8	0.5	Normal	[6.5, 9.5]

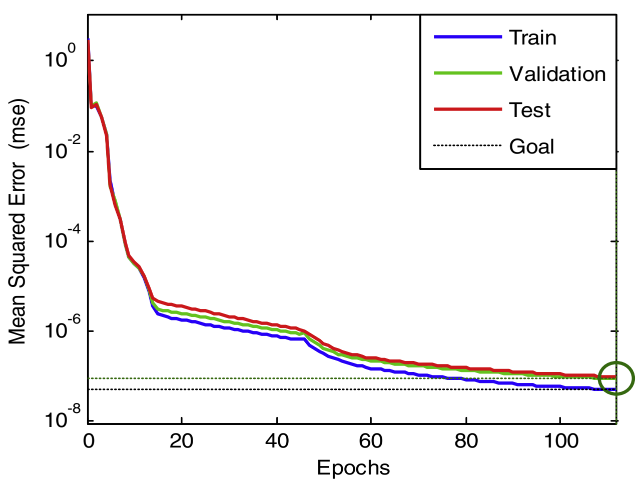

Fig. 18. Training process for the BP neural network.

3.4. Example 4 – a truss system

A linear-elastic truss system with six members [20], shown in Fig. 17, the maximum allowable displacements for nodes 2 and 5 are assumed as 0.0125 and 0.021, respectively, the performance function of the truss system are defined as

$$Z_1 = 0.0125 - |U_{2y}|$$
$$Z_2 = 0.0210 - |U_{5y}|$$

where U_{2y} and U_{5y} denote the displacement of nodes 2 and 5 in y-axis, and both of them are measured in (in).

$A_i(i = 1, 2, \cdots, 6)$ are cross sectional areas measured in in^2. (P_1, P_2) are the applied external loads measured in lb; $E = 1.9 \times 10^6$ is the elastic modulus in lb/in^2. A_5 and A_6 are design variables with the lower and upper bounds of 7 and 9, respectively. The detailed information of truncated random variables is given in Table 7.

In this example, 500 uniformly distributed samples are respectively generated for the design and truncated random variables in the specified intervals and supported intervals, the corresponding system responses and input uniformly distributed samples are used as available data to train the BP neural network. The training process and the probability of failure under different design points are shown in Figs. 18 and 19 and Table 8, respectively.

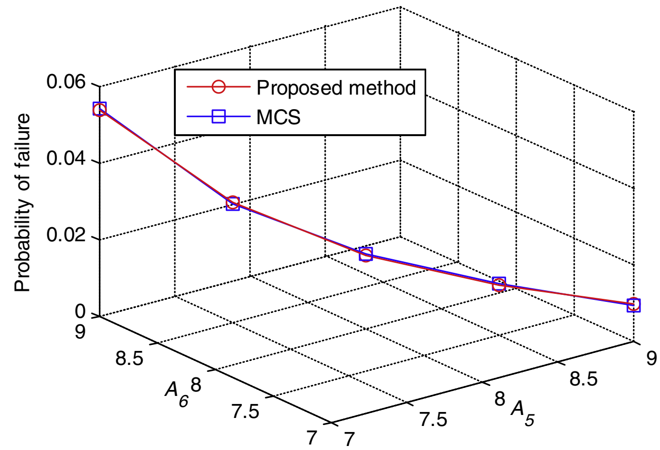

Fig. 19. Probabilities of failure under the different design points.

Table 8
Probabilities of failure under different design points.

A_5(design point)	A_6(design point)	Proposed method P_f^{SYS}	MCS P_f^{SYS}
7	9	0.0540	0.0543
7.5	8.5	0.0314	0.0312
8	8	0.0193	0.0195
8.5	7.5	0.0131	0.0133
9	7	0.0097	0.0095
9	8.5	0.0036	0.0037
9	9	0.0029	0.0031
Total No. of simulations		500	100,000 × 7

This example is a series system with two failure modes, the FEA method should be used to calculate system response because the system performance functions are implicit functions. MCS–FEA method with 100,000 samples is used to calculate system probability of failure under the each design point, as well as references for the accuracy comparisons. From Table 8 and Fig. 19 we know that the results obtained by the proposed method under different design points are almost identical to the results calculated using MCS, while the total No. of simulations of the proposed method are only 500 when compared to the MCS–FEA with 100,000 × 7. It is worth to note that the more design points we calculate, the more function evaluations will be needed for the MCS–FEA method. However, this is not true for the proposed method. This example shows that the computational efficiency of the proposed method for structural systems with implicit functions is higher than the MCS–FEA method significantly.

4. Conclusions

Due to the realistic restricting, truncated random variables are often encountered in many engineering applications. An efficient reliability method for structural systems with truncated random variables is proposed in this paper. The main features of the proposed method are that uniformly distributed samples for truncated random variables in the supported intervals and design variables in the specified bounded intervals are respectively generated, and the generated samples, as well as system responses, are taken as available data to train the constructed BP neural network. Since the uniformly distributed samples can approximate cover the entire uncertain space properly and fully, therefore, the trained BP neural network can approximate the relationships between the inputs and system responses properly in almost entire uncertain space. The trained BP neural network is used as the surrogate model in reliability calculation which not only improves computational efficiency significantly, but also the MPP search is avoided. The four examples have shown that the proposed method is effectiveness because it requires neither repeated function evaluations nor the large samples while keeps high accuracy when compared to MCS. Moreover, the four examples have also shown that the proposed method is robustness because it can be used for structural systems with explicit or implicit performance functions without the MPP search.

It should be noted that all truncated random variables are assumed mutually independent in this paper. Reliability sensitivity analysis for structural systems with dependent truncated random variables will be investigated in our future works.

Acknowledgements

The authors would like to acknowledge the partial support provided by the National Natural Science Foundation of China under the contract number 11272082.

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