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Weighted Fuzzy Risk Priority Number Evaluation of Turbine and Compressor Blades Considering Failure Mode Correlations

Abstract: Failure mode, effects and criticality analysis (FMECA) and Fault tree analysis (FTA) are powerful tools to evaluate reliability of systems. Although single failure mode issue can be efficiently addressed by traditional FMECA, multiple failure modes and component correlations in complex systems cannot be effectively evaluated. In addition, correlated variables and parameters are often assumed to be precisely known in quantitative analysis. In fact, due to the lack of information, epistemic uncertainty commonly exists in engineering design. To solve these problems, the advantages of FMECA, FTA, fuzzy theory, and Copula theory are integrated into a unified hybrid method called fuzzy probability weighted geometric mean (FPWGM) risk priority number (RPN) method. The epistemic uncertainty of risk variables and parameters are characterized by fuzzy number to obtain fuzzy weighted geometric mean (FWGM) RPN for single failure mode. Multiple failure modes are connected using minimum cut sets (MCS), and Boolean logic is used to combine fuzzy risk priority number (FRPN) of each MCS. Moreover, Copula theory is applied to analyze the correlation of multiple failure modes in order to derive the failure probabilities of each MCS. Compared to the case where dependency among multiple failure modes is not considered, the Copula modeling approach eliminates the error of reliability analysis. Furthermore, for purpose of quantitative analysis, probabilities importance weight from failure probabilities are assigned to FWGM RPN to reassess the risk priority, which generalize the definition of probability weight and FRPN, resulting in a more accurate estimation than that of the traditional models. Finally, a basic fatigue analysis case drawn from turbine and compressor blades in aero-engine is used to demonstrate the effectiveness and robustness of the presented method. The result provides some important insights on fatigue reliability analysis and risk priority assessment of structural system under failure correlations.

Keywords: failure mode, effects and criticality analysis, minimum cut sets, risk priority number, Copula, fuzzy probability weighted geometric mean

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1 Introduction

Due to high reliability and safety for most complex systems, failure mode and effects analysis (FMEA)/failure mode, effects and criticality analysis (FMECA) have seen wide applications in various domains such as aerospace, nuclear, power generation, petrochemical, and other industries. FMECA has been proven to be one of the most important preventative initiatives and pro-active measures during the design stage of a system as well as in process and service stages [1]. As the crucial part of aero-engine, blade failures count for 70% of total failures of aero-engine components. For turbine engine components, one of the most failure modes is the fatigue fracture, which is characteristics of high failure rate, multiple failure modes and great harmfulness. Therefore, fatigue analysis is conducted for these high reliability products in FMECA, combining with the structural design of turbine engine components and their condition based maintenances, to prevent the occurrence of malfunctions that may lead to significant losses or even catastrophic failures [2–3].

Traditional FMECA only pays close attention to the effect of single failure modes by considering epistemic uncertainty. For instance, Gargama et al. [4] develop a fuzzy risk priority number (FRPN) method for single failure mode analysis, but the effects of the components on the system with multiples failure modes have not been taken into account. According to the problem, Pickard et al. [5] have taken the consideration of multiple failures within a system. However, multiple failure problems of a large and complex system have not yet been successfully handled. Xiao et al. [6] apply fault tree analysis (FTA) and the minimal cut sets (MCS) to connect complex systems with multiple failure modes. Based on the assumption that individual failure modes are independent with each other, they quantitatively assess the risk priority with the help of probability weighted crisp risk priority number (RPN). Yang et al. [7] proposed a fuzzy rule-based Bayesian reasoning approach for prioritization of failures in FMEA. However, all of these approaches failed to incorporate epistemic uncertainty and multiple correlated failure modes into the RPN metric. Owing to these correlations [8], ambiguous cognition and lack of failure data, ignoring the dependency among the failure modes may affect the accuracy of quantitative analysis for the actual situation. Fortunately, Copula function is found to be a practical and convenient tool to solve the correlation issue [9]. Therefore, in this paper, Copula function is applied to analyze the fatigue reliability for turbine and compressor blades. In particular, FRPN of multiple correlated failure modes is analyzed by using MCS of fault tree analysis, and the lifetime distribution is established based on Weibull function subject to multiple correlated failure modes. Combining the advantages of Copula theory, MCS method, and fuzzy variables in dealing with uncertain problems, a new fuzzy probability weighted geometric average (FPWGM) RPN method is developed.

The rest of the paper is organized as follows: In Section 2, FRPN for single failure mode and new FRPN are evaluated. In Section 3, a new FPWGM RPN algorithm based on Copula theory and MCS is proposed. Section 4 applies the aforementioned method to analyze fatigue failure of turbine and compressor blades.

2 FRPN evaluation and multiple failure modes combination

In this section, MCS is applied to analyze FRPN for multiple failure modes. To connect multiple failure modes, new FRPN is obtained by combining FWGM algorithm, the benchmark adjustment search algorithm (BASA), MCS method, and the Boolean operation. Since the traditional RPN approach is limited to the case of single failure mode and crisp RPN fails to consider epistemic uncertainty under multiple failure modes conditions, a new method for FRPN evaluation is put forward as follow.

2.1 Fuzzy weighted geometric mean RPN evaluation

RPN is a necessary part of FMECA, which is defined as the product of three factors, i.e., the occurrence (O), severity (S) and detection (D): $\mathrm{RPN} = O \times S \times D$. These three factors are evaluated using the ratings (also called rankings or scores) from 1 to 10, as described in [1, 10–11]. However, these three factors of traditional RPN are difficult to be determined precisely because of various uncertainties. Much information in FMECA can be expressed in a linguistic way such as moderate, remote or very high. In addition, the relative importance among O, S and D are not taken into account. Rather these three factors are assumed to have the same importance. This may not be the case when applying to a practical FMECA. As a result, the RPN has been extensively criticized for various reasons [12–13]. To overcome these drawbacks, the comprehensive fuzzy assessment information can be obtained for various failure modes by combining fuzzy relative importance weights of three factors with the fuzzy numbers of the criteria [4] assessed by team members. The concept of FRPN is defined as FWGM RPN. The α-level sets of FRPN can be calculated using the combination of FWGM and BASA. Wang et al. [14] adopt the method of aggregating the FMECA team members' subjective opinions. Since the aggregate method involves tedious procedures, a comprehensive interval number matrix is introduced to simplify and improve aggregate method in this paper.

Suppose there are n types of failure modes, i.e., FM_i ($i = 1, \cdots, n$), to be evaluated and prioritized by a FMECA team member TM_j ($j = 1, \cdots, m$). Let $\tilde{R}_{ij}^O = (R_{ijL}^O, R_{ijM1}^O, R_{ijM2}^O, R_{ijU}^O)$, $\tilde{R}_{ij}^S = (R_{ijL}^S, R_{ijM}^S, R_{ijU}^S)$ and $\tilde{R}_{ij}^D = (R_{ijL}^D, R_{ijM}^D, R_{ijU}^D)$ be the fuzzy ratings of the ith failure modes on O, S and D respectively. $\tilde{w}_j^O = (w_{jL}^O, w_{jM}^O, w_{jU}^O)$, $\tilde{w}_j^S = (w_{jL}^S, w_{jM}^S, w_{jU}^S)$ and $\tilde{w}_j^D = (w_{jL}^D, w_{jM}^D, w_{jU}^D)$ be the fuzzy weights of the three risk factors provided by the jth FMECA team member (TM_j), respectively. These fuzzy grades or fuzzy weights are defined as triangular or trapezoid fuzzy numbers. In addition, h_j ($j = 1, \cdots, m$) are the relative importance weight of the team members, satisfying $\sum_{j=1}^{m} h_j = 1$ and $h_j > 0$ ($j = 1, \cdots, m$), and H is defined as a matrix of h_j ($j = 1, \cdots, m$), let $H = [h_1, \cdots, h_j]$. L, M and U denote the lower, medium, and upper bound, respectively. For a triangular membership functions, M_1 and M_2 denote left and right middle bounds. Based on these assumptions, n failure mode is optimized to evaluate overall risk priority by the following steps. First, the interval number matrix algorithm is used to obtain comprehensive fuzzy assessment information of each failure mode and the relative importance weights of three risk factors from the expert judgments, as is expressed in Eqs. (1)–(6).

$$\tilde{R}_i^O = H\tilde{R}_{ij}^O = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(R_{i1L}^O, R_{i1M1}^O, R_{i1M2}^O, R_{i1U}^O)\\ \cdots \\ (R_{ijL}^O, R_{ijM1}^O, R_{ijM2}^O, R_{ijU}^O)\end{bmatrix} \tag{1}$$

Similarly,

$$\tilde{R}_i^S = H\tilde{R}_{ij}^S = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(R_{i1L}^S, R_{i1M}^S, R_{i1U}^S)\\ \cdots \\ (R_{ijL}^S, R_{ijM}^S, R_{ijU}^S)\end{bmatrix} \tag{2}$$

$$\tilde{R}_i^D = H\tilde{R}_{ij}^D = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(R_{i1L}^D, R_{i1M}^D, R_{i1U}^D)\\ \cdots \\ (R_{ijL}^D, R_{ijM}^D, R_{ijU}^D)\end{bmatrix} \tag{3}$$

$$\tilde{W}_i^O = H\tilde{W}_j^O = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(W_{i1L}^O, W_{i1M}^O, W_{i1U}^O)\\ \cdots \\ (W_{ijL}^O, W_{ijM}^O, W_{ijU}^O)\end{bmatrix} \tag{4}$$

$$\tilde{W}_i^S = H\tilde{W}_j^S = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(W_{i1L}^S, W_{i1M}^S, W_{i1U}^S)\\ \cdots \\ (W_{ijL}^S, W_{ijM}^S, W_{ijU}^S)\end{bmatrix} \tag{5}$$

$$\tilde{W}_i^D = H\tilde{W}_j^D = \left[h_1,\cdots,h_j\right]\begin{bmatrix}(W_{i1L}^D, W_{i1M}^D, W_{i1U}^D)\\ \cdots \\ (W_{ijL}^D, W_{ijM}^D, W_{ijU}^D)\end{bmatrix} \tag{6}$$

Then, the FRPN of each failure mode can be defined as

$$\begin{aligned}\mathrm{FRPN}_i &= (\tilde{R}_i^O)^{\tilde{w}^O/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)} \times (\tilde{R}_i^S)^{\tilde{w}^S/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)}\\ &\times (\tilde{R}_i^D)^{\tilde{w}^D/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)} \quad i=1,\cdots,n\end{aligned} \tag{7}$$

Suppose FRPN_i has been derived by considering relative importance weight and factor fuzzy weight. We utilize the α-level sets to compute the lower and upper bound of failure modes ratings and weights as follows

$$\begin{cases}(R_i^O)_\alpha^L = R_{iL}^O + \alpha(R_{iM1}^O - R_{iL}^O)\\ (R_i^O)_\alpha^U = R_{iU}^O - \alpha(R_{iU}^O - R_{iM2}^O)\end{cases} \tag{8}$$

$(R_i^S)_\alpha^L, (R_i^S)_\alpha^U, (R_i^D)_\alpha^L, (R_i^D)_\alpha^U, (w_i^O)_\alpha^L, (w_i^O)_\alpha^U, (w_i^S)_\alpha^L, (w_i^S)_\alpha^U, (w_i^D)_\alpha^L,$ $(w_i^D)_\alpha^U$ are all adopting the same calculation method as Eq. (8). FWGM can be expressed as Eq. (9)

$$\begin{aligned}\tilde{y}_G &= f_G(\tilde{x}_1,\cdots,\tilde{x}_n;\tilde{w}_1,\cdots,\tilde{w}_n)\\ &= (\tilde{x}_1)^{\tilde{w}_1/(\tilde{w}_1+\tilde{w}_2+\cdots+\tilde{w}_n)}(\tilde{x}_2)^{\tilde{w}_2/(\tilde{w}_1+\tilde{w}_2+\cdots+\tilde{w}_n)}\cdots(\tilde{x}_n)^{\tilde{w}_n/(\tilde{w}_1+\tilde{w}_2+\cdots+\tilde{w}_n)}\\ &= \prod_{i=1}^{n}(\tilde{x}_i)^{\tilde{w}_i/\sum_{j=1}^{n} wj}\end{aligned} \tag{9}$$

where $\tilde{y}_G$ is a fuzzy number which can be calculated using α-level sets and the extension principle [14–15]. Let $\tilde{y}_G = \left[(y_G)_\alpha^L, (y_G)_\alpha^U\right]$ be a α-level set of $\tilde{y}_G$. The following mathematical models by $x = e^{\ln x}$ transformation can be derived as:

$$\begin{aligned}(y_G)_\alpha^L &= \mathrm{Min}\ \exp\left(\frac{\sum_{i=1}^{n} w_i \ln(x_i)_\alpha^L}{\sum_{i=1}^{n} w_i}\right),\\ (y_G)_\alpha^U &= \mathrm{Max}\ \exp\left(\frac{\sum_{i=1}^{n} w_i \ln(x_i)_\alpha^U}{\sum_{i=1}^{n} w_i}\right)\end{aligned} \tag{10}$$

where $(w_i)_\alpha^L \le w_i \le (w_i)_\alpha^U$, $i = 1,\cdots,n$, and exp ( ) denotes the exponential function.

To generate a different α-level sets of FRPN_i by setting different α level, we adopt BASA to derive the minimum and maximum of $\tilde{y}_G$, i.e., $(y_G)_\alpha^L$ and $(y_G)_\alpha^U$. Accordingly, FWGM RPN is calculated by FWGM and BASA algorithms, and the detailed process is presented in literature [4, 14].

2.2 Centroid defuzzification algorithm

The FRPNs are derived from the aforementioned approaches based on α-level sets. It is essential to transform fuzzy numbers into crisp numbers for the purpose of comparison and ranking. Such process is called defuzzification [15–17]. In this paper, centroid defuzzification is adapted to prioritize failure information by the extracted FWGM. $\tilde{x}_0(\tilde{A})$ is the defuzzified value. When the explicit membership function of a fuzzy number $\tilde{A}$ is not known, yet α-level cut sets are available, its defuzzified centroid can be determined by Eq. (11), especially, when $\Delta\alpha = \frac{1}{n}$ and $\alpha_i = \frac{1}{n}$, $i = 0,\cdots,n$, the equation can be simplified as:

$$\tilde{x}_0(\tilde{A}) = \frac{\int_a^b x\mu_{\tilde{A}}(x)dx}{\int_c^d \mu_{\tilde{A}}(x)dx}$$
$$= \frac{1}{3}\frac{((x)^{2U}_{\alpha_0} - (x)^{2L}_{\alpha_0}) + ((x)^{2U}_{\alpha_n} - (x)^{2L}_{\alpha_n}) + 2\sum_{i=1}^{n-1}((x)^{2U}_{\alpha_i} - (x)^{2L}_{\alpha_i}) + \sum_{i=0}^{n-1}((x)^{U}_{\alpha_i}(x)^{U}_{\alpha_{i+1}} - (x)^{L}_{\alpha_i}(x)^{L}_{\alpha_{i+1}})}{((x)^{U}_{\alpha_0} - (x)^{L}_{\alpha_0}) + ((x)^{U}_{\alpha_n} - (x)^{L}_{\alpha_n}) + 2\sum_{i=1}^{n-1}((x)^{U}_{\alpha_i} - (x)^{L}_{\alpha_i})} \tag{11}$$

If one only takes into account the fuzzy risk priority with single failure mode, the defuzzified centroid values of FWGM RPN for each failure mode can be obtained. The bigger the defuzzified centroid value, the higher would be the overall criticality (i.e. risk) and risk priority [18]. However, this is often the case that there are multiple correlated failure modes in practice. Therefore, how to combine multiple failure modes of complex component will be elaborated in the following.

2.3 Multiple failure modes combination

FTA was originally developed in 1962. Since then, the theory and the application of FTA have been developed rapidly and are often used as a failure analysis tool in reliability engineering [19]. In practice, FTA is often combined with FMEA/FMECA to effectively connect multiple failure modes for quantitative reliability analysis.

When fault tree is apply to analyze the reliability of a complex system consisting of n multiple correlated failure modes, if repeated bottom events and multiple failure modes are assumed to be independent of each mode, the probability of top event should be calculated by MCS method instead of by direct probability method. This is because the fault tree of any monotone coherent system can be translated into a bunch of MCS only including AND gate, OR gate and bottom events, and a top event can be regard as a OR gate, and MCS can be viewed as a AND gate. Whether these inputs are independent or dependent failure modes, after a bunch of MCS were derived, top event can be transformed into multiple failure modes only containing MCS. Therefore, the fault tree of a complex system becomes a parallel-series structure system [20].

Each MCS is combined by AND operation. This means that multi-single failure modes can be connected by AND operation using Boolean logic. The calculation rules of AND gate and OR gate connection is redefined as

$$\text{AND:} \quad \text{FRPN} = \prod_{i=1}^{m} \text{FRPN}_i \tag{12}$$

$$\text{OR:} \quad \text{FRPN} = \sum_{i=1}^{m} \text{FRPN}_i \tag{13}$$

Hence, FRPN considering MCS with multiple failure modes in complicated system can be derived using defined AND operation of FRPN.

In this section, the basic flow and steps to derive the FRPN of multiple failure modes is:

1. Establish FMECA table and FTA diagram for complex system. MCS is obtained by analyzing the inherent logical relationship of FTA, as shown in Table 1 and Fig. 1 of Section 4.1.
2. Evaluate the fuzzy ratings of each single failure mode on three risk factors and fuzzy factor weights ratings to derive fuzzy assessment information on single failure mode by FMECA team members, which is described in Section 2.1 and shown in Table 2.
3. Adopt interval number matrix algorithm to derive fuzzy comprehensive assessment information of single failure mode and relative importance weights of three risk factors, as expressed by Eqs. (1)–(6) in Section 2.1 and shown in Table 3.
4. The FWGM algorithm is utilized to define FRPN of single failure mode, and BASA is used to derive the minimum and maximum value of FWGM RPN by setting different α-level of FWGM RPN, as expressed in Eqs. (7)–(10) and shown in Table 4.
5. Using AND operation of Boolean logic to derive FRPN of MCS with multiple failure modes, as shown in Section 2.3 and Table 5.

In the following section, failure probability with correlated failure modes will be considered during the derivation of new RPN rankings.

3 FPWGM RPN based on Copula and MCS

Although multiple failure modes are effectively connected using MCS and Boolean operation, traditional probability weighted RPN assessment in FMECA is mostly used to solve the problem that multiple failure modes are mutually independent, and it is impossible to solve the probability weighted RPN analysis issue with correlated failure mode. At the same time, uncertainty can not be taken into account in the RPN analysis. Fortunately, Copula function is a powerful tool to explore the correlation structure among random variables, which has been applied to correlate failure modeling such as the common failure of redundant structure system and reliability analysis [21] and the reported results are quite encouraging. Although Copula theory has been applied in above field, its application to fatigue FMECA and MCS of FTA has never been seen, in addition, its application for rotor of aero-engine to analyze fatigue fracture failure has not been reported either.

3.1 Definition and properties of *n*-Copula functions

Copula theory was originally proposed by Sklar in 1959 [22]. Nelson [23] gave a rigorous definition for Copula function, i.e., it is a linking function that connect the joint distribution function $F(x_1,x_2,\cdots,x_n)$ of multidimensional random variables $X_1,X_2,\cdots,X_n$ and respective marginal distribution function $F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)$, namely, have a function $C(u_1,u_2,\cdots,u_n)$, let

$$F(x_1,x_2,\cdots,x_n)=C[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)] \tag{14}$$

Suppose $F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)$ is respectively one-dimensional continuous distribution function, and obey uniform distribution on [0, 1]. If there exists a n-Copula function, namely satisfying following properties' function $C(F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n))=C(u_1,u_2,\cdots,u_n)$ ($u_1=F_{X_1}(x_1)$, $u_n=F_{X_n}(x_n)$), and satisfy

(1) Definition domain is I_n, namely $[0,1]^n$;
(2) For each u in definition domain I_n, $C(u_1,u_2,\cdots,u_n)$ exists zero base surface (i.e. there is $F(x_i)\in[0,1]$, $i=1,2,\cdots,n$, make $C[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)]=0$), and is n-dimensional increments.
(3) $C(u_1,u_2,\cdots,u_n)$ has marginal distribution function $C_i(u_i)$ ($i=1,2,\cdots,n$), and satisfying

$$C_i(u_i)=C(1,\cdots,1,u_i,1,\cdots,1)=u_i \tag{15}$$

where $u_i\in[0,1]$ ($i=1,2,\cdots,n$), then $C(F_{X_1}(x_1), F_{X_2}(x_2),\cdots,F_{X_n}(x_n))=C(u_1,u_2,\cdots,u_n)$ is called as a n-Copula function of $F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)$, namely $u_1,u_2,\cdots,u_n$ [24].

According to Sklar Theorem: there is a Copula function $C[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)]$, satisfying $F(x_1,x_2,\cdots,x_n)= C[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)]$. If $[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)]$ is continuous, then $C[F_{X_1}(x_1),F_{X_2}(x_2),\cdots,F_{X_n}(x_n)]$, i.e., $F(x_1,x_2,\cdots,x_n)$ is uniquely determined.

3.2 System reliability modeling based on Copula function

According to the previous definition, determining $R_i(t_i)$, $i=1,2,\cdots,n$ of each subsystem, here $R_i(t_i)$ is continuous, then a Copula function $C(R_1(t_1),R_2(t_2),\cdots,R_n(t_n))$ can be uniquely identified. Let $R(t_1,t_2,\cdots,t_n)=C(R_1(t_1),R_2(t_2),\cdots, R_n(t_n))$, where, $R(t_1,t_2,\cdots,t_k)$ is the joint distribution function of $R_i(t_i)$, $i=1,2,\cdots,k$, namely reliability function of k dependent subsystems. Considering the fact that some failure modes are mutually independent, while others are not, the reliability can be expressed as

$$R(t)=\prod_{i=1}^{m}C_i(t)\prod_{j=1}^{n}R_j(t) \tag{16}$$

In Eq. (16), $C_i(t)$ is the comprehensive reliability for ith group fault related subsystem, $R_j(t)$ is the reliability for jth independent subsystem, and this formula establish the relationship between the system reliability and the subsystem reliability.

According the definition of Copula function and Sklar theorem, the commonly used Copula functions are Gaussian Copula, t-Copula and Archimedes Copula function, in which, Gumbel, Clayton and Frank Copula are three types of popularly used Archimedes Copula function. Gumbel and Clayton Copula function are respectively adapted for positive correlation and negative correlation, while Frank Copula can be used to the two kinds of cases. Since the most mechanical systems are serially connected as shown in OR gate for MCS of FTA, the service life relationships between every component are positively correlated, i.e. during the service time of mechanical systems, the strength degradation of one component often occurs with the strength degradation of another component. For example, vibration agitated by tail gas and trembling vibration often cause the bending vibration of turbine blades. Hence the trend of failure that is greater than the two mutual independent failures. Gumbel function is more suitable for these mechanical systems, and it can be expressed as

$$C(u_1,u_2,\cdots,u_n;\theta)=\exp(-[\sum_{i=1}^{n}(-\ln u_i)^{1/\theta}]^{\theta}),\quad \theta\in(0,1] \tag{17}$$

In this formula, θ is a random variable, when θ is closer to 0, the stronger correlation is, until completely linear correlation. When θ is closer to 1, the weaker correlation is, until $\theta=1$, namely failure probabilities for n subcomponents are mutually independent.

As fault tree is composed of a series of MCS and each MCS consist of a parallel of bottom events, it is necessary to discuss Copula proposition when multiple correlated failure modes are series or parallel. He et al. [25] suppose a system consist of n dependent component or subsystem or MCS. Let the life of ith unit is T_i, then $F_i(t)$ is the distribution function of T_i, and reliability is $R_i(t)=P(T_i>t)=1-F_i(t)$, $i=1,2,\cdots,n$, in which, the life of series system is $T=\min(T_1,T_2,\cdots,T_n)$, the joint distribution function of $T_1,T_2,\cdots,T_n$ is $F(t_1,t_2,\cdots,t_n)=P\{T_1\le t_1,T_2\le t_2,\cdots,T_n\le t_n\}$.

According to Sklar theorem, if there exists an n-dimensional Copula function C, making $F(t_1,t_2,\cdots,t_n)=C^n(F_1(t_1),F_2(t_2),\cdots,F_n(t_n))$. As $F_i(t)$ is continuous, $C^n(F_1(t_1),F_2(t_2),\cdots,F_n(t_n))$ is a unique function. Consequently, reliability model of a series system based on Copula function can be expressed as

$$\begin{aligned} R(t) &= P(\min(T_1,T_2,\cdots,T_n)>t)=P(T_i>t)\\ &=1-\sum_{i=1}^{n}F_i(t)+(-1)^k\\ &\quad\times\sum_{1\le i_1<i_2<\cdots i_k\le n}C(F_{i1}(t),F_{i2}(t),\cdots,F_{ik}(t))\\ &=1-\sum_{i=1}^{n}F_i(t)+(-1)^k\\ &\quad\times\sum_{1\le i_1<i_2<\cdots i_k\le n}C^n(F_{i1}(t),F_{i2}(t),\cdots,F_{ik}(t),\underbrace{1,1,\cdots,1}_{\text{others } n-k}) \end{aligned} \tag{18}$$

In the formula, $2\le k\le n$. At the same principle, the reliability model of parallel system based on Copula function is

$$\begin{aligned} R(t) &= P(\max(T_1,T_2,\cdots,T_n)>t)\\ &=1-P(\max(T_1,T_2,\cdots,T_n)\le t)\\ &=1-C^n(F_1(t),F_2(t),\cdots,F_n(t))\\ &=1-C^n(1-R_1(t),1-R_2(t),\cdots,1-R_n(t)) \end{aligned} \tag{19}$$

When each unit is independent of one another, $R(t)=1-\prod_{i=1}^{n}(1-R_i(t))$, this is identical with reliability model of parallel system assumed as independent.

By constructing a suitable Copula function, we can establish reliability model of system based on Copula function. The reliability of system when failure modes of MCS are correlated can be accurately computed.

3.3 FPWGM RPN algorithm based on Copula and MCS

As is known to all, probability importance of failure is widely applied in FTA, more details are available in [6, 26], and the importance of some events is greater than the others, which is mainly because they contribute more to the occurrence probability of the top events. Thus, to determine reliability index for importance of failure modes, in this paper, FWGM RPN is multiplied by probability weight $f(W_i)$ to derive a new defined Fuzzy Probability Weighted Geometric Mean RPN (FPWGMRPN) by considering the importance of failure probability and assessing their impact on system reliability.

$$\begin{aligned} \mathrm{FPWGMRPN}_i &= \mathrm{FRPN}_i\times f(W_i)\\ &=[(\tilde{R}_i^O)^{\tilde{w}^O/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)}\times(\tilde{R}_i^S)^{\tilde{w}^S/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)}\\ &\quad\times(\tilde{R}_i^D)^{\tilde{w}^D/(\tilde{w}^O+\tilde{w}^S+\tilde{w}^D)}]\times f(W_i) \end{aligned} \tag{20}$$

where W_i is the importance of ith MCS in system, while $f(W_i)$ is the function of variable W_i.

In addition, by utilizing Linear Interval Mapping (LIM), 0 is mapped to 0 and 0.01 is mapped to 1. Assume $x\in[0,0.01]$, $y\in[0,1]$, the mapping between interval [0,1] and interval [0,0.01] is $y = 100 * x$. Here x can be obtained using Copula function to derive failure probability W_i of MCS with multiple correlated failure modes, and y is failure probability weight $f(W_i)$ obtained by LIM in Eq. (20).

$f(W_i)$ is applied to evaluate FWGM RPN for the derived FPWGM RPN based on Copula and MCS by considering the correlation of multiple failure modes.

4 Case study

An example in fatigue failure of FMECA and FTA for aero-engine turbine and compressor blades is illustrated. In this study, the proposed FPWGM RPN algorithm based on Copula and MCS is utilized to perform FRPN quantitative evaluation with multiple failure modes.

4.1 FMECA and FTA

The failure modes for turbine and compressor blades mainly consist of the fatigue failure caused by bending stress results from centrifugal forces; the trembling vibration and torsional resonance as well as bending vibration caused by vibration environment; high temperature fatigue, fretting fatigue and corrosion damage caused by environmental media and the contact state. Because of the complexity of service environment for turbine and compressor blades, the actual fatigue for blades is not one of theses modes, but rather a superposition of several types of failure modes, namely multi-reasons result in "composite" fatigue failure, such as fatigue failure caused by low cycle fatigue, trembling vibration and/or torsional resonance; fatigue failure caused by corrosion fatigue and high cycle fatigue for blade roots [3]. At present, reliability assessment in FMECA for an aero-engine is basically carried out for non-repairable components. Hence, in RPN study, the weight of detection is smaller than repairable components. The categories of severity are determined according to the effect of fault on some agreed levels and are divided into four categories: catastrophic I, fatal II, criticality III, mild IV [10]. According to the specific characteristics of turbine and compressor blades, severity is assigned with larger weights, FMECA and assumed failure mode life distribution [28] of the fatigue failures of turbine and compressor blades are shown in Table 1. Moreover, FMECA would be used to information assessment on eight failure modes in Table 2 in Section 4.2, and life distribution would be utilized to Section 4.3 for failure probabilities calculation with correlated failure modes.

The reliability of an aero-engine is based on the reliability of unit or components. In other words, only when the reliability of turbine engine unit reaches its design target, then the turbine engine can be operated under high reliability conditions. Hence, the correlation of failure mode for each component plays a crucial role on engine reliability. Considering the correlation between individual failure modes, the basic fatigue failure modes for compressor blade and turbine rotor blade can be regarded as a fault tree, as shown in Fig. 1. MCS is obtained by Semanderes algorithm, in which, the top event occurs when any one of MCS which leads to fatigue happened and it needs to be repaired. Meanwhile, the correlated failure mode of each MCS occurs, will cause MCS occur. So, the fatigue failure of whole compressor blade or turbine blade can be regarded as a series-parallel system consisting of MCS.

The top event of failure modes for turbine and compressor blades fatigue is $T = x_1x_7 + x_1x_2x_4 + x_3x_4x_8 + x_5x_7 + x_6$. There are five MCS, they are $\{x_1x_7\},\{x_1x_2x_4\},\{x_3x_4x_8\}$, $\{x_5x_7\},\{x_6\}$, respectively.

4.2 The method to calculate FWGM RPN

Based on those eight failure modes listed in Table 1, a FMECA team consists of five cross functional team members identifies and prioritizes these modes in terms of their three risk priorities so that a initiative corrected action can be taken against the high priority issues. On account of the difficulty in precisely assessing the risk factors and their relative importance weights, the FMECA team members decide to evaluate them using fuzzy language. The five team members from different departments are assumed to be of different importance due to their different field episteme and expertise. To represent their differences in performing FMECA, these five members are assigned with the following relative weights: $h_1 = 10\%$, $h_2 = 25\%$, $h_3 = 30\%$, $h_4 = 15\%$, $h_5 = 20\%$. Based on the ranking of the five team members, we derive interval number matrix fuzzy assessment information of eight failure modes and relative importance weights of three risk factors are shown in Table 3 using Eqs. (1)–(6).

The FWGM and BASA are applied to calculate the FRPNs of the eight failure modes, where the α-levels are set as 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. The results are presented in Table 4.

To rank each failure mode, computing the deffuzified centroid of every failure mode, as shown in the last row but one of Table 4 using Eq. (11). From Fig. 2, it should be noted that FM1 with the least overall risk and FM3 with the maximum overall risk should be given the top risk priority. The overall priority order is FM1 < FM6 < FM5 < FM7 < FM8 < FM2 < FM4 < FM3. Failure modes with dotted line represent the highest and lowest risk, respectively.

4.3 Failure probabilities weighted models based on Copula function

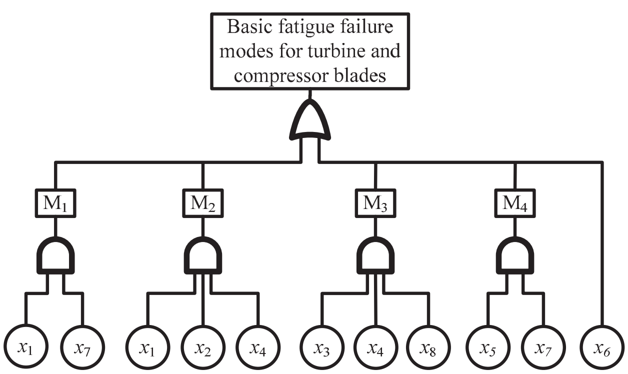
Generally, according to the strength degradation for mechanical systems, the failure probability of components for random failures should be a monotone increasing function of service time t instead of the exponential distribution function. Therefore, the exponential-based modeling method can not be described for the aforementioned degradation case. However, Weibull distribution can describe the monotone increasing or decreasing trend of failure rate and its flexibility is higher than many other distributions [27]. We take the reliability analysis of life distribution as an example, assuming that the life distributions of compressor blade or turbine rotor blade are known, for the fracture failure mode of low cycle fatigue for blade, as shown in the last column in Table 1, failure probability of two parameter Weibull distribution can be estimated [28] by Eq. (21).

Table 1: FMECA of fatigue for turbine and compressor blades

Component	Failure modes	Failure cause	Failure effects on local	Failure effects on system	Severity rank	Failure modes life distribution
Aero-engine turbine and compressor blades	Low cycle fatigue fracture	The dangerous section appears plastic zone if local or total stress of dangerous section for blade approach or exceed the yield strength of materials; or there is a wide range of serious regional defect around dangerous section	Engine damage, engine stall in the air	Aircraft can not be normally operated	IV	$1-e^{-(t/3333)^{3.13}}$
	Trembling vibration fatigue for compressor blade	Poor coupled aerodynamically design for compressor blade lead to blade take alternate loads caused by extra trembling vibration besides take centrifugal loads and aerodynamic loads	Engine damage	Aircraft can not be normally operated	II	$1-e^{-(t/2570)^{3.976}}$
	Bending vibration fatigue	High cycle fatigue failure is more common, in which, the most common fatigue failure caused by the first order bending vibration.	Overall engine damage	Damage aircraft structure, very easily cause catastrophic aircraft accidents	I	$1-e^{-(t/2340)^{4.386}}$
	Reversal resonance fatigue	Failure is usually high cycle fatigue, appear reversal resonance or pitting corrosion for blade surface, or suffer from external object blow so as to damage.	Engine damage is caused by loss corner fracture failure of reversal resonance pitch line	Endanger the safety of flight	III	$1-e^{-(t/3000)^{3.566}}$
	High temperature fatigue and thermal fatigue	Creep damage and fatigue damage are induced by alternating temperature and alternating stress under high temperatures.	Engine damage	Aircraft can not be normally operated	III	$1-e^{-(t/3132)^{3.426}}$
	Fretting fatigue	Fatigue failures are caused by fretting wear for the shroud of shrouded blade and fretting damage of the joint surface of rotor blade and turbine disk serration.	Engine damage	Aircraft can not be normally operated	IV	$1-e^{-(t/4165)^{3.4}}$
	Corrosion fatigue	Pitting corrosion, intercrystalline corrosion, stress corrosion and high temperature corrosion easily cause fatigue.	Engine damage	Aircraft can not be normally operated	IV	$1-e^{-(t/3563)^{3.32}}$
	Blade serration fatigue	Fretting fatigue and big local stress cause fatigue	Engine damage	Greatly threaten flight safety, easily cause a major accident	II	$1-e^{-(t/2390)^{4.03}}$

Table 2: Assessment information on eight failure modes by five FMECA team members

Risk factors	Factor weights	The *i*th failure modes fuzzy number							
		1	2	3	4	5	6	7	8
Occurrence	TM1:(0.25,0.5,0.75)	(1,2,3,4)	(6,7,8,9)	(6,7,8,9)	(6,7,8,9)	(3,4,6,7)	(8,9,10,10)	(6,7,8,9)	(3,4,6,7)
	TM2:(0.5,0.75,1)	(3,4,6,7)	(3,4,6,7)	(8,9,10,10)	(8,9,10,10)	(1,2,3,4)	(6,7,8,9)	(8,9,10,10)	(1,2,3,4)
	TM3:(0.5,0.75,1)	(1,2,3,4)	(6,7,8,9)	(8,9,10,10)	(6,7,8,9)	(3,4,6,7)	(8,9,10,10)	(6,7,8,9)	(1,2,3,4)
	TM4:(0.25,0.5,0.75)	(1,2,3,4)	(6,7,8,9)	(8,9,10,10)	(6,7,8,9)	(1,2,3,4)	(6,7,8,9)	(6,7,8,9)	(3,4,6,7)
	TM5:(0.75,1,1)	(1,2,3,4)	(3,4,6,7)	(6,7,8,9)	(6,7,8,9)	(1,2,3,4)	(6,7,8,9)	(6,7,8,9)	(3,4,6,7)
Severity	TM1:(0.75,1,1)	(5,6,7)	(6,7,8)	(8,9,10)	(6,7,8)	(4,5,6)	(1,2,3)	(2,3,4)	(6,7,8)
	TM2:(0.5,0.75,1)	(4,5,6)	(6,7,8)	(8,9,10)	(5,6,7)	(5,6,7)	(1,2,3)	(3,4,5)	(7,8,9)
	TM3:(0.5,0.75,1)	(3,4,5)	(7,8,9)	(9,10,10)	(6,7,8)	(4,5,6)	(2,3,4)	(2,3,4)	(7,8,9)
	TM4:(0.75,1,1)	(6,7,8)	(6,7,8)	(7,8,9)	(5,6,7)	(5,6,7)	(1,2,3)	(3,4,5)	(7,8,9)
	TM5:(0.75,1,1)	(5,6,7)	(8,9,10)	(7,8,9)	(6,7,8)	(4,5,6)	(1,2,3)	(1,2,3)	(7,8,9)
Detection	TM1:(0,0.25,0.5)	(4,5,6)	(4,5,6)	(4,5,6)	(3,4,5)	(4,5,6)	(4,5,6)	(3,4,5)	(1,1,2)
	TM2:(0,0.25,0.5)	(4,5,6)	(4,5,6)	(3,4,5)	(3,4,5)	(3,4,5)	(3,4,5)	(2,3,4)	(1,2,3)
	TM3:(0.25,0.5,0.75)	(5,6,7)	(5,6,7)	(3,4,5)	(4,5,6)	(4,5,6)	(4,5,6)	(2,3,4)	(2,3,4)
	TM4:(0,0.25,0.5)	(4,5,6)	(4,5,6)	(3,4,5)	(5,6,7)	(5,6,7)	(4,5,6)	(2,3,4)	(3,4,5)
	TM5:(0,0.25,0.5)	(3,4,5)	(5,6,7)	(2,3,4)	(4,5,6)	(4,5,6)	(4,5,6)	(1,2,3)	(2,3,4)



x_1:Low cycle fatigue failure; x_2:Trembling vibration fatigue failure for turbine and compressor blades ; x_3:Bending vibration fatigue failure; x_4:Reversal resonance fatigue failure; x_5:High temperature fatigue and thermal fatigue failure; x_6:Fretting fatigue failure; x_7:Corrosion fatigue failure; x_8:Blade serration fatigue failure

Fig. 1: Basic fatigue failure for turbine and compressor blades

$$F(t)=1-\exp[-(\frac{t}{\eta})^m]=1-\exp[-(\frac{t}{3333})^{3.13}] \tag{21}$$

where η is the scale parameter, M is the shape parameter, and the same principle can be applied to the rest failure probability distribution of fatigue failure modes, as is shown in Table 1. The Gumbel Copula function $C(u_1,u_2,\cdots,u_n;\theta)=\exp(-[\sum_{i=1}^{n}(-\ln u_i)^{1/\theta}]^\theta)$, $\theta\in(0,1]$ is more suitable for mechanical system in the Archimedes function group than the linking function.

For MCS, because the failure mode x_1 and x_7 is connected by AND gate, when the failure probability of two correlated failure modes is calculated using Gumbel algorithm of parallel Copula function, we assume that $\theta=0.4$, when x_1 and x_7 is correlated, by the formula $R(t)=1-C^n(1-R_1(t),1-R_2(t),\cdots,1-R_n(t))$, the reliability can be calculated by

$$\mathrm{R}_{1,7}(t)=1-e^{(-[(-\ln(1-e^{-(t/3333)^{3.13}}))^{2.5}+(-\ln(1-e^{-(t/3563)^{3.32}}))^{2.5}]^{0.4})} \tag{22}$$

When the life is about 1000 hours, to calculate the reliability of blade fracture, $t=1000$ is substituted into Eq. (22) to derive reliability $\mathrm{R}_{1,7}(t)=0.9132$. Meanwhile, the reliability of x_1 and x_7 are assumed as independent and is $R(t)=0.9750$. Therefore, it can be observed from calculated results that assuming independent reliability is bigger than that with considering correlation of n different blade fatigue fault, in other words, the failure probability with considering correlation is bigger than independent. Therefore, the previous failure probability has a certain error if the correlation of each failure mode is not taken into account. Similarly, when x_1, x_2 and x_4 are correlated, the reliability is

$$\begin{aligned}&\mathrm{R}_{1,2,4}(t)\\&=1-e^{(-[(-\ln(1-e^{-(t/3333)^{3.13}}))^{2.5}+(-\ln(1-e^{-(t/2570)^{3.976}}))^{2.5}+(-\ln(1-e^{-(t/3000)^{3.566}}))^{2.5}]^{0.4})}\end{aligned} \tag{23}$$

If substitute $t=1000$ into Eq. (23), then $\mathrm{R}_{1,2,4}(t)=0.9974$. And failure correlated reliability of x_3, x_4 and x_8 is

$$\begin{aligned}&\mathrm{R}_{3,4,8}(t)\\&=1-e^{(-[(-\ln(1-e^{-(t/2340)^{4.386}}))^{2.5}+(-\ln(1-e^{-(t/3000)^{3.566}}))^{2.5}+(-\ln(1-e^{-(t/2390)^{4.03}}))^{2.5}]^{0.4})}\end{aligned} \tag{24}$$

Table 3: Interval number matrix fuzzy assessment information of eight failure modes and relative importance weights of three risk factors

Failure modes	Occurrence	Severity	Detection
1	$\tilde{R}_1^O=(1.5,2.5,3.75,4.75)$	$\tilde{R}_1^S=(4.3,5.3,6.3)$	$\tilde{R}_1^D=(4.1,5.1,6.1)$
2	$\tilde{R}_2^O=(4.65,5.65,7.1,8.1)$	$\tilde{R}_2^S=(6.7,7.7,8.7)$	$\tilde{R}_2^D=(4.5,5.5,6.5)$
3	$\tilde{R}_3^O=(7.4,8.4,9.4,9.7)$	$\tilde{R}_3^S=(7.95,8.95,9.65)$	$\tilde{R}_3^D=(2.9,3.9,4.9)$
4	$\tilde{R}_4^O=(6.5,7.5,8.5,9.25)$	$\tilde{R}_4^S=(5.6,6.6,7.6)$	$\tilde{R}_4^D=(3.8,4.8,5.8)$
5	$\tilde{R}_5^O=(1.8,2.8,4.2,5.2)$	$\tilde{R}_5^S=(4.4,5.4,6.4)$	$\tilde{R}_5^D=(3.9,4.9,5.9)$
6	$\tilde{R}_6^O=(6.8,7.8,8.8,9.4)$	$\tilde{R}_6^S=(1.3,2.3,3.3)$	$\tilde{R}_6^D=(3.75,4.75,5.75)$
7	$\tilde{R}_7^O=(6.5,7.5,8.5,9.25)$	$\tilde{R}_7^S=(2.2,3.2,4.2)$	$\tilde{R}_7^D=(1.9,2.9,3.9)$
8	$\tilde{R}_8^O=(1.9,2.9,4.35,5.35)$	$\tilde{R}_8^S=(6.9,7.9,8.9)$	$\tilde{R}_8^D=(1.8,2.7,3.7)$
Importance weights	$\tilde{w}^O=(0.4875,0.7375,0.9375)$	$\tilde{w}^S=(0.6125,0.8625,1)$	$\tilde{w}^D=(0.075,0.325,0.575)$

Table 4: Computed α-level sets and defuzzified centroid of the FRPNs for eight failure modes

α	Failure modes							
	1	2	3	4	5	6	7	8
0	[2.337,5.840]	[5.121,8.390]	[5.508,9.432]	[5.119,8.407]	[2.613,6.004]	[2.292,6.148]	[2.728,6.601]	[2.715,7.280]
0.1	[2.504,5.719]	[5.251,8.249]	[5.726,9.295]	[5.269,8.249]	[2.770,5.857]	[2.487,5.997]	[2.884,6.379]	[2.880,7.052]
0.2	[2.670,5.597]	[5.382,8.110]	[5.942,9.155]	[5.418,8.093]	[2.927,5.767]	[2.680,5.799]	[3.041,6.162]	[3.047,6.830]
0.3	[2.835,5.475]	[5.514,7.973]	[6.156,9.012]	[5.566,7.938]	[3.082,5.623]	[2.872,5.614]	[3.200,5.949]	[3.217,6.615]
0.4	[2.999,5.353]	[5.647,7.837]	[6.368,8.866]	[5.712,7.785]	[3.236,5.528]	[3.061,5.440]	[3.361,5.742]	[3.390,6.405]
0.5	[3.161,5.230]	[5.780,7.703]	[6.578,8.717]	[5.857,7.633]	[3.388,5.393]	[3.248,5.242]	[3.523,5.538]	[3.565,6.200]
0.6	[3.321,5.107]	[5.915,7.570]	[6.785,8.565]	[6.002,7.482]	[3.539,5.280]	[3.433,5.067]	[3.687,5.338]	[3.744,6.001]
0.7	[3.480,4.984]	[6.050,7.439]	[6.990,8.410]	[6.145,7.333]	[3.689,5.166]	[3.616,4.890]	[3.853,5.143]	[3.925,5.806]
0.8	[3.638,4.860]	[6.186,7.309]	[7.193,8.252]	[6.287,7.184]	[3.837,5.053]	[3.796,4.711]	[4.021,4.950]	[4.110,5.615]
0.9	[3.794,4.736]	[6.323,7.181]	[7.394,8.091]	[6.428,7.037]	[3.985,4.939]	[3.975,4.530]	[4.190,4.762]	[4.298,5.427]
1	[3.949,4.612]	[6.461,7.052]	[7.592,7.926]	[6.569,6.891]	[4.130,4.825]	[4.151,4.347]	[4.362,4.576]	[4.489,5.244]
Deffuizified centroid	4.170	6.746	7.596	6.750	4.373	4.243	4.572	4.915
Priority ranking	8	3	1	2	6	7	5	4

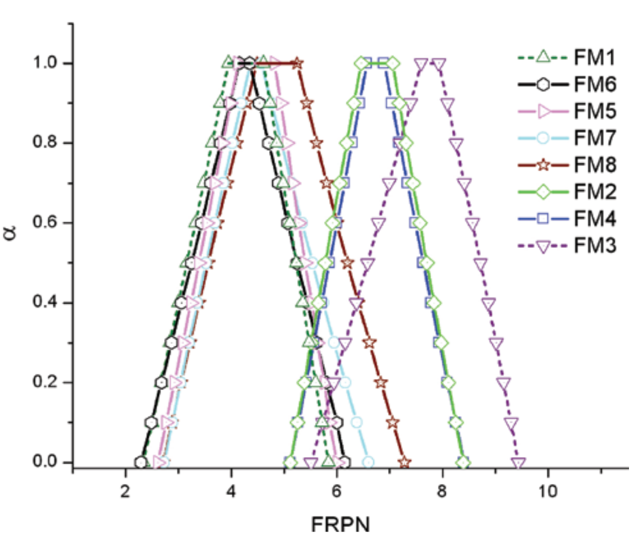


Fig. 2: FRPNs of eight failure modes

When $t=1000$, we can derive $R_{3,4,8}(t)=0.9970$. By the same way, when x_5 and x_7 are correlated, the reliability is

$$R_{5,7}(t)=1-e^{(-[(-\ln(1-e^{-(t/3132)^{3.426}}))^{2.5}+(-\ln(1-e^{-(t/3563)^{3.32}}))^{2.5}]^{0.4})} \tag{25}$$

When $t=1000$, we derive $R_{5,7}(t)=0.9954$. When only have x_6 bottom event, reliability is expressed as $R_6(t)=1-e^{(-[(-\ln(1-e^{-(t/4165)^{3.4}}))^{2.5}]^{0.4})}$, i.e. when the life $t=1000$ for blade serration fatigue, $R_6(t)=0.9922$ is obtained.

4.4 FPWGM RPN evaluation and analysis

In multiple failure mode assessment, based on Zadeh's extension principle [29], trapezoidal fuzzy numbers are calculated. The new FMECA report and FRPN of MCS for

Table 5: New FMECA report

Failure mode	Potential failure mode	Potential failure cause	FRPN
Top event occurrence	$\{x_1,x_7\}$	x_1	(6.376,17.222, 21.105,38.550)
		x_7	
	$\{x_1,x_2,x_4\}$	x_1	(61.260,167.576,224.136,411.936)
		x_2	
		x_4	
	$\{x_3,x_4,x_8\}$	x_3	(76.561,223.866,286.414,577.246)
		x_4	
		x_8	
	$\{x_5,x_7\}$	x_5	(7.128,18.016,22.076,39.631)
		x_7	
	$\{x_6\}$	x_6	(2.292,4.151,4.347,6.148)

Table 6: New RPN rankings

Potential failure mode	Old RPN	Old RPN rank	Failure probability weights	New RPN	New RPN rank
$\{x_1,x_7\}$	20.741	4	5	103.704	3
$\{x_1,x_2,x_4\}$	215.232	2	2.6	559.604	2
$\{x_3,x_4,x_8\}$	289.791	1	3	869.372	1
$\{x_5,x_7\}$	21.609	3	4.6	99.401	4
$\{x_6\}$	4.243	5	7.8	33.098	5

fatigue failure of blades are shown in Table 5 by using Boolean operation from Eqs. (12) and (13).

The centroid value of each FRPN listed in Table 5 is calculated [16] and regarded as the old RPN. The new RPN and their rankings are derived by multiplying failure probability weight, as shown in Table 6.

It can be observed from Table 6 that new risk priority ranking has changed after considering relatively probability weight, in which, the risk priority ranking of MCS $\{x_1, x_7\}$ rise from four to three, namely, the correlative risk of x_1 and x_7 failure modes increase after considering failure probability weight of MCS based on Copula function. Instead, the risk of MCS $\{x_5, x_7\}$ has reduced. This is more consistent with the actual situation that the blade fatigue caused by high temperature and thermal stress is seldom seen, however, corrosion is a more common failure mode.

5 Conclusions

In this paper, Copula theory has been successfully incorporated into FMECA and MCS theory to quantitative evaluate FRPN under multiple failure modes. Compared to the case where only single or independent failure mode is considered, the proposed quantitative analysis approaches improve the precision of reliability and risk assessment for complex systems. As illustrated by the case study, a new FPWGM RPN method based on Copula and MCS extends the definition and application scope of FRPN. By multiplying a correlated failure probability weighted parameter to characterize the importance of failure cause meanwhile remaining the characteristics of FMECA. The proposed method is applicable not only to the quantitative analysis for turbine and compressor blades, but also to many other complex mechanical systems. Future research will focus on the discussion and quantitative analysis of dynamic FTA considering failure mode correlations.

Nomenclature

FMECA	Failure mode, effects and criticality analysis
RPN	Risk priority number
FTA	Fault tree analysis
MCS	Minimum cut sets
FPWGM	Fuzzy probability weight geometric mean
FRPN	Fuzzy risk priority number
O	Probability of occurrence
S	Severity
D	Detectability
FWGM	Fuzzy weighted geometric mean
BASA	Benchmark adjustment search algorithm
LIM	Linear interval mapping

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