DE GRUYTER DOI 10.1515/tjj-2013-0043 — Int. J. Turbo Jet-Engines 2014; 31(2): 167–174

Hong-Zhong Huang*, Ping-Liang Cui, Weiwen Peng, Hui-Ying Gao and Hai-Kun Wang

Fatigue Lifetime Assessment of Aircraft Engine Disc via Multi-source Information Fusion

Abstract: Fatigue lifetime assessment for aircraft engine discs is an important issue for the operation and health management of aircraft engines. Due to the lack of field test data, traditional methods can hardly meet the requirements of fatigue lifetime assessment of aircraft engine discs. By combining a multi-source information fusion method with a Bayesian inference technique, this paper develops a practical approach for fatigue lifetime assessment of aircraft engine discs. Subjective information and historical data are combined coherently with the sparse test data to generate a credible fatigue lifetime assessment of aircraft engine discs. Methods for quantifying subjective information, checking different experts' information, and fusing multiple prior distributions are presented to facilitate the implementation of fatigue lifetime assessment. An illustrative example is presented to demonstrate the procedures and the implication of the proposed method.

Keywords: fatigue lifetime assessment, aircraft engine disc, Bayesian inference, multi-source information fusion

PACS® (2010). 02.50.Tt

***Corresponding author: Hong-Zhong Huang:** School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China. E-mail: hzhuang@uestc.edu.cn
Ping-Liang Cui, Weiwen Peng, Hui-Ying Gao, Hai-Kun Wang: School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

1 Introduction

Due to the fast development of modern aircraft industry and the continual growth of market demand, aircrafts are putting on the market in an ever-increasing pace. Manufacturers and companies are under great pressure to deliver aircrafts with high safety and reliability under limited time. Moreover, the evolving of customer demands has laid a great imperative for manufacturers and companies to develop new types of aircrafts efficiently. Increased expectation with lower cost and high reliability has become critical issue in aircraft industry. Aircraft engine discs has been recognized as one of the critical components that contribute to the failure of aircrafts. Fatigue lifetime assessment of aircraft engine discs has consequently become a key issue both for the development of aircrafts with high reliability in limited time and for the operation of aircrafts with high safety under limited cost. Commonly, fatigue lifetime assessment of aircraft engine discs is carried out based on the test data gathered from relevant reliability tests. However, due to the high reliability and cost of the aircraft engine discs, it is hard to implement reliability test of the aircraft engine discs with large sample size and long test time. As a result, common methods for fatigue lifetime assessment which are mainly based on the abundance of test data can hardly lead to a credible result. Fatigue lifetime assessment of aircraft engine discs is consequently encountered the difficulty introduced by small sample size and sparse failure data.

In fact, other than the data gathered from reliability test, various types of reliability information can be gathered and integrated for the fatigue lifetime assessment of the aircraft engine discs. Thanks to the continue evolving and the gradual development of the aircraft engines, there are plenty of subjective information and historical data for the reliability analysis of aircraft engines. In detail, for the aircraft engine discs, subjective information can be gathered from experts who design the discs and from the engineers who repair the engines. Moreover, historical data can be obtained from test data of similar discs and from the maintenance records of similar engines. These types of reliability information can be incorporated and combined with the reliability test data to mitigate the difficulty discussed above. Accordingly, a coherent method is needed to implement the subjective information quantification and multi-source information fusion.

To handle the problems introduced by small sample size and spare test data, Bayesian method has been frequently used in fatigue lifetime assessment. A coherent framework can be constructed for the incorporation of multiple sources information using Bayesian method [1–2]. Subjective information and historical data can be quantified and incorporated through the proper derivation of prior distribution. The information contained in the sparse test data is described by the relevant likelihood function. These various types of information are then be combined through the Bayesian formulation. The posterior distribution is obtained based on the prior distributions and likelihood function. Various studies have discussed the reliability analysis with different data types and reliability information using Bayesian method [3–5]. Peng et al. [6] introduced a framework for reliability analysis of multicomponent system by incorporating multilevel heterogeneous data, such as pass/fail data, lifetime data, and degradation data. Reese et al. [7] proposed a Bayesian framework for lifetime assessment of complex system with multiple level lifetime data. Wang et al. [8] introduced a method for reliability assessment and prediction with evolving, insufficient, and subjective data set based on the Bayesian theory. Huang et al. [9] presented a framework for reliability assessment using Bayesian method by considering the situation of fuzzy lifetime data. Liu [10] applied various qualitative and quantitative methods to synthesize multiple sources of information. Liu [11] discussed information pretreatment, including collecting and converting prior information, and methods of distribution type test of prior information.

Based on the literature review presented above, one practical way to handle the issues introduce by the fatigue lifetime assessment of aircraft engine discs is to construct a coherent framework by incorporating the Bayesian theory. In this paper, a theoretically sound model and a practically usable method are build up based on the Bayesian theory for the fatigue lifetime assessment of the aircraft engine discs. Three critical expects are considered in the proposed method, i.e. (1) quantification of subjective information gathered from multiple experts, where a probability encoded method is used, (2) checking of prior consistency for information obtained from different experts, where a common recognition extent is defined, (3) fusion of multiple prior distribution obtained from different sources, where a linear weighted combining method is used. The proposed method is demonstrated with a step-by-step implication to the fatigue lifetime assessment of aircraft engine discs.

The remainder of this paper is presented as follows. Section 2 briefly introduces the multi-source information fusion. Section 3 presents the Bayesian method for the fatigue lifetime assessment with specific detail on the construction of Bayesian model, the discussion of subjective information quantification, multiple experts' information checking, and multiple prior distributions fusion. The proposed method is then demonstrated through a fatigue lifetime reliability assessment of an aircraft engine disc in Section 4. We then conclude the paper in Section 5 with some remarks on the future research.

2 Multi-source information fusion

Multi-source information fusion method was originated from multi-sensor fusion method. The fundamental idea of multi-source information fusion is to combine the information gathered from various sources, where the issues of information quantification, coherency checking, and distribution fusion are handled properly [4]. The implication of multi-source information fusion in fatigue lifetime assessment is aimed to mitigate the insufficient of spare reliability test data by incorporating information obtained from various sources as discussed in the introduction part. In order to obtain more reliable and accurate information than a single source of information, it makes use of different sources of information through a theoretically sound algorithm.

Researchers have done a significant amount of work to apply the information fusion method to analyze the reliability and lifetime of aircraft engine discs [12–14]. Zhang [15] applied information fusion theory to perform accuracy evaluation and reliability assessment of the weapon systems, and proposed an effective solution to the reliability information fusion for similar system in different environments and different conditions. Feng [16] made a comprehensive analysis on reliability information fusion method applied to a complex system with small samples, and developed the weighted fusion method by synthesizing multi-source prior information.

To construct a coherent framework for the fatigue lifetime assessment of aircraft engine discs using the multi-source information fusion, the following aspects need to be handled properly.

1. A practical model is chosen for the modeling of fatigue lifetime of aircraft engine disc. It is chosen based on the fitness of the reality of the disc by the suggestion of experts in that domain or through the goodness-of-fit checking of test data. Moreover, to facilitate the implementation of multi-source information fusion, a parametric model is preferable than a non-parametric model, especially for the one with practical meaning of model parameters such as Weibull model [17].
2. A usable method is needed for the quantification of subjective information gathered from experts. Since the subjective information can be presented in various forms, a method is needed to unify the procedure for

subjective information derivation [18]. Moreover, probability distribution for the model parameters chosen above can be properly derived using this method.

3. A proper method is required for the consistency checking of information gathered from multiple experts. The information obtained from different experts can be consistent or conflict with each other. Moreover, some information from particular experts is more valuable than the others. As a result, a proper method should be introduced to handle the issue of consistency checking.
4. An efficient method is needed for the fusion of multiple probability distributions. The ultimate target for the implication of multi-source information fusion for the fatigue lifetime assessment is to obtain the joint probability distribution for model parameters chosen above. In detail, different probability distributions need to be compressed into a joint probability distribution. A criteria needs to be introduced for the evaluation of the importance of the distributions for the joint distribution. Moreover, the efficiency of generation samples from this joint distribution needs to be guaranteed for the calculation of Bayesian inference.

Based on the discussion about multiple information fusion highlighted above, the proposed method for the fatigue lifetime assessment of aircraft engine discs is presented step by step in the following sections.

3 Reliability assessment of aircraft engine discs

3.1 Bayesian model

Bayesian method is used to integrate the subjective information and historical data with the sparse lifetime data which come from reliability test with limited sample size and test time. The fundamental principle of Bayesian method can be described as

$$P_{\theta|T}\left(\theta|T\right)=\frac{\pi_{\theta}\left(\theta\right)l_{T|\theta}\left(T|\theta\right)}{\int\limits_{\theta}\pi_{\theta}\left(\theta\right)l_{T|\theta}\left(T|\theta\right)\mathrm{d}\theta} \tag{1}$$

where θ are parameters of the model chosen for the fatigue lifetime modeling of an aircraft engine disc. T denotes the sparse lifetime data obtained from reliability test of the aircraft engine disc. $\pi_{\theta}\left(\theta\right)$ is the prior distribution for model parameters. The multi-source prior information are quantified and formulated as this prior distribution. The information contained in the sparse lifetime T is delivered through the likelihood function $l_{T|\theta}\left(T|\theta\right)$, which is obtained based on the probability density function of the chosen fatigue lifetime model. All of the information is integrated and presented in the joint posterior distribution of model parameters $P_{\theta|T}\left(\theta|T\right)$. The fatigue lifetime assessment and posterior analysis are based on this joint posterior distribution.

For the fatigue lifetime of an aircraft engine disc, a two-parameter Weibull distribution is generally chosen based on experts' testimony and historical experience. The corresponding Bayesian model is given as:

$$\begin{aligned} &P_{\alpha,\beta|t}\left(\alpha,\beta|T\right)\\ &=\frac{\pi_{\alpha,\beta}\left(\alpha,\beta\right)L_{t|\alpha,\beta}\left(T|\alpha,\beta\right)}{\int\limits_{\alpha,\beta}\pi_{\alpha,\beta}\left(\alpha,\beta\right)L_{t|\alpha,\beta}\left(T|\alpha,\beta\right)\mathrm{d}\alpha\mathrm{d}\beta}\\ &=\frac{\pi_{\alpha,\beta}\left(\alpha,\beta\right)\prod\limits_{i=1}^{n}\left\{\frac{\beta}{\alpha}\left(\frac{t_i}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{t_i}{\alpha}\right)^{\beta}\right]\right\}}{\int\limits_{\alpha,\beta}\pi_{\alpha,\beta}\left(\alpha,\beta\right)\prod\limits_{i=1}^{n}\left\{\frac{\beta}{\alpha}\left(\frac{t_i}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{t_i}{\alpha}\right)^{\beta}\right]\right\}\mathrm{d}\alpha\mathrm{d}\beta} \end{aligned} \tag{2}$$

where $P_{\alpha,\beta|t}(\alpha,\beta|T)$ is joint posterior distribution of model parameters. $L_{t|\alpha,\beta}(T|\alpha,\beta)$ is the likelihood for the collected lifetime data T. $\pi_{\alpha,\beta}(\alpha,\beta)$ is the prior distribution which is generated from historical data and subjective information.

3.2 Quantitative method for subjective information

Derivation of prior distribution is one of the key questions in a Bayesian analysis of lifetime data. In this paper, prior distribution is derived from information of relevant experts and historical data of the same product lines. Primary prior information for an aircraft engine disc is expert experience and knowledge. Subjective information and opinions from expert can be used to increase the sample size, which will be very helpful for improving the accuracy of reliability assessment in the small sample situation.

The prior distribution is a random variable with a certain distribution in Bayesian method while actual subjective data is highly personal representation. It is our priority to quantify subjective information. Probability encoding method is simple and manipulable [19]. It is a process to obtain statistical information in the form of probability density functions, discrete probability values, or occurrence rates. Spetzler et al. [20] provide an overview of probability encoding methods.

The inverse transformation of a parametric cumulative distribution function (CDF) can fully fit the process of changing the encoded survey data into parameter data of the relative prior distribution [19]. The inverse transformation of normal cumulative probability function can be described as follows:

$$P_\alpha = \int_{-\infty}^{V_\alpha} f\left(\alpha; \mu_\alpha, \sigma_\alpha\right) \mathrm{d}\alpha \tag{3}$$

where P_α and V_α are obtained from experts, and σ_α is known, and the only unknown hyper-parameter μ_α can be obtained by the inverse transformation of the CDF. There are some basic steps to implement the probability encoding method [20].

- Step 1: Specify uncertain quantities interested.
- Step 2: Choose probability encoding methods.
- Step 3: Interview subjects and collect subjective data sets.
- Step 4: Choose suitable Bayesian updating models.
- Step 5: Transfer the subjective data to model parameter data by inverse CDF analysis.
- Step 6: Update model parameters by integrating model parameter data using the Bayesian updating technique.

An illustration of the probability encoding method is presented in Section 4.

3.3 Multiple expert information checking

In the actual assessment process, more than one expertise knowledge can be collected [9]. Expert information is quantified as prior probability distribution through the probability encoding method. The entire distribution will be gotten by fusing all prior distributions. Since simple weighted method cannot meet requirements, a more suitable method needs to be found.

Assuming two prior probability distributions obtained from two experts is $f_a(x)$ and $f_b(x)$, we can define a common recognition extent (CRE) as the fusion of two expert information:

$$CR\left(f_a, f_b\right) = \frac{\int \left\{\min\left[f_a(x), f_b(x)\right]\right\} dx}{\int \left\{\max\left[f_a(x), f_b(x)\right]\right\} dx} \tag{4}$$

where the numerator represents the intersection of the area in which two probability distributions are surrounded by the X-axis. The denominator represents the union of the area in which two probability distributions are surrounded by the X-axis. The ratio is the degree of common recognition extent.

If fatigue life distribution function transformed from expert information E_i is assumed to be $f_i(\theta)$, the matrix CR can be obtained according to the degree of coincidence between the expert information.

$$CR = \begin{pmatrix} 1 & \cdots & CR_{1n} \\ \vdots & \ddots & \vdots \\ CR_{n1} & \cdots & 1 \end{pmatrix}$$
$$CR\left(E_i\right) = \sum_{\substack{i=1 \\ j \neq i}}^{n} CR_{ij}, \left(i = 1, 2, \cdots, n\right) \tag{5}$$
$$\omega_i = \frac{CR\left(E_i\right)}{\sum_{i=1}^{n} CR\left(E_i\right)}, \left(i = 1, 2, \cdots, n\right)$$

where $CR(E_i)$ means the sum of coincidence degree that expert I with respect to the other N-1 experts. After normalization, we can calculate the weight ω_i which shows the importance of each expertise. If $\omega_i = 0.10$, it means that expert I has a large conflict with other experts and should be removed. In particular case, a standard is required according to the requirements.

3.4 Multiple prior distribution fusion

In addition to the information gathered from the expert knowledge, there is also historical information from the similar disc. We can use data from the similar disc to obtain prior distribution through Bayesian model with non-informative priors. Different prior distributions have different goodness of fit to the experimental data [21]. How to effectively reflect the prior distribution that fits field test is considered in the paper.

The reference [22] provides a method to combine two groups of prior distributions. It is one of the most widely used approaches for combining probability distribution. It is given as:

$$f\left(\theta\right) = \lambda f_E\left(\theta\right) + \left(1 - \lambda\right) f_C\left(\theta\right) \tag{6}$$

where $f_E\left(\theta\right)$ is prior distribution obtained from expertise. $f_C\left(\theta\right)$ is prior distribution obtained from similar disc. λ and $1 - \lambda$ are the weights assigned for the corresponding two priors, and $f\left(\theta\right)$ represents the combined probability distribution.

The idea of this method is a measure of prior distribution fitting to field data. The prior distribution that fits the field data more closely will be assigned a higher weight in the linear pooling. The combined distribution inherits major belief of two distributions and maintains characteristics of relevant field data.

4 Case study

4.1 Problem definition

In order to demonstrate the proposed method, the fatigue lifetime of a type of aircraft engine disc is estimated based on multi-source information fusion. The fatigue lifetime model is assumed to follow the two-parameter Weibull distribution. The field data is listed in Table 1.

Table 1: Field fatigue lifetime data of aircraft engine disc

Field fatigue lifetime data (cycles)					
11,200	14,000	15,530	17,330	18,850	19,740

4.2 Quantifying the subjective information

The interested parameters are shape parameter β and scale parameter η, which are assumed to be statistically independent and to follow the normal distribution:

$$\begin{aligned}&\beta_E : N(\mu_\beta,\sigma_\beta),\ \ \sigma_\beta=0.1,\quad \mu_\beta : N(5.125,0.05)\\&\eta_E : N(\mu_\eta,\sigma_\eta),\ \ \sigma_\eta=20,\quad \mu_\eta : N(17009,17)\end{aligned} \tag{7}$$

As subjective information is collected and translated, these distributions can be updated. The uncertain quantities are the mean values of the parameters β and η. The standard deviations are fixed. Probability encoding method is used in this example, and two experts are interviewed regarding the values (V_β, V_η) of the scale parameter and shape parameter and corresponding probabilities (P_β, P_η). The survey results are listed in Table 2. The corresponding mean value of each survey point can be obtained by using the inverse CDF transformation method as depicted in Eq. (3). By taking the first data set of scale parameter [16750, 0.07] from Expert 1 as example, the computational process is shown as follows:

Table 2: Results of the survey and inverse CDF analysis

	V_β	P_β	μ_β	V_η	P_η	μ_η
Expert 1	5.24	0.05	5.40	16750	0.07	16779
	4.86	0.20	4.95	17250	0.18	17268
	5.33	0.60	5.30	17150	0.60	17145
	5.38	0.85	5.28	17180	0.85	17159
	5.51	0.95	5.35	17200	0.95	17167
	5.50	0.96	5.30	17220	0.96	17185
	5.52	0.98	5.32	17230	0.98	17189
Expert 2	4.50	0.12	4.67	16820	0.10	16846
	5.10	0.25	5.17	16780	0.15	16802
	4.65	0.65	4.69	16850	0.55	16853
	4.70	0.75	4.77	16790	0.80	16807
	4.75	0.90	4.93	16840	0.95	16807
	4.80	0.95	5.01	16860	0.98	16819
	4.90	0.99	5.13	16870	0.99	16823

$$\begin{aligned}&P_\eta=\int_{-\infty}^{V_\eta} f(\eta;\mu_\eta,20)\,d\eta\\&0.07=\int_{-\infty}^{16750}\frac{1}{\sqrt{2\pi\times 20^2}}\exp\left(-\frac{(\eta-\mu_\eta)^2}{2\times 20^2}\right)d\eta\end{aligned} \tag{8}$$

where μ_η can be acquired by solving the Eq. (8), here it equals to 16779. The other results are given in Table 2.

To expert I, if prior distribution of μ_η is assumed to follow the normal distribution $N(17150, 20)$. Treating encoded values in Table 2 as field data together with the assumed prior, posterior distribution for μ_η is updated to normal distribution $N(17130, 6.33)$ using Bayesian method.

Similarly, we can obtain all posterior distributions of the subjective information as shown in Eq. (9):

$$\begin{aligned}&\beta_{E_1} : N(\mu_\beta,\sigma_\beta),\ \sigma_\beta=0.1,\ \mu_\beta : N(5.27,0.03)\\&\eta_{E_1} : N(\mu_\eta,\sigma_\eta),\ \sigma_\eta=20,\ \mu_\eta : N(17130,6.33)\\&\beta_{E_2} : N(\mu_\beta,\sigma_\beta),\ \sigma_\beta=0.1,\ \mu_\beta : N(4.87,0.23)\\&\eta_{E_2} : N(\mu_\eta,\sigma_\eta),\ \sigma_\eta=20,\ \mu_\eta : N(16820,5.65)\end{aligned} \tag{9}$$

Therefore, the probability density function of prior distributions derived from expert's subjective knowledge is:

$$\begin{aligned}&f_{E_1}(t)=\frac{5.27}{17130}\left(\frac{t}{17130}\right)^{4.27}\exp\left\{-\left(\frac{t}{17130}\right)^{4.27}\right\}\\&f_{E_2}(t)=\frac{4.87}{16820}\left(\frac{t}{16820}\right)^{3.87}\exp\left\{-\left(\frac{t}{16820}\right)^{3.87}\right\}\end{aligned} \tag{10}$$

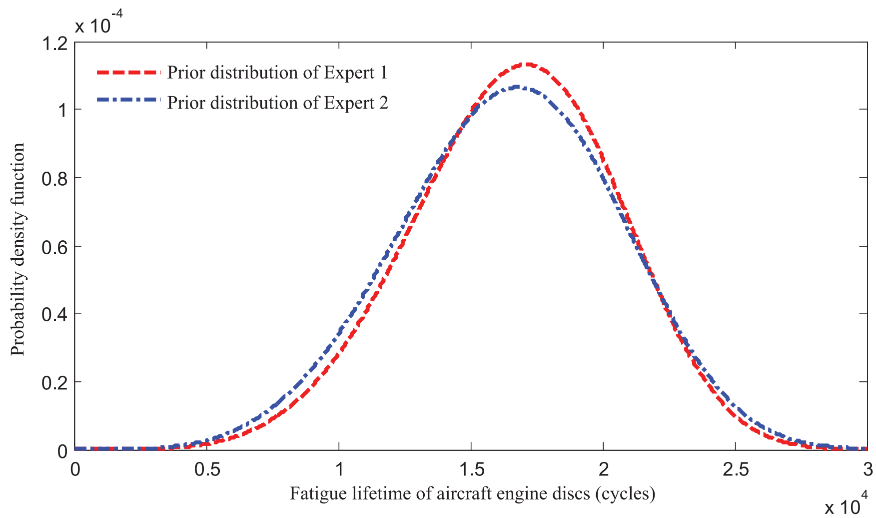

Fig. 1: Probability distribution function of priori distribution

Clearly, the probability distribution function of prior distribution is derived and shown in Fig. 1.

It should be noted that parameters of prior distribution deprived from different expert knowledge are different, which means different experts have different perspectives to fatigue lifetime distribution. Their common recognition extent (CRE) needs to be found, which can be calculated by Eq. (4) as follows:

$$CRE\left(f_{E_1}, f_{E_2}\right) = \frac{\int\left\{\min\left[f_{E_1}(t), f_{E_2}(t)\right]\right\}dt}{\int\left\{\max\left[f_{E_1}(t), f_{E_2}(t)\right]\right\}dt} = 0.9082 \quad (11)$$

By using computational formula in Eq. (5), the weights $\omega_1 = \omega_2 = 0.5$ can be calculated which shows the importance of each expertise. Therefore, the fusion of multi-source expert information priori probability density parameter β and η is

$$\beta_E : N(5.07, 0.10) \qquad \eta_E : N(16975, 20) \quad (12)$$

4.3 Multiple prior distribution fusion

Historical information from the similar disc is shown in Table 3.

The data of the similar disc is used to obtain the prior distribution through the non-informative prior Bayesian method.

$$\beta_C : N(4.8236, 0.3118) \qquad \eta_C : N(14368, 648) \quad (13)$$

Table 3: Historical fatigue lifetime data from similar types of aircraft engine discs

Historical fatigue lifetime data (cycles)					
6,820	7,740	8,850	10,500	12,850	13,130
17,420	18,950	19,600	24,510	28,500	32,000

To incorporate distribution shown in Eq. (12) and Eq. (13), the multiple prior distribution fusion method is used.

The Bayesian goodness-of-fit for these two types of prior distributions are obtained using the method mentioned in [18]. The information fusion factor in this stage is then obtained as

$$B_p{}^E = 0.9023, \quad B_p{}^C = 0.6784, \quad \lambda = \frac{B_p{}^E}{B_p{}^E + B_p{}^C} = 0.5708 \quad (14)$$

Finally, the prior distribution is obtained as follows:

$$\pi(\beta, \eta) = 0.5708\pi_E(\beta, \eta) + 0.4292\pi_C(\beta, \eta) \quad (15)$$

4.4 Reliability assessment using the Bayesian method

The field data is given in Table 1 and the prior distribution is shown in Eq. (15). Using Bayesian method, the estimated results based on the prior distribution and the field data are listed in Table 4.

Therefore, fatigue lifetime model of aircraft engine disc is:

Table 4: Estimation results of the Bayesian method

Parameter	Mean	SD	2.5%	97.5%
β	4.95	0.63	4.81	5.16
η	16825	126	15624	17265

$$F(t)=1-\exp\left\{-\left(\frac{t}{16825}\right)^{4.95}\right\} \tag{16}$$

The fatigue lifetime can be gotten using above model with certain reliability degree.

5 Conclusions

This paper proposed a fatigue lifetime assessment method based on the multi-source information fusion. Our study extends the application of Bayesian inference theory to fatigue lifetime assessment for aircraft engine discs. Firstly, to make the fatigue lifetime assessment more reliable, the probability encoding method is used to quantify the subjective information. A common recognition extent is used to get a multiple prior distribution. Secondly, by fusing the actual field data and other available information, the difficulty introduced by small sample size for lifetime analysis is mitigated to a certain extent. Finally, the Bayesian goodness-of-fit method is used to confirm the consistence degree of field data and expert knowledge, which effectively mitigate the subjectivity of fatigue lifetime estimation. However, there are some works need to be handled properly in our future work such as a prior distribution considering more than two expertise, the uncertainty quantifying to improve the degree of accuracy.

This research was supported by the National Natural Science Foundation of China under the contract number 11272082, and Specialized Research Fund for the Doctoral Program of Higher Education of China under the contract number 20120185110032.

Received: November 25, 2013. Accepted: December 2, 2013.

References

[1] M.S. Hamada, H.F. Martz, C.H. Reese, et al. Bayesian Reliability. Springer, New York, 2008.
[2] J.O. Berger. Statistical Decision Theory and Bayesian Analysis. Springer, New York, 1980.
[3] N.Z. Gebraeel, M.A. Lawley, R. Li, et al. Residual-life distributions from component degradation signals: a Bayesian approach. IIE Transactions, 2005, 37(6): 543–557.
[4] A.G. Wilson, L.A. McNamara, G.D. Wilson, et al. Information integration for complex systems. Reliability Engineering and System Safety, 2007, 92(1): 121–130.
[5] N. Chen, K.L. Tsui. Condition monitoring and remaining useful life prediction using degradation signals: revisited. IIE Transactions, 2013, 45(9): 939–952.
[6] W.W. Peng, H.Z. Huang, M. Xie, et al. A Bayesian approach for system reliability analysis with multilevel pass-fail, lifetime and degradation data sets. IEEE Transactions on Reliability, 2013, 62(3): 689–699.
[7] R.C. Shane, A.G. Wilson, J.Q. Guo, et al. A Bayesian model for integrating multiple sources of lifetime information in system-reliability assessments. Journal of quality technology, 2011, 43(2): 127–141.
[8] P. Wang, B.D. Youn, Z. Xi, et al. Bayesian reliability analysis with evolving, insufficient, and subjective data sets. Journal of Mechanical Design, 2009, 131(11): 111008–111011.
[9] H.Z. Huang, M.J. Zuo, Z.Q. Sun. Bayesian reliability analysis for fuzzy lifetime data. Fuzzy Sets and Systems, 2006, 157(12): 1674–1686.
[10] X.R. Liu. Research on method of aero-engine disc reliability estimation. Master Dissertation, Nanjing University of Aeronautics and Astronautics, 2011.
[11] H. Liu. Research on methods of reliability assessment with small sample test on the basis of Bayes theory. Master Dissertation, National University of Defense Technology, 2006.
[12] S.P. Zhu, H.Z. Huang, Z.L. Wang. Fatigue life estimation considering damaging and strengthening of low amplitude loads under different load sequences using fuzzy sets approach. International Journal of Damage Mechanics, 2011, 20(6): 876–899.
[13] S.P. Zhu, H.Z. Huang, V. Ontiveros, et al. Probabilistic low cycle fatigue life prediction using an energy-based damage parameter and accounting for model uncertainty. International Journal of Damage Mechanics, 2012, 21(8): 1128–1153.
[14] S.P. Zhu, H.Z. Huang, L.P. He, et al. A generalized energy-based fatigue-creep damage parameter for life prediction of turbine disk alloys. Engineering Fracture Mechanics, 2012, 90: 89–100.
[15] X.P. Zhang. Researches on the small sample statistical inference and fusion theory and its application to the assessment of weapon system. Ph.D. Dissertation, National University of Defense Technology, 2003.
[16] J. Feng. Research on methods and applications of reliability information fusion of complex system with small sample test. Ph.D. Dissertation, National University of Defense Technology, 2004.
[17] D.N.P. Murthy, M. Xie, R. Jiang. Weibull Models. John Wiley & Sons, 2004.
[18] A. O'Hagan, C.E. Buck, A. Daneshkhah, et al. Uncertain Judgements: Eliciting Experts' Probabilities. John Wiley & Sons, New York. 2006.
[19] H.Q. Li, R. Yuan, W.W. Peng, et al. Bayesian inference of Weibull distribution based on probability encoding method. Quality, Reliability, Risk, Maintenance, and Safety Engineering Conference, 2011: 365–369.

[20] C.S. Spetzler, S.V. Holstein, S. Carl-Axel. Probability encoding in decision analysis. Management Science, 1975, 22(3): 340–358.
[21] V.E. Johnson. A Bayesian χ^2 test for goodness-of-fit. The Annals of Statistics, 2004, 23(6): 2361–2384.
[22] W.W. Peng, H.Z. Huang, Y.F. Li, et al. Life cycle reliability assessment of new products – A Bayesian model updating approach. Reliability Engineering and System Safety, 2013, 112: 109–119.